



Sustainability of the plastron on nano-grass-covered micro-trench superhydrophobic surfaces in high-speed flows of open water

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This paper studies the sustainability of plastrons on superhydrophobic (SHPo) surfaces made of longitudinal micro-trenches covered by nano-grass with the main interest on hydrodynamic friction drag reduction in high-speed flows of open water, which represent the operating conditions of common watercraft. After revising the shear-driven drainage model to address the air diffusion for SHPo surfaces, the existing theories are combined to reveal the trends of how the immersion depth, air saturation level and shear stress affect the maximum attainable plastron length. Deviations from the theories by the dynamic effect at the two ends of the trench, the interfacial contaminations and turbulent fluctuation are also discussed. A combinatorial series of well-defined SHPo trench surfaces (4 cm \times 7 cm in size with varying trench widths, depths, lengths and roughnesses) is microfabricated and attached underneath a 4 m long motorboat on seawater in turbulent flows up to 7.2 m s^{-1} (shear rate $\sim 83\ 000\ \text{s}^{-1}$ and friction Reynolds number ~ 5500). Because the plastron can provide a substantial slip only while its air-water interfaces are pinned (or only slightly depinned) at the trench top, two underwater cameras are employed to differentiate the pinned (and slightly depinned) interfaces from the depinned (and no) interfaces. In addition to achieving pinned plastrons on 6 cm long trenches aligned

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to high-speed flows in open water, the experimental results corroborate the theoretical estimations, supporting the design of SHPo surfaces for field applications.

Key words: drag reduction, MEMS/NEMS, contact lines

1. Introduction

Superhydrophobic (SHPo) surfaces have been one of the most popular topics in science and engineering over the last two decades because of their unique potentials, such as hydrodynamic drag reduction (Ou, Perot & Rothstein 2004; Choi & Kim 2006), self-cleaning (Barthlott & Neinhuis 1997), anti-icing (Cao et al. 2009), anti-biofouling (Marmur 2006) and anti-corrosion (Liu et al. 2007). Among them, drag reduction of watercraft has been cited as a motivating factor in nearly every publication on SHPo surfaces for its global-scale impact on energy saving and environmental protection (Park, Choi & Kim 2021). When a SHPo surface is completely immersed in water, a substantially continuous layer of air, commonly called a plastron (Brocher 1912), may be formed on it and produce a slip boundary that reduces skin friction drag. While all numerical studies over the years (Min & Kim 2004; Fukagata, Kasagi & Koumoutsakos 2006; Martell, Perot & Rothstein 2009; Park, Park & Kim 2013; Rastegari & Akhavan 2015; Im & Lee 2017) and many experimental studies in the 2010s (Daniello, Waterhouse & Rothstein 2009; Bidkar et al. 2014; Park, Sun & Kim 2014; Gose et al. 2018) have reported a significant drag reduction, successful drag-reduction experiment in fully turbulent flows in open water, which represents the field condition of watercraft, has not been reported until 2020 (Xu et al. 2020b, 2021). These most recent successes have eased the skepticism that had grown against the SHPo drag reduction, after two decades of research without any successful field experiments. Importantly, the successful reports strongly suggested that most of the inconsistent experimental results in the past may have been simply due to the loss or deterioration of the plastron. In other words, the original notion of SHPo drag reduction is valid as far as the plastron remains in a good shape. The tortuous path to the current state of knowledge also indicates how difficult yet important it is to accurately monitor the state of the plastron during experimental studies of SHPo drag reduction, leading to the two-camera observation technique by Yu et al. (2021). Focusing on longitudinal micro-trench SHPo surfaces, which have been the most effective for drag reduction (Park et al. 2021) and to help the design of SHPo surfaces capable of reducing the drag for watercraft, this paper aims to understand the range of trench geometries that can maintain a pinned or slightly degraded plastron, which has its air-water interfaces pinned or slightly depinned at the top edges over the entire or nearly entire trench length so that much of the pristine slip capability is preserved, in high-speed open-water flows.

Over the years, hydrostatic pressure and air diffusion have been found to affect the plastron stability in stationary (Bobji *et al.* 2009; Poetes *et al.* 2010; Samaha, Vahedi Tafreshi & Gad-el-Hak 2012; Lv *et al.* 2014; Xu, Sun & Kim 2014) and flowing (Ling *et al.* 2017; Kim & Park 2019) water. More recently, the shear-driven drainage (Wexler, Jacobi & Stone 2015; Liu *et al.* 2016), which was developed to understand the loss of infused oil on liquid infused surfaces (LIS) (Wong *et al.* 2011) caused by the shear in flowing water, has been borrowed to explain the plastron loss on trench SHPo surface in very high shear flows (Xu *et al.* 2021). However, while the diffusional loss of the infused oil on LIS in water is negligible and was justifiably ignored in the shear-drainage model, the diffusional loss of the trapped air in water cannot be ignored and would require a revised theory applicable to SHPo surfaces. To establish a theoretical model that can describe the plastron morphology

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on the longitudinal micro-trench SHPo surfaces in high-speed flows under water, in this paper we utilize both (i) the plastron stability theory based on the hydrostatic pressure and air diffusion and (ii) the shear-drainage theory modified for plastron (i.e. not oil) stability. In addition, the water pressure on the plastron is expected to deviate from the theory at the front and rear end of the trench by the dynamic effect of flows, leading to a negative effect when compounded with the interfacial contaminations such as surfactants (Landel *et al.* 2020) that accumulate at the rear end.

