2.12 A SEARCH FOR VARIATIONS IN THE INTENSITY OF THE OPTICAL PULSES FROM NP 0532

D. HEGYI*
NASA Goddard Institute for Space Studies, New York, N.Y., U.S.A.

R. NOVICK
Columbia Astrophysics Laboratory,
Physics Department, Columbia University, New York, N.Y., U.S.A.

and

P. THADDEUS
Goddard Institute for Space Studies, New York, N.Y., U.S.A.

Abstract. A search for short-time variability of NP 0532 using the 82 and 107 in. telescopes at McDonald Observatory is described. Observations were made of the mean intensity, the mean square of the intensity and the mean autocorrelation function of the pulsar light. Limits are placed on the variability of the optical pulsar.

1. Introduction

A search for variations in the optical pulses from NP 0532 in the Crab Nebula – the only known optical pulsar – is prompted by the large and erratic variations observed in the intensity of its radio pulses, (Graham *et al.*, 1970; Heiles *et al.*, 1970; Staelin and Sutton, 1970) and in the several types of irregularities found in those of other pulsars. Since our understanding of the optical emission mechanism of NP 0532 is, if anything, even more speculative than that of its radio emission, the detection of such variations is potentially of considerable interest, and should be pursued not only on the time scale of milliseconds, but on much shorter and longer scales as well. With modern photoelectric and digital techniques it is possible to do this all the way from the scale of seconds or minutes associated with conventional photometry to nanoseconds.

Some limits have already been set on optical pulse variation, but the data are fragmentary, and occasionally contradictory. Various authors, ourselves included, have shown that no extreme fluctuations occur in the pulsar light on the scale of nanoseconds or microseconds (Duthie *et al.*, 1969; Anderson *et al.*, 1969; Ögelman and Sobieski, 1969; Hegyi *et al.*, 1969; Jelley and Willstrop, 1969); observations for the most part, however, were made with small telescopes, and as the following discussion will show, this severely limits the sensitivity that can be attained on a very short time

* National Research Council Postdoctoral Research Associate, currently at the Department of Physics and Astronomy, Boston University.

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scale. The possible existence on a scale of milliseconds or a fraction thereof of optical analogues to the 'marching subpulses' observed in several pulsars (Drake and Craft, 1968; Sutton *et al.*, 1970) or the 'giant' radio pulses of NP 0532 itself has received little discussion; presumably rather restrictive limits to such structure could be extracted from the large existing body of optical timing and polarization observations. Finally, on a scale of minutes rather extreme variations in the optical intensity of a given component of polarization have been reported (Freeman *et al.*, 1969; Cocke *et al.*, 1969), but there now seems to be widespread agreement among observers that this effect is spurious (cf. Cocke *et al.*, 1970).

Even with a large reflecting telescope only 10–50 photoelectrons are received over the 33 msec period of NP 0532, and it is not possible from the record of individual pulses to determine their shape to any great precision. The usual way to study the structure in greater detail is to coherently add together in a multichannel recording instrument the photoelectric counts received from many thousands of pulses; but this technique cannot distinguish between the case where all pulses are identical, and the more general situation where the intensity of individual pulses $I(t)$, is a *random variable* whose mean value $\langle I(\phi)\rangle$, as a function of phase ϕ, is the quantity observed.

To decide between these alternatives it is necessary to observe higher moments of the probability distributions which define a random process. Here we will describe observations of the mean intensity, the mean square of the intensity, and the mean autocorrelation function of the pulsar light which we have made with the 82 and 107 in. telescopes at McDonald Observatory. From intercomparison of these quantities it is found that little if any random variation in the pulse structure exists, or in other words that the individual optical pulses of NP 0532 appear to be identical, one to another. Limits are also imposed on gradual secular variation of the intensity of the optical pulsar, but these results are not as sensitive as those which can be obtained by standard photometry.

A few elementary quantitative considerations will make these points more precise, and will indicate the way in which the data have been analyzed. Let $W_1(I, t)$ and $W_2(I_1, t_1, I_2, t_2)$ be the first and second probability distributions which describe the postulated random intensity. These are defined in the usual way such that $W_1(I, t)\,\mathrm{d}I$ is the probability of finding the intensity between I and $I+\mathrm{d}I$ at time t, and $W_2(I_1, t_1, I_2, t_2)\,\mathrm{d}I_1\mathrm{d}I_2$ is the joint probability of finding a pair of values of I in the ranges I_1, $I_1+\mathrm{d}I_1$ and I_2, $I_2+\mathrm{d}I_2$ at the respective times t_1 and t_2. In the subsequent discussion it will be assumed that these two distributions are periodic with the pulsar period of 33 msec, so that the pulsar phase ϕ, rather than t, becomes the appropriate independent variable. The quantities which are yielded by the observations described below are the first and second moments of the first probability distribution, i.e., the mean intensity

