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AN ANALYSIS OF SCALING METHODS FOR STRUCTURAL COMPONENTS IN THE CONTEXT OF SIZE EFFECTS AND NONLINEAR PHENOMENA

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Abstract
Similitude theory helps engineers to investigate system properties and behaviour with scaling methods. The application of such methods reduces the time for product development and production of prototypes. With increasing component size, the impact of size effects and nonlinear phenomena becomes more important in reduced scale model testing. The aim of this paper is to provide an overview of the scaling methods and their applicability with regard to size effects and nonlinear phenomena as well as a procedure to support the selection of a suitable method for the scaling task of structures.

Keywords: similitude methods, size effects, design methods, product development, modelling

1. Introduction

With scaling methods and similarity laws, complex technical problems can be solved by model tests. The application of such methods in product development leads to a reduction of development time and costs for the production of prototypes. Methods for the application of similarity laws in general physics and fluid mechanics are common practice. Similarity key ratios, for example, are used to methodically support solution finding and evaluation. In many areas of engineering it is necessary to have a suitable mechanism for the transmission of dimensional variables such as force, time, mass, length, etc. From a design point of view, there are approaches that describe the scaling of components with basic geometries, but are not suitable for the transfer of nonlinear phenomena and size effects during scaling. Concerning such constructions, where material properties and behaviour cannot be linearly scaled to different sizes, the developed methods and approaches from similarity mechanics contain deficits and do not allow reliable predictions about designs to be scaled. Especially with regard to the analysis of effects, which may be present in large and small designs of the structural components, but which cannot be captured or do not occur in the scaled design. Examples of such effects on a large scale are the effects of static and dynamic loads, realizable surface roughness, material inhomogeneity, plastic deformations, damping parameters and much more. Not only increasing scale is a challenge for the design process, also the environmental conditions, for example for locating huge wind energy parks (sea, low temperature, ice, storms, corrosion etc.) are becoming more challenging.
For this reason, the methodology for the scalability of nonlinear, multiscale and discontinuous effects for large components has to be investigated and suitable methods for the transfer of such effects

has to be developed. For this purpose, an analysis of effects occurring in large components as well as their handling in the design and mechanical layout of the components to be scaled must be taken into account. Consequently, this article discusses the known theories on scaling of components and shows the limits of applicability. In this context, the term of large products refers to XXL products, such as components from wind turbines, according to an initial definition by Behrens and Nyhuis (2014).

2. State of the art

2.1. Scaling methods in literature

There are many methods in the literature that enable physical and dimensional scaling. These methods are the research topic of many authors who have laid the foundations for the scaling process. In physics and engineering, the Buckingham pi theorem is advantageously used to obtain and control new functional relationships. This process of modelling supported by the pi theorem is called dimensional analysis. In classical engineering, the efficiency of dimensional analysis is impressively demonstrated in a wide variety of applications in difficult questions in the fields of fluid mechanics, heat transfer and mass transport. This theory states that a dimensionless potency product relationship can be found for any fully dimensional homogeneous relationship (Bridgman, 1932). The result of this so-called "dimensional analysis" is a group of dimensionless products that are valid for the respective problem. The aim is to replace singular solutions with dimensionless quantities. A relevance list is to be created for the application of dimensional analysis, where all important variables are used, which describe the object/system to be scaled. Dimensionless products are to be derived from a dimension matrix. Two phenomena are similar when the dimensionless products of a complete set have the same value (Stichlmair, 1990). If all certain dimensionless potency products are constant over the entire area of application, the problem under consideration can be completely transferred from a model to a real size. The disadvantage of this method is that it requires a deep understanding of the whole physical problem. This leads to the wrong determination of the dimensionless potency products. The first application of dimensional analysis for structural components is applied by Goodier and Thomson (1944). These authors present a systematic procedure for determining similarity conditions by means of dimensional analysis. Another common used method is the similarity theory using differential equations. Kline (1986) shows a way to create similarity conditions with the application of dimensional analysis and also the direct use of governing differential equations. This method bases on the definition of scale factors which are successively inserted into the governing differential equations to derive similarity conditions.