To evaluate the model experimentally, we prepare a combinatorial series of SHPo surfaces and monitor their plastron status in fully turbulent flows under a motorboat at various flow speeds on seawater. For drag-reduction applications, ideally one would like to have a pinned plastron, where the trapped air fills the entire depth and length of trench. To enable differentiating pinned and slightly degraded plastron, which can retain an acceptably substantial amount (e.g. >30%) of the drag reduction induced by pinned plastron, from degraded and no plastron, which is left with an unacceptably small amount (e.g. <30%) of the drag reduction by pinned plastron, we devise a new observation scheme using two underwater cameras by applying the approach of Yu et al. (2021) to the current goal. Since the plastron was observed to be intact at all speeds (i.e. $2.3 \text{ m s}^{-1} < \text{flow}$ seed $< 5.1 \text{ m s}^{-1}$ or 10 000 s⁻¹ < shear rate $< 62 \text{ }000\text{s}^{-1}$; see supplementary movie S1 is available at https://doi.org/10.1017/jfm.2023.184 with Appendix A) in the previous boat experiments (Xu et al. 2020b) but found to be depleted at the higher speeds (i.e. $6.1 \text{ m s}^{-1} < \text{flow speed} < 10.1 \text{ m s}^{-1}$, or 55 000 s⁻¹ < shear rate < 140 000 s⁻¹) in the towing tank experiments while using similar trench SHPo surfaces (Xu et al. 2021), we modify the boat to increase its top speed to 7.2 m s⁻¹ (or shear rate up to ~83 000 s⁻¹) so that the plastron can be depleted by shear-driven drainage (see supplementary movie S1 with Appendix A) in the current boat study.

2. Theories and deviations from the model

2.1. Acceptable and unacceptable plastron for drag reduction

Let us consider a micro-trench SHPo surface immersed in water, as illustrated in figure 1, which also defines the pitch p, width w and depth d of the trenches. The gas fraction of the surface is defined as w/p. Although most numerical studies of SHPo drag reduction assume air-water interfaces (or menisci) to be flat and pinned on the trench top edges, in reality menisci are rarely flat and may not be pinned on the top. For watercraft applications, which usually involve open water in nature (whose air saturation level hovers around 100%) and hydrostatic pressure, menisci would either be pinned and concave, as shown in figure 1(a-1), or depinned-in and concave, as shown in figure 1(a-2). Note that the amount of depinning is expressed as the water intrusion depth h, which is the distance between the trench top and the meniscus contact line. Compared with the pinned-and-flat meniscus, the pinned-and-concave meniscus would degrade the slip only slightly, but the depinned-in meniscus (even if flat) would degrade the slip significantly, as summarized in Lee, Choi & Kim (2016). Both numerical (Ng & Wang 2009) and analytical (Crowdy 2021) studies have predicted that the slip length, which determines the drag-reducing ability of a SHPo surface, on longitudinal trenches would decrease by $\sim 50 \%$ when the contact line slides down from the trench top by merely 10 % of the trench width, i.e. h/w = 0.1, and by nearly 70% when h/w = 0.2, for the trench with 0.9 of gas fraction (w/p = 0.9). Such a small amount of depinning has been unnoticeable in previous high-speed flow experiments, where the only practical way to confirm the plastron was by observing its silvery sheen, which indicates its existence but not thickness. Note that even a significantly



Figure 1. Schematic illustration of plastron being compromised on a SHPo surface made of micro-trenches with vertical sidewalls. (a) Since the water pressure is usually higher than the trapped air pressure, the air–water interface is concave when pinned (a-1). If the water pressure is large enough to make the contact angle of water on the trench sidewall exceed the advancing contact angle θ_a , the contact line is depinned from the top edges and slides into the trench (a-2) until the trench is fully wetted (a-3). ((b) Although not common, if the water pressure is lower than the trapped air pressure, the meniscus is convex when pinned (b-1). If the contact angle of water on the trench top decreases below the receding contact angle θ_r , the contact line is depinned from the top edges and lets the neighbouring air pockets merge (b-2). The merged air may form isolated bubbles off the surface, shrinking the plastron (b-3), which grows back to the pinned state (b-1).

depinned interface, e.g. figure 1(a-2), may still appear bright on trench SHPo surfaces, as demonstrated by Yu et al. (2021). Compounded by the fact that even a marginal depinning would lead to a substantial decrease in slip length, the common practice of confirming the existence of plastron only with its brightness helps explain the frustratingly inconsistent experimental results even with trench SHPo surfaces (Henoch et al. 2006; Woolford et al. 2009) that have been hampering the progress of SHPo drag-reduction research. Considering the stringent condition that little depinning is allowable for a successful drag reduction, let us define an acceptable plastron as one with contact lines pinned or slightly depinned on the trench top edge (e.g. h/w < 0.1, or h/w < 0.2). Note this new definition of acceptable plastron, which is useful for drag reduction, differs from the common definition of plastron lifetime that includes all shades of plastron until the meniscus hits the trench bottom (Emami et al. 2013; Xu et al. 2014) and instead resembles the stringent definition of plastron lifetime that includes only pinned interfaces (Piao & Park 2015). Assuming the depinning-caused loss of drag reduction by up to 70 % (which means down to 15 % of drag reduction if the pinned plastron was to provide 50 % of drag reduction) is acceptable in this study (somewhat arbitrarily), we devise and implement a new observation scheme that can differentiate $h/w \le 0.17$ (acceptable plastron) from h/w > 0.17 (unacceptable plastron), as explained in the experimental sections.