$$\langle I(\phi)\rangle = \int_0^\infty I W_1(I, \phi)\,\mathrm{d}I, \tag{1}$$

and the mean intensity squared

$$\langle I^2(\phi)\rangle = \int_0^\infty I^2 W_1(I, \phi)\,\mathrm{d}I, \tag{2}$$

and in addition the mean autocorrelation function, which in terms of the second probability distribution we define to be

$$G(\tau) = \frac{1}{T}\int_0^T\int_0^\infty\int_0^\infty I_1(\phi-\tau)\,I_2(\phi)\,W_2(I_1, \phi-\tau, I_2, \phi)\,\mathrm{d}I_1\,\mathrm{d}I_2\,\mathrm{d}\phi, \tag{3}$$

where $T \approx 33$ msec is the pulsar period.

It now follows directly from consideration of the variance $\langle (I - \langle I\rangle)^2\rangle$ that

$$\langle I^2(\phi)\rangle \geqslant \langle I(\phi)\rangle^2, \tag{4}$$

the equality holding when all pulses are identical. This is the fundamental condition on which our most sensitive search for pulse variations will be based, the results applying to time scales from roughly microseconds to tens of minutes. Information on the pulse structure on the scale of nanoseconds will be provided by measurement of $G(\tau)$, which in the limit of identical pulses and $\tau \to 0$ reduces to

$$G_0 = \frac{1}{T}\int_0^T \langle I(\phi)\rangle^2\,\mathrm{d}\phi. \tag{5}$$

2. Apparatus

A. FOR COMPARISON OF $\langle I^2\rangle$ WITH $\langle I\rangle^2$

A schematic illustration of the apparatus in the 'intensity squared' mode is given in Figure 1. Light from an aperture in the focal plane of the telescope was split into two beams of approximately equal intensity by front surface reflection from an aluminized right-angle prism. The beams were then detected by two fast Amperex 56DVP photomultipliers located 50 in. apart in order to reduce fast (nanosecond) cosmic ray coincidences to a negligible level. Each tube was followed by conventional stages of amplification and discrimination, but the discriminators were turned off for 10 μsec, following each photoelectron pulse, by a 'deadtime' circuit to eliminate the effects of phototube afterpulsing. The total number of counts received in each channel during an observing run was recorded by the scalers which, in Figure 1, are shown adjacent to the discriminators.

Following discrimination, pulses from phototube 1 triggered a single pulse generator ('one-shot') which in turn opened a coincidence gate for a predetermined length of time $\Delta\tau$. Pulses from tube 2 which passed through the gate were then stored in a 511 channel multichannel scaler which was synchronized to the pulsar frequency. All

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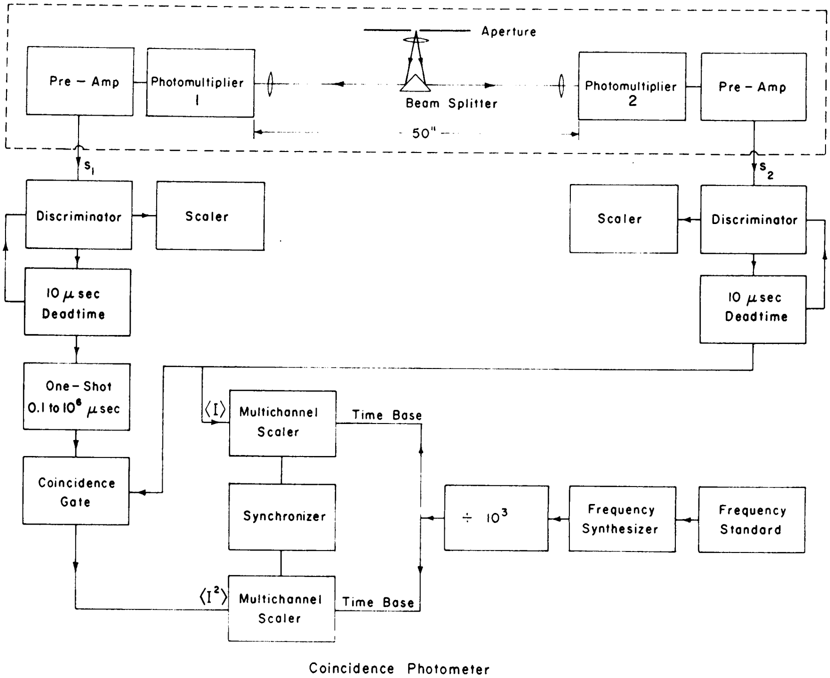

Fig. 1. Schematic diagram of the apparatus.

pulses from tube 2, whether they passed through the gate or not, were stored in a second multichannel scaler also driven synchronously with the pulsar.