These existing traditional similarity methods shows serious errors when applying dimensional analysis and transferring properties from model to large-scale execution. These errors cannot be quantified and are caused by distortions categorized into geometric, functional, parametric, and experimental distortions by the authors Cho, Wood and Dutson. Distortions reduce the possibility of making correct statements about a product with the help of an associated model. Wood and Dutson have presented a different approach based on this problem and to overcome the problems of dimensional analysis, the Empirical Similitude Method (ESM). With this method a separate scaling of material properties and geometric properties should be realized (Cho et al., 2005).The state of the product (full-size structure) is predicted by experimental investigations on a model and two other structural parts, model specimen and product specimen. The model is the scaled version of the product, while the specimens are geometrically simplified and distorted versions of the model and the product. The behaviour of the systems is measured at different geometrical points or under different load cases. The measurement data are mapped as vectors. By testing model and model specimen, the state changes due to the change of the geometric form or shape can be obtained. For this, however, material properties, size ratios and load cases must be the same for both. Model specimen and product specimen can be tested to obtain the state change due to changes in material properties, size and load cases. Size change at this point means parametric scaling and not shape change (Dutson, 2002). If changes in geometry affect material behaviour, the change in material behaviour must be identical in both systems (model and model specimen, product and product specimen). Material behaviour and geometric shape must therefore be independent of each other for the application of the ESM.

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Apart from the methods mentioned above, alternative methods based on an energy approach exist. The concept of these procedures is that a potential total energy of a similarly scaled model has to be proportional to that of a full-size structure and corresponds to the principle of energy conservation (Kasivitamnuay et al., 2005). Another energy approach is described by De Rosa et al. (2005) and is known as Asymptotical Scaled Modal Analysis, which was developed to reduce the spatial extent in order to save time during FE-simulations. Shokrieh and Askari (2013) presented a method known as sequential similitude method. The sequential similitude method indicates that for a structure subjected to different stress situations with permanent effects, similarity can be established assuming that each stress situation is simulated separately. Luo et al. (2015) showed how distorted scaling laws of the thin-walled annular plate are obtained by combining the governing equation and sensitivity analysis.
Many other authors published on the topic of similitude techniques. Pahl et al. (2013) focused with the formulation of the series development on product development. Deimel (2007) applied similarity ratios in the design field for the optimization and evaluation of design solutions. Examples of his work are ratios in design catalogues and the graphic representation of similarity. In Koschorrek's (2007) work, a method for the use of dimensionless ratios and similarities is presented which allows comparative statements about design solutions to be made already during the conceptual phase of development. Wolniak et al. (2018) proposed a method for building a knowledge-based engineering system to scale structural components. The benefit of this proposed method is the defined outcome as a configuration tool for the scaling of components with the flexibility of an individual adaption of the geometry and of given design features. This allows a quick initial estimation of the feasibility of the desired scaling (Wolniak et al., 2018).
Rudolph (2002) worked on the transfer of similarity theory into various new research fields, such as artificial intelligence, data mining and computer-aided-engineering. In the domain of reliability engineering Krä (1988) used the model of the statistical size effect with the help of the Basquin equation to derive a function that can be used to specify the failure probability of components subjected to vibrational stress. Based on the evaluation of known tests, the failure probability of other samples can be predicted regardless of their shape and size. The prerequisite for this is the use of the same material and the same load-time function (Krä and Heckel, 1989). The challenge to transfer fatigue life data with the statistical size effect, apart from simple specimen geometries, lies in determining a correction function which takes into account both the geometry of the component and the crack as well as the location of the crack and the stress distribution located there (Huster, 1988).
Figure 1 summarizes methods of scaling in a chronological overview from 1915 until today.