For the contact line to stay pinned as in figures 1(*a*-1) and 1(*b*-1), the pressure difference between the water above and the air inside the plastron, $\Delta P = P_{water} - P_{air}$, should be balanced by the Laplace pressure of the air–water interface ΔP_{σ} at the trench top, $\Delta P = \Delta P_{\sigma}$. Since the trench geometry determines the minimum and maximum value of ΔP_{σ} possible at the trench top, the range of pressure difference allowable for pinning can be expressed as

$$\Delta P_{\sigma,\min} < \Delta P = \Delta P_{\sigma} < \Delta P_{\sigma,\max}, \qquad (2.1a)$$

$$\Delta P_{\sigma,min} = -\frac{2\sigma\cos(\theta_r - 90^\circ)}{w},\tag{2.1b}$$

$$\Delta P_{\sigma,max} = -\frac{2\sigma\cos\theta_a}{w},\tag{2.1c}$$

where σ is the surface tension of water. If the water pressure is higher than the plastron pressure by more than the maximum Laplace pressure, $\Delta P > \Delta P_{\sigma,max} = -2\sigma \cos \theta_a/w$, the contact line will be depinned in and slide into the trench, as illustrated in figure 1(*a*-2). Note that the above ranges of Laplace pressure were based on the simple trench geometry with vertical sidewalls. If one adds re-entrant edges to the micro-trenches, the maximum Laplace pressure increases to $\Delta P_{\sigma,max} = 2\sigma/w$, expanding the pinned state, as introduced in the previous open-water drag-reduction experiments (Xu *et al.* 2020*b*). On the other hand, if the water pressure is lower than the plastron pressure by more than the minimum Laplace pressure, $\Delta P < \Delta P_{\sigma,min} = -2\sigma \cos(\theta_r - 90^\circ)/w$, the contact lines will be depinned out and let neighbouring air pockets merge, as illustrated in figure 1(*b*-2). The latter case, i.e. figure. 1(*b*), may occur when a SHPo surface is placed shallow in supersaturated water. While the merged air pockets may grow large and leave by buoyancy in static water as shown in figure 1(*b*-3), in fast flowing water, the overgrown plastron is mostly prevented by the shear.

2.2. The effect of hydrostatic pressure and air diffusion on plastron morphology

Diffusion of air between the plastron and surrounding water on a hydrophobic trench in stationary water has been well studied using a two-dimensional model and experimentally verified (Xu *et al.* 2014). Based on Henry's law, the partial pressure of dissolved air in water is $p = k_H c$, where k_H is Henry's constant and c is the concentration of dissolved air. The partial pressure of dissolved air in water can also be expressed as $p = sP_{atm}$, where s is the pressure ratio of the dissolved air in the water to the atmospheric air above the water or the percentage saturation of air in water (Mortimer 1956), also simply called the air saturation level. The volumetric diffusion rate of air into the plastron can be approximated by Fick's law as

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = \int k_p [sP_{atm} - P_{air}(x, t)] \,\mathrm{d}A(x, t), \tag{2.2}$$

where V is the volume of air in the plastron, k_p is the mass transfer coefficient of air across the air–water interface, P_{air} is the air pressure in the plastron, A is the air–water interfacial area, x is the position along the trench and t is time. In static water, where the condition is uniform along the trench so that $P_{air}(x, t) = P_{air}(t)$ and A(x, t) = A(t), the above diffusion rate can be simplified as

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = k_p A(t) [sP_{atm} - P_{air}(t)].$$
(2.3)

If the plastron is in equilibrium (i.e. at a steady state) so that dV/dt = 0, the air pressure in plastron equals the partial pressure of dissolved air in the surrounding water:

$$P_{air,st} = sP_{atm},\tag{2.4}$$

where the subscript *st* indicates static water as opposed to the dynamic water, which flows and imposes a shear stress on the plastron. Since in static water, the water pressure on the



Figure 2. Pressure distributions along the trench. For the air–water interface to stay pinned on the trench top at x, the pressure difference between the water and the plastron, $\Delta P(x) = P_{water}(x) - P_{air}(x)$ (blue vertical arrows), should be sustainable by the Laplace pressure of meniscus ΔP_{σ} or $\Delta P_{\sigma,min} < \Delta P(x) < \Delta P_{\sigma,max}$. (a) The effect of immersion depth H and air saturation level s. In static water, the water pressure on the trench surface (thick green line) is $P_{water,st} = P_{atm} + P_H$. The partial pressure of air dissolved in water $P_{air,st}$ is sP_{atm} (thick red line), which equals P_{atm} if the water at the free surface (in contact with ambient air) is saturated with the atmospheric air. (b) The effect of shear stress by water τ_w . In flowing water, the shear stress τ_w makes the air pressure in the plastron $P_{air}(x)$ (thick red line) increase linearly with x, decreasing the pressure difference $\Delta P(x)$ along the trench. (c) When immersed in flowing water, the two trends of (a) and (b) are combined to suggest a more general trend. (d) The above trend may be deviated by the dynamic effects of water flow near the front and rear end of trench.

plastron is $P_{water,st} = P_H + P_{atm}$, where P_H is the hydrostatic pressure at immersion depth H, (2.4) allows the pressure difference between two sides of the meniscus to be expressed in the following way:

$$\Delta P_{st} = P_{water,st} - P_{air,st} = P_H + (1-s)P_{atm}.$$
(2.5)

To help conceptualize how the pressure difference is determined on a longitudinal trench SHPo surface by multiple factors, (2.5) is graphically presented in figure 2(*a*), which visualizes how the pressure difference $\Delta P(x)$ (vertical arrows in the figure) is determined by the hydrostatic pressure P_H and air saturation level *s*. If $\Delta P(x)$ is larger than the largest sustainable Laplace pressure, i.e. $\Delta P_{\sigma,max}$, or smaller than the smallest sustainable Laplace pressure, i.e. $\Delta P_{\sigma,min}$, by the air–water interface, depinning would occur at location *x*.