The time base for both multichannel scalers was provided by the apparatus shown in the lower right hand corner of Figure 1. A frequency synthesizer stabilized by the rubidium frequency standard at McDonald Observatory was manually adjusted at about 5 minute intervals to precisely 511 000 times the calculated* pulsar frequency. Division by 1000 then provided a sequence of timing pulses which advanced the multichannel scalers from one channel to the next. It is believed that phase slippage during the longest observations ($\sim$ 1 h) amounted to only a fraction of a channel.

To understand the essential operation of the device it is best to consider only low light intensities, such that the probabilities $s\,\Delta\tau$ and $s\,\Delta\tau_D$ of counts arriving within the deadtime $\Delta\tau_D = 10$ μsec, and the coincidence gate resolution time $\Delta\tau$, are infinitesimal. In fact, these probabilities while always small, were not always negligibly so, and it was necessary to make small corrections to the recorded counts in order to compare $\langle I\rangle^2$ with $\langle I^2\rangle$. For the sake of brevity the details of these corrections will be omitted from the following discussion.

Suppose now that $s_1(t)\,\mathrm{d}t = \alpha_1 I(t)\,\mathrm{d}t$ and $s_2(t)\,\mathrm{d}t = \alpha_2 I(t)\,\mathrm{d}t$ are the probabilities, in channels 1 and 2 respectively, of the arrival of a photoelectron count in the interval

* Based on measurements of the NP 0532 frequency and its time derivatives made by Boynton *et al.* (1969).

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between t and $t+\mathrm{d}t$. These probabilities, although comparable, are not precisely equal because of the differences in the phototube efficiencies and inaccuracy in splitting the optical beam; we will only assume that their ratio, and hence the ratio of α_1 to α_2, is constant over an observing run. The probability $c(t)\,\mathrm{d}t$ of a pulse passing through the coincidence gate during the interval t, $t+\mathrm{d}t$ is then $s_2(t)\,\mathrm{d}t$ times the probability P_0 that the gate is open, which in turn is

$$P_0(t) = \int_0^{\Delta\tau} s_1(t-\tau)\,\mathrm{d}\tau\,. \tag{6}$$

It is now desirable to either assume that $I(t)$, and hence $s_1(t)$, possess no structure on a scale finer than $\Delta\tau$, or better still, to redefine $I(t)$ to be the instantaneous intensity averaged over a time of order $\Delta\tau$, so that Equation (6) simplifies to

$$P_0(t) = s_1(t)\,\Delta\tau\,, \tag{7}$$

and consequently

$$c(t) = s_1(t)\,s_2(t)\,\Delta\tau = \alpha_1\alpha_2\,\Delta\tau I^2(t)\,. \tag{8}$$

The number of counts C_i stored in the i'th channel of the $\langle I^2\rangle$ multichannel scaler is then simply $c(t)$ integrated over the channel width $\Delta\tau_c = T/N_c$, (where $N_c = 511$ is the total number of channels), and summed over all the NP 0532 pulses in the observing run of duration T_r. The result is

$$C_i = \frac{\alpha_2^2 N_1\,\Delta\tau T_r}{N_2 N_c}\,\langle I^2(\phi_i)\rangle\,, \tag{9}$$

where N_1 and N_2 are the total counts recorded by the two scalers, and it has been assumed that $\alpha_1/\alpha_2 = N_1/N_2$, a relation which neglects phototube dark counts, which were very low in the phototubes used. Similarly, the number of counts stored in the i'th channel of the $\langle I\rangle$ multichannel scaler is found to be

$$S_i = \frac{\alpha_2 T_r}{N_c}\,\langle I(\phi_i)\rangle\,. \tag{10}$$

Thus from Equations (9) and (10) we see that the fundamental condition $\langle I^2(\phi)\rangle \geqslant \langle I(\phi)\rangle^2$ is equivalent to

$$C_i \geqslant \frac{N_1\,\Delta\tau N_c}{N_2 T_r}\,S_i^2\,. \tag{11}$$

Since all the quantities occurring in Equation (11) are well defined, being either parameters of the apparatus ($\Delta\tau$, N_c), or numbers furnished by the counters (C_i, S_i, N_1, N_2), analysis of the data is straightforward.