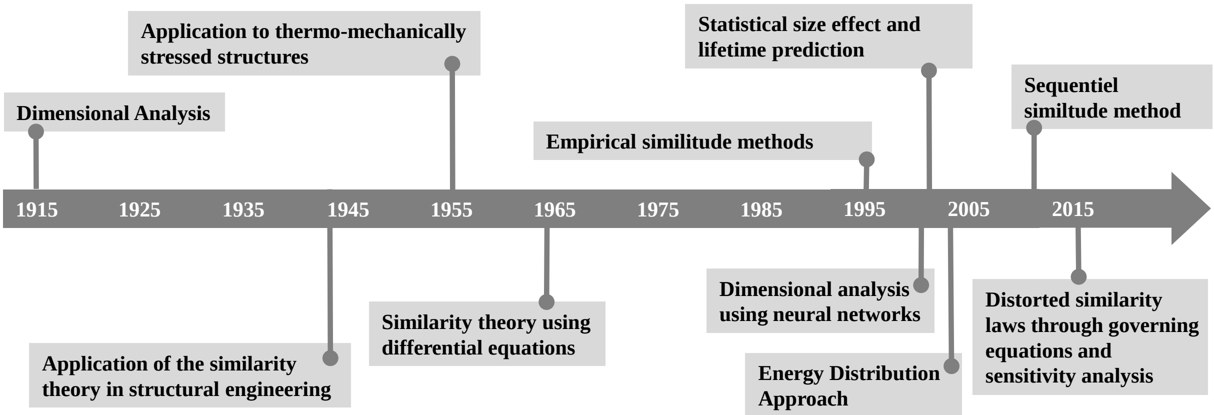

Figure 1. Time overview of scaling methods

2.2. Size-effects and nonlinearity in scaling of structural components

The methods described in Section 2.1 are limited in their applicability in case of scaling large components. The reason for this is, that the material behaviour and the assumed homogeneity can no longer be considered idealized with larger components. Since the behaviour of real structures can rarely be described by linear equations, especially for large components, the consideration of nonlinearities in scaling is

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essential. In this context, it is necessary to classify the terms size effect and nonlinearities in relation to the scaling of components. After an introduction of these two terms, the similitude techniques, described in section 2.1, will be discussed with regard to their applicability and consideration of size effects and nonlinearities.
When the size is changed, there are usually influencing mechanisms on the strength (Huster, 1988). With regard to the scaling of sample geometry and microstructure, the dimensions to which size effects can be attributed can principally be divided into four classes (Henning, 2008):

1. Absolute geometric dimensions
2. Relative geometric dimensions, such as specimen thickness - specimen width
3. Absolute microstructural dimensions
4. Relative microstructural dimensions, such as specimen thickness - grain size

The traceability of a measured effect to one of the four classes is not possible without further ado, because for example a scaling of the sample geometry with constant grain size always changes the ratio of sample geometry to grain size. Therefore, it is often not possible to deduce the cause of the effect from a single test series. For this reason, geometry and grain size are scaled separately in many experiments (Henning, 2008). A decisive function in addition to the shape of the component is the size. If the size increases, the lifetime is reduced despite similar stress. Many studies deal with this size effect (Krä, 1998; Huster, 1988; Köhler, 1975; Ziebert, 1976). There are basically three types of size effects:

- **Geometrical size-effect**: Refers to all size effects resulting from an inhomogeneous stress gradient (Scholz, 1988). This model assumes that a higher fatigue life for smaller specimens results from different stress gradients for specimens of various thicknesses at the edge of the component, combined with the supportive influence of the less stressed areas which are located more inwards (Little, 1966)
- **Technological size-effect**: Reasons for the influence are based on the specific effects of the used manufacturing process. This includes the size-dependent effects of all influencing parameters caused by the technology on the fatigue life, such as edge hardness, degree of purity and forging, graphite form, etc. (Huster, 1988; Scholz, 1988)
- **Statistical size-effect**: This size effect can be explained with Weibull's model of defects (Weibull, 1959). The failure behaviour is significantly influenced by micro- and macrostructural material inhomogeneity. These can be interpreted as statistically distributed defects in the material with local loads. A crack then starts from these points, which are responsible for the component failure. Under the assumption that the component failure is determined only by the material element of lowest strength (Weakest-Link-Concept), the failure probability of a vibrating stressed component can be described satisfactorily by the Weibull distribution function. (Diemar et al., 2004)

According to Kloos (1979), the relationships between the various size effects can be illustrated as follows in Figure 2:

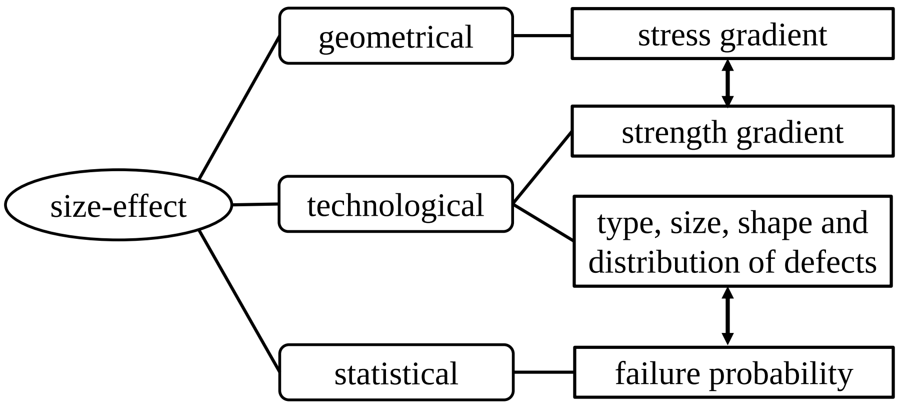

Figure 2. Relationships between size effects

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In addition to the effects listed above, the following circumstances are often not considered during component testing in experimental environments: *Deformation velocity, surface topography and grain size distribution* (Henning, 2008).

- **Deformation velocity**: The velocity of dislocations may not be proportional to the acting shear stress, as in the case of Fe-3%Si (Saka et al., 1972). In modelling, this strain rate dependence manifests itself in the form of an internal length contained in the constitutive equations, thus implying a size-dependent behaviour (Needleman, 1988). For this reason, the time scale must be scaled correspondingly in dynamic experiments, according to the similarity theory, so that the strain rate is maintained (Pawelski, 1964).
- **Surface topography**: The surface roughness can strongly influence the friction between specimen and tool, and thus the test result (Staeves, 1998). Irrespective of friction, a reduction in surface roughness can, for example, lead to increased formability of the sample (Yamaguchi et al., 1995). The absolute surface roughness has a different effect on the test result depending on the sheet thickness, and therefore a consideration relative to the sheet thickness appears to be more suitable. (Henning, 2008)
- **Grain size distribution**: The commonly used arithmetic average of grain size is not an explicit measure. The same mean value can be linked to different grain size distribution functions, which in turn influences the mechanical properties differently (Berbenni, 2008).

As components become larger, nonlinear phenomena, as already described above, play an increasingly significant role. In the following, types of nonlinearity will be shown, which in addition to size effects, must also be taken into account when scaling large components. There are mainly three types of nonlinearities in continuum mechanics: *Geometric nonlinearities, material nonlinearities and Nonlinearities due to boundary conditions*. (Knothe and Wessels, 2017)

- **Geometric nonlinearities**: This is associated with problems where large displacements and twists with small distortions have to be taken into account, e.g. for structural elements such as ropes, beams or membranes (Wriggers, 2008). If a structure has a significantly different geometric shape or a different type of equilibrium to the unloaded state under load, geometric nonlinearities may be relevant (Gebhardt, 2018). Figure 3 shows a rigid beam with a torsion spring and transverse force as an example of such a nonlinearity. The nonlinear relationship for the force is: $F = (c \cdot \varphi)/(l \cdot cos\varphi)$. For small angles φ is $cos\,\varphi = 1$ and you get a linearized equation. For large angles, the result of the linearized equation differs greatly from the exact solution, as can be seen in the first row of Figure 3.
- **Material nonlinearities**: For most materials, the assumption of a linear correlation between stress and strain is only an approximation and only valid under certain simplifying conditions, e.g. assumption of small strains. (Wriggers, 2008) Figure 3 (second row) illustrates this context graphically. An example of nonlinear material behaviour is plasticity: When a ductile steel is loaded, it initially exhibits an easy-to-describe behaviour: With increasing loading, the deformation increases proportionally. This applies up to the yield point. As soon as this load level is reached, the steel begins to yield without any further increasing load.
- **Nonlinearities due to boundary conditions:** The most common cause for this is changing boundary conditions during the deformation process, e.g. during load increasing. An example of this are contact problems in which the contact zone between two objects changes during the deformation, whereby the ingress of one object into the other is excluded. (Wriggers, 2008) Contacts which can either open or close, change the stiffness of an assembly depending on the force size (Figure 3, third row) and direction (Rust, 2016).