2.3. The effect of shear by water flow on plastron morphology

SHPo surface vs. LIS: if water is not static, the flowing water will drag the trapped air with it, causing a shear-driven flow of air inside the plastron, hence increasing the air pressure toward the rear (trailing) end of the trench. The increased air pressure at the rear end, in turn, will cause a pressure-driven flow of air in the opposite direction to the water flow. In accordance with the pressure distribution, the plastron morphology can be depicted as shown in figure 3(a), which is drawn for a simple trench with length *L*, width *w* and depth *d* and assuming $L \gg w \sim p$. While the work of Wexler *et al.* (2015) and Liu *et al.* (2016) was developed to understand the loss of oil on LIS, for which there is no oil diffusion across the oil–water interface, our analysis for the loss of air on SHPo surfaces starts by noting there exists air diffusion across the air–water interface, as will be elaborated in this section.



Figure 3. A hydrophobic micro-trench submerged in longitudinally flowing water with the contact lines pinned on top. (*a*) An exemplary illustration of plastron morphology. The arrow in the trench shows the air circulation inside the plastron. (*b*) Profiles of the *x*-direction air flow inside plastron. The net air flow profile consists of three different flow profiles: shear driven, Laplace pressure driven and air diffusion driven. The first two profiles follow Wexler *et al.* (2015) developed for LIS, and the third profile is newly introduced to account for the air diffusion across the air–water interface, which varies along the *x*-direction. The air flux (in the *x*-direction) induced by the air diffusion varying along *x* turns out to be small for the flow conditions of this study.

By combining the three types of air flows (i.e. shear driven, Laplace pressure driven and air diffusion driven), as indicated in figure 3(b), we will obtain the air pressure in the plastron along the trench and the resulting meniscus morphology.

Water shear-driven flux: to analyse the air flux driven by the flowing water, we assume (i) the air flow inside the plastron is laminar, and (ii) the meniscus is flat. Based on Liu *et al.* (2016), shear-driven flux q_s can be expressed as

$$q_s = \frac{2D}{1+2DN} \frac{c_{sl}\tau_w w^3}{\mu_{air}},\tag{2.6}$$

where *D* is the normalized maximum local slip length on the plastron (Schönecker, Baier & Hardt 2014) that is determined by trench aspect ratio d/w and gas fraction w/p (D = 0.201 if d/w = 1 and w/p = 0.9, which are the typical parameters used for the experiments in this study); *N* is viscosity ratio, which is $N = \mu_{water}/\mu_{air} = 55$ for SHPo surfaces, where μ_{water} and μ_{air} are dynamic viscosities of water and air, respectively; c_{sl} is a factor determined by d/w (if d/w = 1, $c_{sl} = 0.108$); τ_w is the shear stress of flowing water applied on the SHPo surface.

Laplace pressure-driven flux: in addition to the above assumptions, for simplicity, we further assume (iii) the air pressure in the plastron changes linearly with x. Following Liu *et al.* (2016), the Laplace pressure-driven flux inside the trench is

$$q_p = -\frac{c_p w d^3}{\mu_{air}} \frac{\mathrm{d}P_{air}(x)}{\mathrm{d}x},\tag{2.7}$$

where c_p is a factor determined by trench aspect ratio; for d/w = 1, $c_p = 0.0351$.

Air diffusion-driven flux: when analysing the shear-driven flux for LIS, the infused oil was considered to not diffuse into the surrounding water (Wexler *et al.* 2015). While such an assumption was reasonable due to the insolubility of silicone oil in water, the same

assumption is not reasonable for SHPo surfaces, for which the solubility of air in water is appreciable, e.g. ~0.8 mM (Sander 1999), compelling us to analyse how the air diffusion between the plastron and flowing water would affect the plastron morphology. Note the diffusion rate across the meniscus varies along the trench because the pressure of trapped air varies along the trench, as indicated with 'air diffusion' in figure 3(b). At a steady state, for example, air would diffuse into the plastron on the leading half of the trench and diffuse out from the plastron on the trailing half of the trench, inducing a new air flux q_d that we call air diffusion-driven flux, as shown in figure 3(b). The varying air pressure along the trench would also change the meniscus curvature, and thus, the meniscus area, which would affect the air diffusion rate across the meniscus. However, we would ignore the curvature effect for simplicity here, leaving it for a future study. In any case, interestingly and somewhat surprisingly, we found that the air diffusion-driven flux, although clearly relevant to SHPo surfaces, is negligibly small when compared with the shear-driven and pressure-driven flow for typical flow conditions of watercraft, as analysed in Appendix B:

$$q_d \ll q_s; \quad q_d \ll q_p. \tag{2.8a,b}$$

The net flux: following the above approximation, the net flux of the air in the trench is practically zero,

$$q_s + q_p + q_d \approx q_s + q_p = 0, \tag{2.9}$$

which means the shear-drainage model for LIS can be used as a good approximation for SHPo surfaces as well. By integrating (2.6) and (2.7) into (2.9), we can get the gradient of air pressure along the trench as

$$\frac{dP_{air}(x)}{dx} = \frac{2D}{1+2DN} \frac{c_{sl}w^2}{c_n d^3} \tau_w.$$
(2.10)