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B. FOR COMPARISON OF $G(\tau)$ WITH G_0

The configuration of the apparatus in the mode used to determine the 'mean auto-correlation function' $G(\tau)$ has been previously described (Hegyi *et al.*, 1969). In this mode the coincidence gate shown in Figure 1 is replaced by a time-to-amplitude converter (TAC), which is started by photoelectric counts from phototube 1 and stopped by counts from phototube 2 which have been delayed ~ 30 nsec by a length of line. The TAC provides an output pulse whose height is proportional to the time elapsed between the arrival of the start and the arrival of the stop counts, but only if it is stopped before reaching the end of its range; if it runs off range no output pulse is produced. Nominal ranges of 100 nsec, 300 nsec, 1 μsec,..., 30 μsec are provided.

It is now easy to see that pulse height analysis of the TAC output yields essentially $G(\tau)$. The probability in the time interval $t, t+\mathrm{d}t$ of receiving an output pulse from the TAC corresponding to a time delay in the range $\tau, \tau+\Delta\tau$ is $s_1(t-\tau)s_2(t)\,\mathrm{d}t\,\mathrm{d}\tau = \alpha_1\alpha_2 I(t-\tau)I(t)\,\mathrm{d}t\,\mathrm{d}\tau$. Hence the total number of counts C_i received in the delay range $\tau, \tau+\Delta\tau$ over a run of duration T_r, and stored in the i'th channel of the pulse height analyzer, is

$$C_i = \alpha_1\alpha_2 T_r\,\Delta\tau \left\langle \int_0^T I(\phi-\tau)\,I(\phi)\,\mathrm{d}\phi \right\rangle \cdot \tag{12}$$

Since the average over many periods of NP 0532, indicated by the $\langle\ \rangle$ in Equation (12), must, by the hypothesis that the distribution functions are periodic with the pulsar, yield the same result as the ensemble average of Equation (3). Equation (12) becomes

$$C_i = \alpha_1\alpha_2 T_r\,\Delta\tau G(\tau) \tag{13}$$

If again N_1 and N_2 are the total counts recorded during a run by the two scalers, and the S_i are the total number of counts recorded in the i'th channel of the $\langle I\rangle$ multichannel recorder shown in Figure 1 (there is of course no $\langle I^2\rangle$ analyzer in this mode of operation), then by Equation (10) the limit $G(\tau)=G_0$ [Equation 5)], is equivalent to

$$C_i = \frac{N_1 N_2\,\Delta\tau\xi}{T_r}, \tag{14}$$

where the enhancement factor ξ, which is unity for steady light, is defined by

$$\xi = N_c \sum_i S_i^2 \Big/ \Big(\sum_i S_i\Big)^2 . \tag{15}$$

All the terms appearing in Equation (14) are well defined experimentally, so that analysis of the data simply consists in comparing the left with the right hand side of the equation. ξ should remain constant under ideal observing conditions for a given aperture; it was found to change slowly by up to a factor of two in the course of a night during our observing period because of a varying haze layer which, by scattering

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moonlight, enhanced the background light relative to that of the pulsar. Unfortunately, for these observations, only a single multichannel instrument was available (Northern Scientific NS-550 'Digital Memory Oscilloscope'), and this did double service as both a multichannel scaler and pulse height analyzer. It was therefore not possible to determine C_i and ξ simultaneously, as would have been most desirable; instead the S_i and hence ξ were determined by a brief (5 or 10 min) intensity run just before and, when possible, immediately after the generally longer observation of the C_i.

3. Results

A. $\langle I^2 \rangle$ VS. $\langle I \rangle^2$

Observations of the mean intensity squared of NP 0532 were made at the Cassegrain focus of the 82 inch telescope on the nights of 13, 14, and 15 February 1970 UT. The moon passed near the Crab Nebula toward the end of this period, and the observations were occasionally interrupted by clouds, but useful data were obtained on all three nights.

A typical observation lasted from 5 min to 1 h. In order to exclude scattered moonlight and background nebular light, observations were made with the smallest aperture that the seeing allowed; on a few exceptional occasions a 3.5″ aperture was tried with some success, but generally a 4″ one was the smallest used under good seeing conditions. Since slow modulation of the pulsar light, owing for example to guiding irregularities or clouds, can be as effective as short term fluctuations in enhancing $\langle I^2 \rangle$ with respect to $\langle I \rangle^2$, the mean counting rate (time average about 1 sec) of one of the phototubes was continuously monitored by a chart recorder. In this way it was established for the best runs that long term atmospheric or guiding effects probably caused an enhancement of $\langle I^2 \rangle$ of less than 1%, and can therefore be safely neglected in the data analysis. Observation of field stars confirmed this conclusion, and showed that short term seeing fluctuations were negligible as well.