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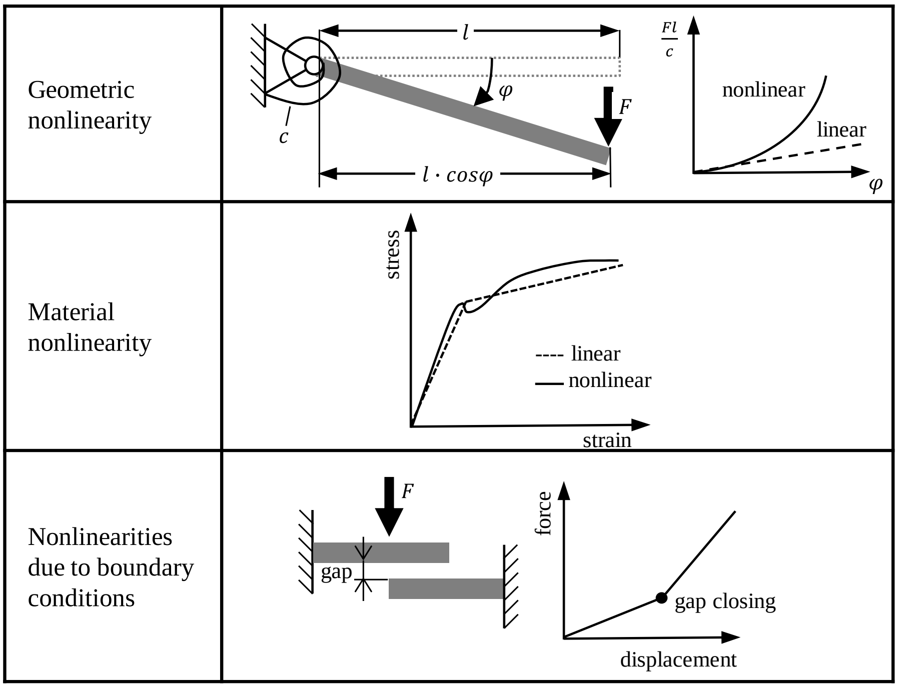

Figure 3. Examples of nonlinearities

3. Discussion of scaling methods with regard to their applicability and consideration of size effects and nonlinearities

Most of the methods described in Section 2.1 are based on linear assumptions concerning material properties and behaviour or use linear functions to transfer similarity from small to large components and vice versa. The use of dimensionless potency products or key ratios also does not consider nonlinear phenomena of mechanics. These ratios are transferred linearly by simple multiplication to quantities to be scaled. An analytical relationship to include physical meaning is only considered in the methods through governing differential equations and energetic methods. However, many phenomena in mechanics, are not analytically detectable or rather not available. The applied geometries are mostly simple in the context of structural scaling. The most commonly studied components are beams, plates, shells and cylinders. It is not the purpose of this paper to show the applications up to date. There are many review papers in this context, which already reviewed applications as well as methods. Especially the works of Casaburo et al. (2019) and Coutinho et al. (2016). The Empirical Similitude Method allows more complex and distorted geometries, but only for functionally decoupled parameters. The complexity of the components is further limited by the fact that only those components can be scaled in which material behaviour and geometric shape are independent of each other (Dutson, 2002). The statistical size effect method is limited to the transfer of the lifetime of components in different sizes. However, this method is not validated for large scale products. The acquisition of correction functions for the transfer of the lifetime function based on the Basquin equation is a challenge still to be accomplished. Sensitivity analysis is not an independent method for scaling, it only supports the methods through differential and energetic equations. The sequential similitude method has so far only been performed by using the example of an impacted composite laminate and needs further investigation to validate the method for large scale products and more complex structures. The methods from the field of artificial intelligence mainly transfer the theory of dimensional analysis and the concept of similarity into new fields of development, thus they are currently not applied in the sense of model experiments with real structures. However, when applying scaling methods, it is recommended to check whether size effects or nonlinear phenomena are to be taken into account in the considered circumstances or whether the influence of these are negligibly small. The analysis of the state of the art with a point of view on size effects and nonlinear phenomena results in Table 1 for the classification of the scaling methods.