The shear introduces a linear increase of air pressure, as shown in figure 2(*b*), for a given micro-trench geometry (i.e. *w*, *d*, c_{sl} , c_p) and fluid properties (i.e. *D* and *N*). As the flow speed increases (along with the shear stress), depinning would occur at the front end of trench when $\Delta P(0) > DP_{\sigma,max}$ (figure 1(*a*-2)) or at the rear end when $\Delta P(L) < DP_{\sigma,min}$ (figure 1(*b*-2)) depending on which one would occur first. Based on (2.10), decreasing the trench depth *d*, increasing the trench width *w* or increasing the shear stress of water τ_w on the trench would lead to a larger pressure gradient of air, which promotes depinning on the leading or the trailing end of the trench, as shown in figure 2(*b*). A simple scaling analysis indicates the air circulating inside the micro-trench is laminar for most flow conditions of relevance. While further supporting the air flow inside the trench is laminar, the three-dimensional simulation of turbulent boundary layer flow summarized in Appendix C also verifies the obtained air pressure aligns with (2.10).

General: to understand the state of plastron on a trench SHPo surface covering the hull of a travelling watercraft, one should consider all three – the flow speed, the immersion depth of the position of interest on the hull and the air saturation level of the water. For this more general situation of interest, figure 2(c), which combines figures 2(a) and 2(b), is presented to help one understand the trends of how the three main factors affect the plastron stability.

2.4. Deviations by water dynamic pressure, interfacial contamination and turbulent fluctuation

The above subsections focused on the effects of water pressure and shear stress and ignored the effects of trench boundaries. The water pressure was assumed to be uniform on the trench, i.e. $P_{water}(x) = P_{water}$, ignoring the effect of solid surfaces before (x < 0) and after (x > L) the trench for simplicity. However, the water pressure would decrease and increase momentarily as water flows past the front (leading) and rear (trailing) end of the trench, where the boundary condition changes from no slip to slip and from slip to no slip, respectively. Such a dynamic pressure effect has been studied on SHPo surfaces with posts (Seo, García-Mayoral & Mani 2015) but not on longitudinal trenches, which are typically modelled to be infinitely long. For now, let us present a qualitative analysis of the dynamic pressures to understand their effects on plastron morphology, as illustrated in figure 2(d). The pressure difference between the water and the plastron at the front end would be smaller than the expected, suppressing the depinning-in at the front end. In other words, as the shear stress of water flow increases, depinning-in would start to occur slightly away from the front end. On the other hand, the pressure difference at the rear end would be larger than the expected, promoting the depinning-in at the rear end. While qualitative and two-dimensional, the current discussion on dynamic pressure is supported by the numerical simulation in Appendix C and the experimental results later in this paper. Dedicated investigations would be needed in the future to quantitatively assess how the dynamic pressure affects the plastron morphology on longitudinal trench SHPo surfaces.

In the above subsections, the surface tension of the air-water interface was assumed to be constant, ignoring the effects of potential contaminants inevitable in the environmental water. Surfactants in water can adsorb onto the air-water interfaces, where they can be advected by the shear and accumulate at the rear end of trench (Landel et al. 2020). The accumulation may lead to the formation of a stagnant-cap region, where the surfactant reaches its maximum interfacial concentration and reduces the surface tension by $\sim 50\%$ for a typical surfactant such as sodium dodecyl sulphate (SDS) (Menger & Rizvi 2011). Although the surfactant effect may dominate and practically eliminate the drag reduction for some cases (Landel et al. 2020), the detrimental effect by the stagnant cap is confined to a relatively short range (e.g. ~ 1 mm) at the rear end of trench for typical flow conditions. Accordingly, the surfactant effect is relatively small for the long (>10 mm) trenches used for drag reduction in turbulent flows (Daniello et al. 2009; Park et al. 2014; Xu et al. 2020b, 2021). Nevertheless, the surfactant may induce a premature depinning at the rear end when the lowered surface tension is compounded by the dynamic pressure. Lastly, we would like to note numerous other effects, such as the small particles and micro-organisms that may accumulate on the meniscus and decrease surface tension (Zhang, Wang & Levänen 2013), the impact of solid particles onto the meniscus (Hokmabad & Ghaemi 2017) and the influence of salinity level of seawater (Ochanda et al. 2012). These and other unforeseeable environmental effects are important motivations behind performing flow experiments in a field condition, such as a passenger motorboat on natural seawater for this study.

Furthermore, the above subsections considered steady-state flows with time-averaged values. For the typical flow conditions of watercraft, however, the turbulent pressure fluctuations are significant. The water pressure in turbulent flow is

$$P_{water,turb} = \bar{P}_{water} \pm P'_{water}, \qquad (2.11)$$

where \overline{P} is the time-averaged pressure and P' is the pressure fluctuations. In contrast, the circulating air inside the trench is laminar and assumed not to generate pressure fluctuation. Since the air is confined in the trench, the fluctuation in water would compress and decompress the trapped air, inducing a reactive fluctuation in air that opposes the fluctuation of water. Accordingly, the pressure difference across the air–water interface for turbulent water flow over the air trapped in micro-trench may be expressed as

$$\Delta P(x) = P_{water,turb} - P_{air}(x) = \bar{P}_{water} - \bar{P}_{air}(x) + P'_{water} - P'_{air}, \qquad (2.12)$$