The results of a good (but short) observation, a 5 min run starting at 3:39, 13 February 1970 UT, for which the coincidence gate resolution $\Delta\tau$ was 30 μsec, are given in Figure 2; in the units of Equation (11) the lightly plotted line is $\langle I^2 \rangle$, and the heavily plotted one is $\langle I \rangle^2$. Data from only about 30% of the instrumental channels – corresponding to the 30% of the NP 0532 period centered on the main pulse – are shown in the figure; the omitted data show comparable noise and agreement of the two curves for the interpulse.

Neither in Figure 2 nor in any other observation at whatever $\Delta\tau$ was any overall enhancement of $\langle I^2 \rangle$ with respect to $\langle I \rangle^2$ discernible. But the question remains as to whether the actual complexion of the noise in $\langle I^2 \rangle$ can be attributed to purely statistical fluctuations in the number of received photoelectrons, or whether structure is present which might result from phase-dependent rapid fluctuation in the pulsar light. For many runs there was a temptation for the eye to pick out structure on the leading edge of the main pulse, but it has been concluded that the noise is always of a purely statistical origin, since it was found that the records in question could

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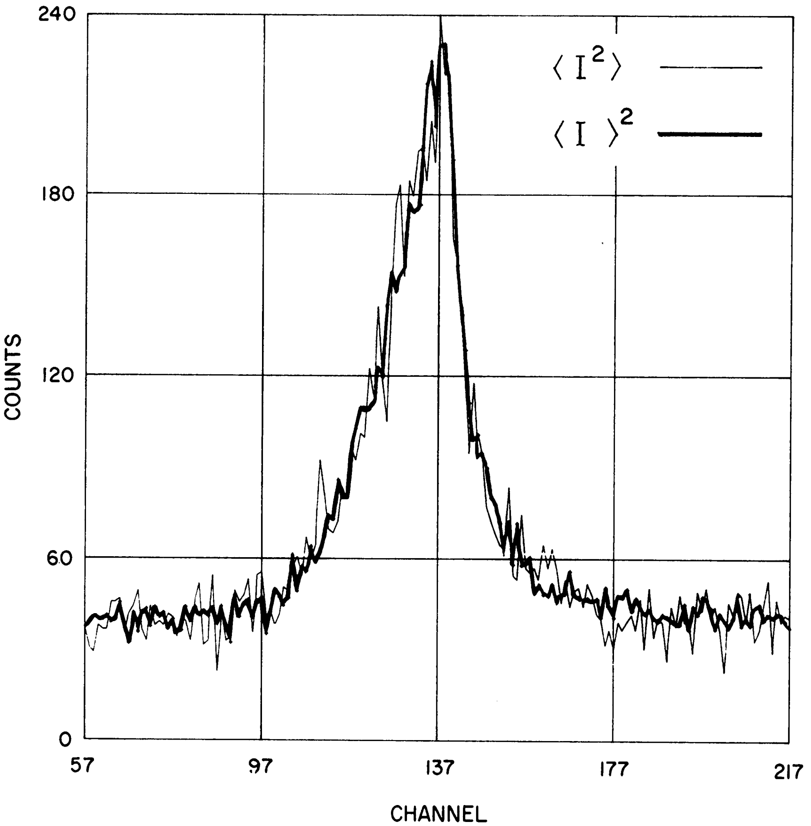

Fig. 2. Comparison of $\langle I^2 \rangle$ with $\langle I \rangle^2$, – a single 5 min observation. Only the main NP 0532 pulse is shown.

never be distinguished with any confidence from fictitious ones whose noise was generated by Monte Carlo technique on a digital computer. Also, none of the supposed structure remained when the best observations taken at a given $\Delta\tau$ were added together coherently in pulsar phase*.

