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Professor J. E. A. Steggall, M.A., President, in the Chair.

On Lines of Magnetic Force.

By Professor J. E. A. Steggall.

In my laboratory we make great use of the method of tracing lines of force described in Glazebrook and Shaw's *Practical Physics.* The field is modified in various ways; for instance, the fixed magnets are sometimes placed so as to somewhat resemble in their disposition the field magnets of a dynamo: sometimes a circular piece of soft iron is placed in the field, and the effect of its induced magnetization examined. The students like these exercises, and the results (of which some are exhibited) are very beautiful. We use steel bars about 8 c.m. long for fixed magnets, and small rather heavy lozenge-shaped needles, about one to two c.m. long, as what I shall call pointers. When we wish to eliminate the directive effect of the earth's magnetism we use two boards, of which the upper can rotate easily on the lower; on the upper the paper, magnets, and pointer are placed, and it is rotated until the pointer lies due North and South: this is determined by the coincidence of the direction of a light aluminium wire, attached to the pointer at right angles to its magnetic axis, with a thread or line from East to West. In this case we know that the force of all the influencing magnets, as well as of the earth, is directed North and South. The process is somewhat laborious, but when applied to a single magnet the resulting lines of force agree with remarkable exactness with those given by theory for an indefinitely thin magnet.

In making these experiments it occurred to me to examine whether any graphic methods similar to those given by Lloyd and others could be applied to determine the theoretical form of the lines of force due to the joint influence of a bar magnet and the earth. A result of a very simple kind was arrived at in one case, namely, when the magnet lies in the magnetic meridian; and having drawn the curves of a bar magnet alone to scale, it was possible to deduce these for the same magnet lying as described, with the result that these lines also agreed most minutely with those determined by experiment. The method used can only be applied in the case cited.

Let A, B, denote the poles of a magnet in the earth's meridian, P any point, co-ordinates x, y, m the intensity of a pole, H the earth's horizontal force, $AP=r_1$, $BP=r_2$, angle $PAB=\theta$, angle $PBA=\pi-\theta_2$, then if s be the arc of a line of force, resolving normally,

$$\frac{m}{r_1^2} \cdot \frac{r_1 d\theta_1}{ds} - \frac{m}{r_2^2} \cdot \frac{r d\theta_2}{ds} + \mathrm{H} \cdot \frac{dy}{ds} = 0$$

and $$r_1 \sin\theta_1 = r_2 \sin\theta_2 = y,$$

therefore $$m \sin\theta_1 \frac{d\theta_1}{ds} - m \sin\theta_2 \frac{d\theta_2}{ds} - \mathrm{H} y \frac{dy}{ds} = 0$$

or $$m(\cos\theta_1 - \cos\theta_2) + \tfrac{1}{2}\mathrm{H}y^2 = \text{constant}$$

is the equation to the lines of force.

If the magnet is not along the meridian the equation cannot be integrated as it stands.

Now if we trace the systems of curves

$$m(\cos\theta_1 - \cos\theta_2) = \text{constant},$$
$$\tfrac{1}{2}\mathrm{H}y^2 = \text{constant},$$

of which the former represents the lines of force due to a magnet alone, the latter a series of parallel straight lines, and if the constants form arithmetical progressions of equal differences, the consecutive intersections give the lines of force due to the disposition of the magnet considered.

The numerical ratio of m and H can be at once obtained by measuring the distance of the axial points of equilibrium from the ends of the magnet: this gives, a, b being these distances

$$m\left(\frac{1}{a^2} - \frac{1}{b^2}\right) = \mathrm{H}.$$

In a similar way to the above equi-potential lines may be drawn by having a short aluminium lozenge fixed at right angles to the pointer.