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where P'_{air} is related to P'_{water} via the volume of the trapped air (determined by w, d, L and meniscus shape) and air compressibility factor. Although it will require additional research to describe the plastron with the above equation to account for the turbulent fluctuations, at this point we can point to a couple of reports in the literature. For example, Rastegari & Akhavan (2019) studied the probability density function of the wall pressure fluctuations in turbulent channel flows of water on longitudinal trench SHPo surfaces, assuming a shear-free interface ($\tau_{w,air} = 0$) and infinitely deep and long trench (i.e. $d \to \infty, L \to \infty$), and showed an estimate of the upper limit for P'_{water} to be

$$P_{water}^{\prime +} = 4(P_{water}^{\prime})_{rms}^{+} = 4\{2.32\ln(\{w^+\}^{3/4}) + 2.31\ln(Re_{\tau}) - 14\},$$
(2.13)

for $-w^+ \gtrsim 5$ with 99.75 % confidence. The pressure fluctuation was normalized by the wall shear stress of the SHPo surface τ_w , i.e. $P'_{water} = P'_{water}/\tau_w$. The trench width was normalized by the wall unit of the turbulent boundary layer, i.e. $w^+ = w/\delta_v$, where the wall unit is defined as $\delta_v = v(\tau_w/\rho)^{-1/2}$, v is kinematic viscosity of water and ρ is the density of water. The friction Reynolds number is defined as $Re_\tau = \delta/\delta_v$, where δ is the boundary layer thickness. Because the infinitely deep trench would have no air compression, making $P'_{air} = 0$ in (2.12), (2.13) may be viewed as an extreme case of (2.12). On the other hand, Piao & Park (2015) studied how the pressure fluctuation in water affect the lifetime of plastron on a longitudinal trench SHPo surface, which have a finitely deep (limited d) and infinite length $(L \to \infty)$ trench, by considering the gas compression (i.e. $P'_{air} \neq 0$ in (2.12)) and viscous dissipation induced by the fluctuation. Using the fluctuation data reported by Tsuji *et al.* (2007) for common high Reynolds number flows and assuming the trench geometry similar to the current study, they found the fluctuation would not affect the plastron stability in the shallow water used for the current flow experiments.

3. Experiments and methods

3.1. The boat and underwater cameras

The motorboat (13 foot Boston Whaler) retrofitted for the drag-reduction research by Xu *et al.* (2020*b*) was used for the current study. Since shear-induced wetting, which was not observed in the boat test of Xu *et al.* (2020*b*), was found during the high-speed tow tank test by Xu *et al.* (2021) for similar SHPo surfaces, the boat was revamped to increase its top speed. By adding a hydrofoil stabilizer (Doel-Fin Hydrofoil, Davis Instruments) to the outboard motor, the boat top speed was increased from 10 knots to 14 knots, increasing the maximum shear rate attainable on the sample surface from ~5500 to ~8300 s⁻¹. A test well, which replaces a portion of the boat hull with a testing unit including sample surfaces, was installed on the boat as shown in figure 4(*a*) (similarly to Xu *et al.* 2020*b*). A custom-developed shear-stress sensor (UCLA-TAMNS; Xu *et al.* 2020*a*) was used, as shown in figure 4(*b*), to measure the shear stress on the SHPo surface during the boat test with uncertainties of $0.1\tau_{w0}$, where τ_{w0} is the measured shear stress on a smooth surface. An overall picture of the retrofitted boat is shown in figure 4(*c*).

Two miniature underwater cameras with waterproof rating IP67 (TODSKOP 5.5 mm WiFi Borescope) were used to monitor the plastron status on the SHPo surface during the boat test, following the observation strategy by Yu *et al.* (2021). Each camera was held in its own 3D-printed housing with a streamlined profile and installed as shown in figure 4(*d*) (one black and one white) to observe the sample from a specific distance and direction, so that together, the two cameras can accurately monitor the plastron states over the entire sample surface. The side camera observed the SHPo surface in the spanwise direction of the trench with an elevation angle $\beta = 10 \pm 2^\circ$, which is the angle between



Figure 4. Experimental set-up. (a) Schematic cross-section view of boat set-up. (b) Schematic cross-section view of the testing unit, including shear sensor and camera set-up. (c) Picture of the boat. (d) Picture of the bottom of testing well, taken by looking up from below in air.

the sample surface and the camera central axis. When the sample is observed from this specific elevation angle $\beta = 10 \pm 2^{\circ}$, the regions with $0 \le h/w \le 0.17 \pm 0.04$ (i.e. pinned and slightly depinned interface) appeared bright with the well-known silvery sheen, while the regions with $h/w > 0.17 \pm 0.04$ (i.e. depinned and no interface) appeared dark. The smallest detectible depinning is determined by the minimum elevation angle, which is limited by the camera's depth of focus and the size of the surface to observe. On the other hand, from the rear camera, which observed the surface in the parallel direction of the trench, the regions with h/w < d/w (i.e. any plastron) appeared bright, while the regions with h/w = d/w (i.e. no plastron) appeared dark. For the experiments in this study, if a type of trenches appears bright from the side camera, it has a pinned or slightly degraded plastron (i.e. deemed acceptable for drag reduction). If a type appears dark from the side camera, it has a degraded or no plastron (i.e. deemed unacceptable). Although not used to determine the acceptable and unacceptable plastron, the rear camera helped us understand how the plastron is morphed inside the trench by differentiating the depinned interface from no interface along the trench length.