This is shown in Figures 3 and 4, which are the result of synthesizing the best observations taken at $\Delta\tau = 30$ and 3 μsec, respectively, and where the excellent agreement of $\langle I^2 \rangle$ with $\langle I \rangle^2$ is manifest. When the counts due to background are subtracted off, the limits which these observations provide on the intrinsic enhancement factor $\xi = \langle I^2 \rangle / \langle I \rangle^2$ of the pulsar light are then summarized in Table I. These limits pertain to the 21 channels centered on the peak of the main NP 0532 pulse; the first entry ($\Delta\phi = 65$ μsec) for each $\Delta\tau$ is the maximum value of ξ permitted for any channel within this range; the second results when all channels are summed together and

* Absolute phase was not maintained from one run to the next by the apparatus shown in Figure 1, but it proved possible to reconstruct it to an accuracy of about one-third of a channel – corresponding to 21 microseconds – by cross correlation of different runs over the main pulse.

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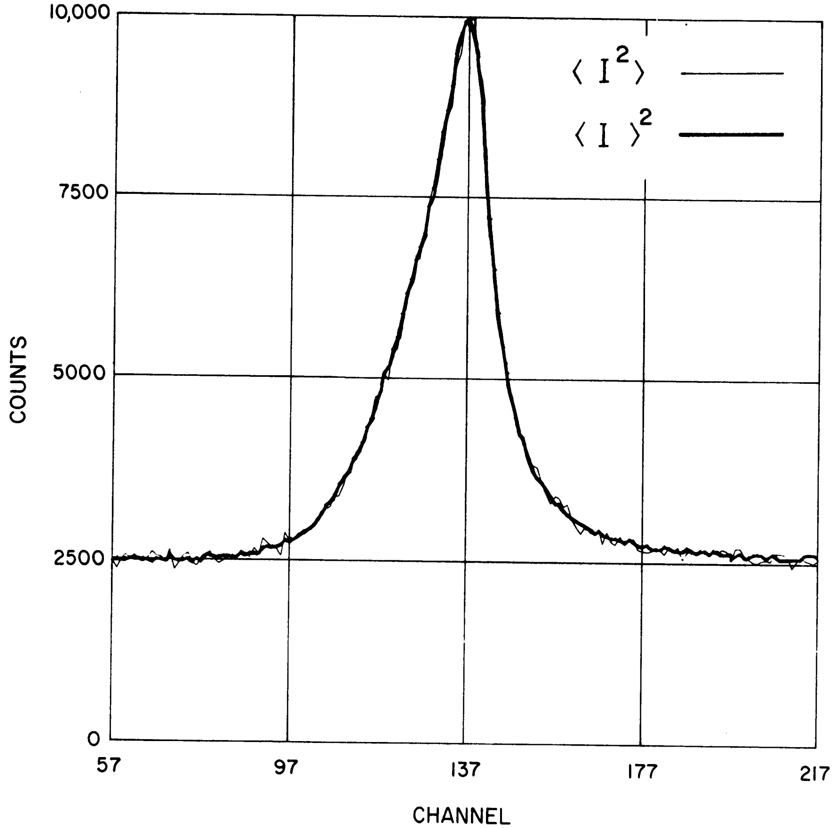

Fig. 3. Synthesis of $\Delta\tau = 30$ μsec observations. Here, as in Figures 2 and 4, the two curves have not been fit but result from analysis of wholly independent data.

TABLE I

Upper limits to the enhancement factor

Number of Observations	Total obs. time (min)	$\Delta\tau$ (μsec)	$\Delta\phi$ (μsec)	ξ
6	192	30	65	< 1.04
			1356	< 1.02
2	61	3	65	< 1.14
			1356	< 1.04

treated as a single channel of width $\Delta\phi = 1356$ μsec. Naturally, somewhat less constrictive limits than those listed in the table apply to the wings of the main pulse or the interpulse of NP 0532.

It is now of interest to briefly consider the restrictions on specific kinds of pulse variation set by the data in Table I.