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Table 1. Overview of scaling methods and their consideration of size effects and nonlinear phenomena

Method	Description	Main scope	Consideration of size effects	Consideration of nonlinearity
Dimensional analysis	Dimensionless key ratios are derived from a dimension matrix based on a relevance list with system-specific variables.	Many fields of engineering, from fluid mechanics up to structural engineering. Systems with unknown behavior.	No	No
Differential Equations	Based on the definition of scale factors which are inserted into the governing differential equations to derive similarity conditions	Similar to dimensional analysis (more physical meaning). Any system with available governing equations.	No	For simple case studies
Empirical Similitude	Transformation matrices are used to merge the empirically determined data of the geometrical and material changes. Separate scaling of material properties and geometric properties	Rapid Prototyping of models.	Partially	No
Energetic Approach	Using energy equations. Potential total energy of a similarly scaled model has to be proportional to that of a full-size structure and corresponds to the principle of energy conservation	Linear Static deflection and free vibration. Use of relationships between mode shapes, natural frequencies and damping loss factors.	Partially	No
Statistical Size Effect	Transfer of fatigue life data with the statistical size effect and the Basquin-equation to predict lifetime of scaled components	Reliability Engineering	Yes	Yes
Artifical Intelligence	Parameter reduction and associated simplification of equations and contexts that result from the dimensional analysis	Genetic algorithms, case-based-resonning, design evaluation, neural networks and pattern recognition.	No	No
Sequential Similitude Method	Similarity conditions can be established for a structure subjected to different loading situations, provided that each loading event is simulated independently.	Structures subjected to sequentially loading situations.	Yes	Yes
Sensitivity Analysis	Combining the governing equation and sensitivity analysis to derive similitude conditions for distorted models	Linear static and frequency analysis	No	No

4. An approach for identifying a suitable scaling method for structural components

Based on the information from Table 1, a flowchart has been built up, which shows a procedure for orientation while selecting a suitable scaling method (Figure 4). Once again, it is shown that there is no method that can take all size effects and nonlinear phenomena into account. The method Sequential similitude method is not listed, as it has so far only been validated by using the example of an impacted composite laminate. An area that has not been investigated so far is represented by a dashed

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arrow. This refers to the possibility of using artificial neural networks as functional approximators for specific nonlinear effects (e.g. the damping coefficient, which is a function of material and geometry at the same time). Experimentally determined data related to an effect in small and large scale could be used as training data for an artificial neural network. This network could then be applied to approximate intermediate sizes.