3.2. Preparation of SHPo surface samples

A series of SHPo surface samples were prepared, as shown in figure 5. To test different Laplace pressure limitations, 3 different roughness types of longitudinal trenches shown in figure 5(a) were prepared. The first roughness type was micro-trenches with a re-entrant shape at the top edge of the trench (named RE), which is the type used by



Figure 5. The SHPo samples prepared for the experimental verification. (a) Illustration of 3 different trench types depending on the edge shape and surface roughness. The SEM pictures reveal the top edges of the cross-cleaved trenches as well as the nano-grass. (b) Each sample carries 10 parallel sections each containing 30 or 42 trenches. All trenches in this study have a gas fraction w/p = 0.9. The 40 mm × 70 mm sample has a 30 mm × 60 mm micromachined surface surrounded by a smooth surface. The micromachined region has repeated sections of longitudinal trenches with $p = 75 \,\mu\text{m}$ (drawn blue) and $p = 100 \,\mu\text{m}$ (drawn red) combined with L = 2.5, 5, 10, 30, 60 mm. The inset SEM picture shows a spanwise divider which partitions a 60 mm trench into shorter trenches. The same arrangement was used for all the 12 samples (3 roughness types × 4 trench depths), providing 120 different trench geometries with one photomask. The SEM pictures of cleaved samples show the trenches of two different pitches and one depth $d = 67.5 \,\mu\text{m}$.

Xu *et al.* (2020*b*). The second roughness type was micro-trenches without a re-entrant edge but covered with nano-grass (named NG). The third roughness type had both the re-entrance and nano-grass (named RE + NG). For each roughness type, 4 different trench depths were prepared, making a total of 12 samples (40 mm \times 70 mm in size) each diced out from a 4 inch silicon wafer. Since there are 10 different combinations of trench widths and lengths on each sample, we may use a descriptive name for each trench geometry. For example, NG_d90-p75L30 points to the section filled with trenches of 75 µm pitch and 30 mm length on the sample of the nano-grass (but no re-entrance) type and 90 µm trench depth.

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Type of surface	θ (deg.)	θ_a (deg.)	θ_r (deg.)	
FDTS-coated smooth Si FDTS-coated Al ₂ O ₃ nano-grass	$\begin{array}{c} 110\pm1\\ 166\pm1 \end{array}$	$\begin{array}{c} 116\pm3\\ 166\pm1 \end{array}$	$\begin{array}{c} 101\pm1\\ 165\pm2 \end{array}$	
Table 1. Contact angles of water on FDT	S-coated n	ano-grass an	d smooth su	face.

The micro-trenches were made on silicon wafer by developing 3 different fabrication processes of micro electro-mechanical systems (MEMS) based on photolithography, deep reactive ion etching (DRIE), and atomic layer deposition (ALD). For the 3 roughness types shown in figure 5(a), the first type (RE) was micro-trenches with re-entrance at the top edge of the trench. This type was used for the boat study by Xu et al. (2020b) and tested for a comparison in this study. The DRIE recipe was modified to create a \sim 250 nm of undercut below the \sim 500 nm thick silicon dioxide layer on top of trenches, thus creating the re-entrance, which is shown in the top scanning electron microscope (SEM) images of figure 5(a). The sawtooth-like sidewall below the re-entrance is by how DRIE works and should be considered smooth in nanometre scale. The second type (NG) was removed of the re-entrant edge by adding hydrofluoric wet etching after the DRIE. Following the wafer dicing, the surface was conformally coated with a \sim 55 nm thick Al₂O₃ layer by ALD and then immersed in a 60 °C deionized water bath for 10 minutes to roughen the Al₂O₃ into a nano-grass. The middle SEM picture of figure 5(a) shows the top edge with no re-entrance and the entire surfaces uniformly covered with nano-grass with ~ 100 nm of roughness. The third type (RE + NG) had both the re-entrance and nano-grass, as shown in the bottom SEM picture of figure 5(a), by omitting the hydrofluoric wet etching in the processing steps of the second type. For each of the three roughness types, 4 samples with increasing trench depths (i.e. $d = 50.6, 67.5, 90, 153 \,\mu\text{m}$) were prepared by increasing the etching time of DRIE. Hence, each of the 12 samples has a unique roughness type and trench depth. Once the trenches were formed, all the samples were cleaned by O_2 plasma and then coated uniformly with the self-assembled monolayer of 1H,1H,2H,2H-perfluorodecyltrichlorosilane (FDTS) in a custom-made vapour-based coater to achieve superhydrophobicity. The contact angles of water on FDTS-coated smooth silicon and Al₂O₃ nano-grass were measured with an in-house contact angle measurement apparatus and summarized in table 1. On each sample, trenches with a combination of 2 different pitches ($p = 75, 100 \,\mu\text{m}$) and

On each sample, trenches with a combination of 2 different pitches (p = 75, 100 µm) and 5 different lengths (L = 2.5, 5, 10, 30, 60 mm) were fabricated, as schematically shown in figure 5(b). A sample was cleaved along the vertical broken line drawn on the schematic to obtain the two SEM pictures (p75 and p100), which show the two different pitches. The gas fraction of all trenches was kept at 90%, i.e. w/p = 0.9. The 30 mm × 60 mm micromachined area in the middle was divided into 10 parallel sections each ~3 mm wide and containing 42 or 30 parallel trenches of $p = 75 \,\mu$ m (shaded blue) or 100 μ m (shaded red), respectively. The section width was, in part, designed based on the resolution of the side camera. To provide the 5 different trench lengths, 8 of the 10 parallel sections were further divided into multiple (2, 6, 12 or 24) shorter trenches, the top SEM showing one such partition. The smooth area (grey) outside the micromachined area (blue and red) was to prevent the flow disturbances by the gap between the sample and the surrounding plate, as observed by Xu *et al.* (2021). Since 12 different samples were fabricated to provide combinations of 3 roughness types (i.e. RE, NG and RE + NG) and 4 trench depths (i.e. d = 50.6, 67.5, 90, 153 μ m), a total of 120 different trench geometries have been prepared for flow experiments.