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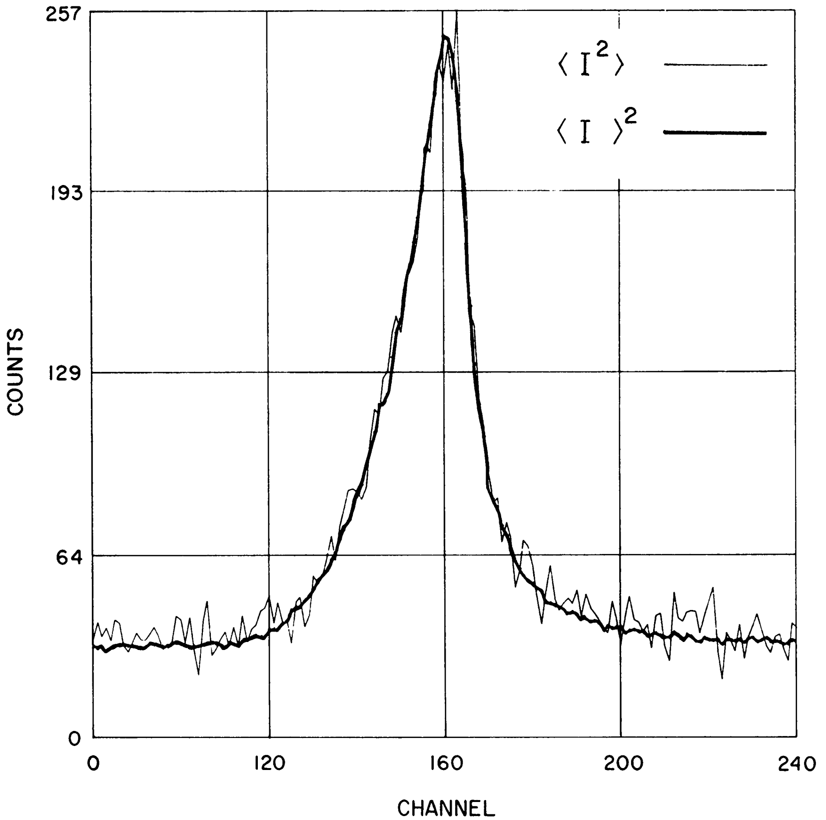

Fig. 4. Synthesis of $\Delta\tau = 3$ μsec observations.

1. *Slow Secular Variation*

Suppose that the intensity of the pulsar varies linearly with time during the course of an observation. If the total change in intensity is a fraction f of the mean, then one readily calculates that $\xi = (1 + f^2/12)$. The most restrictive limit in Table I, $\xi < 1.02$, then implies that $f < 0.33$, which is equivalent to a change in intensity of 0.44 mag. It is evident from this that a comparison of $\langle I^2 \rangle$ with $\langle I \rangle^2$ is not as sensitive as conventional photometry in detecting fluctuations of this kind.

2. *Sudden Enhancement or 'Giant' Pulses*

Suppose that one or more pulses, comprising a small fraction g of the total number, are more intense than the rest by a factor F. As long as F is less than about 20 (see below), the enhancement factor is

$$\xi = 1 + Fg^2 . \tag{16}$$

Thus the limit $\xi = 1.02$ implies that for g equal to 10^{-1}, 10^{-2}, 10^{-3}, and 10^{-4}, F is,

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respectively, less than 0.5, 1.4, 5, and 14. These figures clearly show that the present technique has a real edge over conventional photometry in the detection of fluctuations of this kind, since a 14-fold increase in the intensity of a single pulse would, for example, produce only a 0.05 magnitude increase in the mean intensity of NP 0532 for photometric observation over an interval as short as 10 sec.

For 'giant' pulses, with F much greater than 20, the instrument saturates, either because of the 10 μsec deadtime $\Delta\tau_D$, or the coincidence gate resolution time $\Delta\tau$, whichever is greater: a maximum of $\Delta\tau_c/\Delta\tau_D \sim 6$ counts per channel in the $\langle I^2 \rangle$ multichannel counter will result from a very intense pulse, and the limits which can be set on infrequent giant pulses are therefore not as restrictive as Equation (16) would indicate.

Referring to the counts shown in Figure 4, we now note that two such giant pulses occurring during the course of the $\Delta\tau = 3$ μsec observations would have produced an enhancement factor of $(250+12)/250 = 1.05$, greater, according to Table I, than the upper limit of 1.04 obtained from the twenty-one channels centered on the pulsar main peak, and therefore marginally detectable. It is thus possible to conclude that giant optical pulses (if of the same width as the usual ones) occurred at a rate of less than about 2/h during the period of our observations.

3. *Marching Subpulses*

The limits just set on sudden enhancement of entire pulses apply to marching subpulses as well, if g now denotes the ratio of the subpulse length to the subpulse repetition period (i.e. g is the fraction of the time that the subpulse is on).

B. $G(\tau)$ VS. G_0

Observations of $G(\tau)$ were made at the Cassegrain focus of the 107 inch telescope on the nights of 12, 13, and 14 December 1969 UT, for the most part under photometric sky conditions with seeing in the range 1–2″. Figure 5a shows the result of a 55 min observation begun at 8:21 UT on 14 December, with a 4″ aperture and the time-to-amplitude converter set on a full range of ~ 100 nsec. Counts have been summed over successive blocs of 20 channels of the pulse height analyzer to reduce statistical fluctuations. The enhancement factor ξ used to calculate G_0 via Equations (14) and (15) was derived from a 5 min determination of $\langle I \rangle$ taken immediately before the run; because of the small aperture, the good seeing, and the absence of moonlight, the background counts were low, and the resulting enhancement factor $\xi = 2.10$ was about as large as was ever obtained. Consequently, most of the counts contributing to $G(\tau)$ in Figure 5a come from the pulsar itself and not the background.