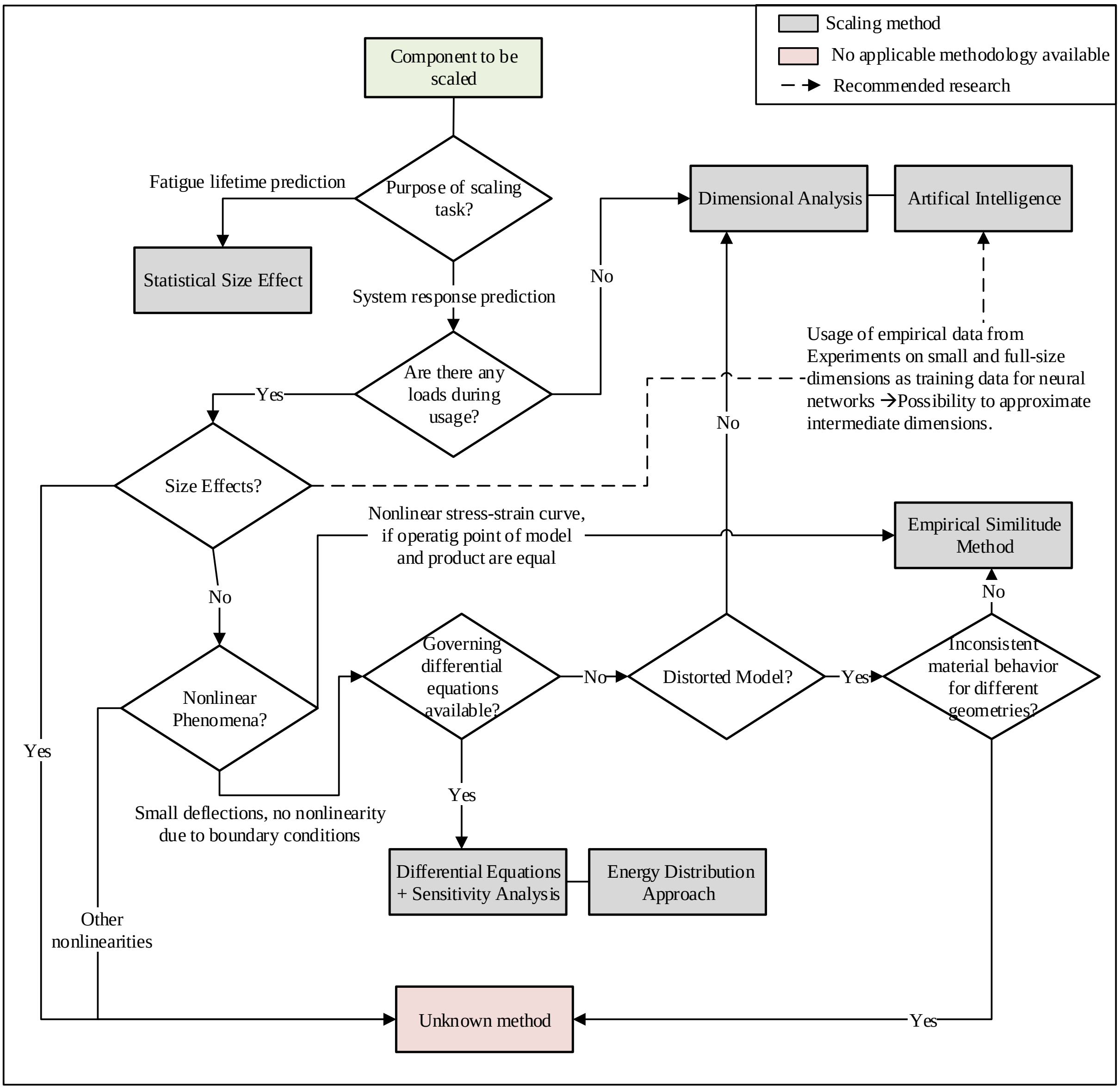

Figure 4. Procedure for identifying a scaling method

5. Conclusion and future work

The purpose of this article was to examine the state of the art of common scaling methods with respect to their applicability to large components and the particular challenges this brings. When considering large components, size effects and nonlinear phenomena have a major impact. Idealized assumptions which are taken for small sizes are not always valid for large components. The literature review has shown that the inability to predict size effects is a common problem of many scaling methods. For design practice, this means that previously known methods cannot simply be transferred to large components. The change in strength properties and their effect on system responses under load conditions are rarely or not at all considered. Only the method "Statistical size effect" considers one of three size effects using statistical distribution functions. However, this method has so far often been applied to small and

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unspectacular specimen geometries. For large component applications, the difficulty is to determine correction functions to ensure the transfer of fatigue life data from small to large scale.
One result of this work is the tabulated summary of common scaling methods and their evaluation with regard to the consideration of size effects and nonlinear phenomena. Furthermore, a procedure has been created to assist development engineers in choosing a suitable method for their scaling task. In many applications concerning large components, the process unavoidably ends with not being able to select a suitable method. Thus, there is still a need to develop new methods and to further investigate those that have already been developed. Since nonlinear phenomena and size effects are difficult to analytically capture, empirical methods have greater potential to overcome these difficulties. In particular, methods of artificial intelligence, such as neural networks, could be investigated for the development of new empirical methods. This article proposes to use experimental data as training data for artificial neural networks in order to approximate functional relationships of an effect in different scales.

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