Figures 5b and 5c are similarly the result of observing runs taken again with a 4″ aperture and with the TAC on nominal full ranges of, respectively, 3 μsec and 30 μsec. The 3 μsec full-range observations began at 7:30 UT, 14 December 1969 and lasted 30 min; the counts as before have been summed over many channels – 10 in this case – to reduce statistical fluctuations. The 30 μsec full-range observations began at 9:51 UT, 14 December and lasted 50 min; there has been no attempt to reduce

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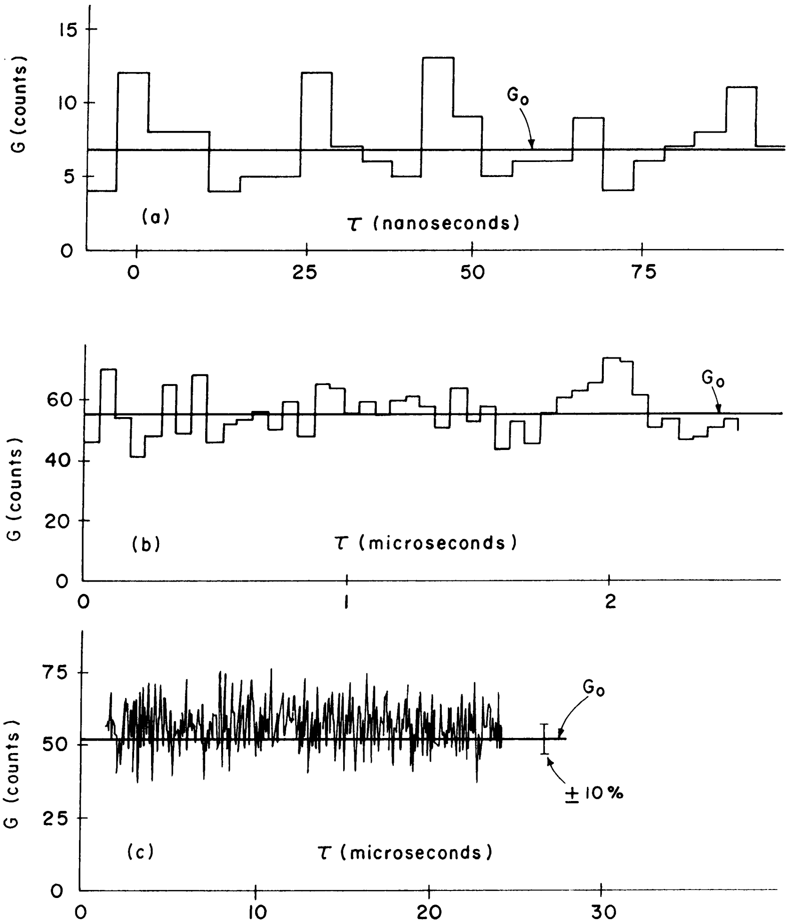

Fig. 5. Comparison of $G(\tau)$ with G_0, for three ranges of the time-to-amplitude converter.

statistical fluctuations at the expense of resolution in delay time τ: counts received in each channel of the pulse height analyzer are shown.

The most important conclusion which we draw from Figure 5 and similar data is that $G(\tau)$ is essentially flat over the interval of delay times τ from about 1 nsec to 30 μsec, the observed noise being purely statistical in character; therefore little if any variation in the pulsar light occurs on a time scale too short to have been detected by the observations described in Section 3A. Fluctuations on a much longer time scale will of course enhance $G(\tau)$ over G_0 in the way that they enhance $\langle I^2 \rangle$ with respect to $\langle I \rangle^2$, but the data at hand adds nothing in this respect to the limits set in Section 3A – entirely owing to the uncertainty of roughly 10% introduced into the determination of ξ by the sequential observation of the C_i and S_i. To within this uncertainty, as Figure 5 shows, $G(\tau)$ and G_0 are in good agreement.

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Discussion

J. Nelson: Concerning the shape of the light curve: we find the phase angle between the main pulse and the interpulse is constant to within our errors of $\pm 0.03°$ (or ± 2.5 μsec). Also the shape of the light curve (averaged over 15 minutes) does not change during our observation period.

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