3.3. The flow experiments

To comprehensively compare the effects of hydrostatic pressure, air diffusion and shear stress on different SHPo samples, we performed all the flow tests in brackish water with air saturation level at 100 %-101 % in the mouth of a creek (Ballona Creek, Los Angeles, California, USA) meeting the Pacific Ocean. The air saturation level was monitored regularly by a total gas sensor (Point FourTM tracker, PENTAIR), and the specific testing area was determined for each test based on the air saturation level within the 2 mile range inside the creek. One end of the range was the creek's entry point into the ocean, where the air saturation level tended to be 104 %-106 % due to the wind and waves on the ocean, while the other end was the farthest upstream point allowed by the transportation rules, where the air saturation level was measured to be constantly below 99 %. The air saturation level gradually decreased away from the ocean but varied significantly by the tide and wind conditions, requiring us to measure the air saturation level regularly and often. At high tide, the ocean water would enter the creek, increasing the air saturation in the upstream end to as high as 100 %-101 %, while at low tide the ocean water would retreat from the creek, decreasing the air saturation in the downstream end (i.e. the entrance point) to as low as 99 %-100 %.

Each sample was tested with boat speeds varying from 2 to 7.2 m s⁻¹ with \sim 0.5 m s⁻¹ intervals. For each test, the boat remained stationary at first, then accelerated to the target speed in \sim 5 seconds and maintained the target speed for \sim 40 seconds for observation. The sample was kept under water during the entire test trial (typically 30–40 min), and its immersion depth was measured to be 0.15 ± 0.03 m for all tests. The boat was carefully trimmed (i.e. weight distributed carefully) to maintain a $\sim 3^{\circ}$ running (tilting) angle, measured by an inclinometer (H4A1-45 Inclinometer, RIEKER), and a constant waterline at all the target speeds. To estimate the shear stress on the SHPo surface for a given boat speed, a smooth 40 mm \times 70 mm silicon sample, diced from a 4 inch bare silicon water, was attached to the shear-stress sensor (Xu et al. 2020a) and its shear stress τ_{w0} was measured at different speeds multiple times. Based on the shear-stress versus speed data, we derived the relation between smooth surface shear stress τ_{w0} and boat speed U, using the power regression method, as summarized in Appendix D. The shear stresses on the SHPo surface were, then, estimated from that on the smooth surface from $\tau_w \sim 0.7 \tau_{w0}$, which was found in the previous research using similar surfaces and the same boat (Xu et al. 2020b). After the flow experiments, the samples were cleaned, dried, and examined under SEM to confirm their integrity including the nano-grass structures.

4. Results and discussions

4.1. Image pairs collected, plastron length measured and key trends confirmed

The images from the side and rear cameras were analysed as pairs to determine the state of plastron along the trench: (i) pinned or slightly depinned interface (i.e. $h/w \le 0.17$ in this study, limited by the underwater cameras availability), (ii) depinned interface (i.e. 0.17 < h/w < d/w) and (iii) no interface (i.e. h/w = d/w). The plastron length L_p was obtained by measuring the length of plastron in the first state. In other words, the depinned interface is excluded when defining L_p in this study. If a trench is filled with the plastron

Sustainability of the plastron

of the first state of interface (i.e. $h/w \le 0.17$) over the entire length (i.e. $L_p = L < L_{ss}$), the trench is deemed to have a pinned or slightly degraded plastron, which is acceptable for our interest of drag reduction. For all other cases (i.e. $L_p = L_{ss} < L$), the trench is deemed to have degraded or no plastron, which is unacceptable. We have analysed all the sample images obtained from the boat tests – a pair of images at each of ~10 different boat speeds for each of the 12 samples, i.e. a total of ~120 image pairs with each covering 10 different trench types, producing ~1200 data points of L_p . Among them, 4 sets of images for 4 selected flow speeds, with each set collecting the image pairs of all the 12 samples, are presented in figures 10–13 of Appendix E, where coloured outlines are often used on the two types ($p = 75 \,\mu$ m and 100 μ m) of 60 mm long trenches to assist readers in identifying the state of plastron.

Throughout the collected data, the plastron length L_p increased with trench depth d and decreased with trench width w (or pitch p) and boat speed U, as expected from the theory. Several sample images were selected in figure 6 to reveal key trends. The selected ones were more often RE samples because the loss of plastron was rare (i.e. difficult to spot trends) on NG and RE + NG samples. Figure 6(a) shows a rear-view picture of an RE sample with $d = 67.5 \ \mu \text{m}$ (i.e. RE d67.5) at $U = 5.5 \ \text{m s}^{-1}$. The image revealed trenches with $p = 75 \,\mu\text{m}$ had longer plastron than those with $p = 100 \,\mu\text{m}$ on 60 mm long trenches, indicating a stronger plastron stability on narrower trenches, as expected. Figure 6(b)presents 4 side-view pictures of 4 RE samples with $d = 153 \ \mu m$ (i.e. RE d153) taken at 4 different flows speeds ($U=3.8, 4.6, 5.5, 6.7 \text{ m s}^{-1}$). The images of 60 mm long trenches revealed pinned or slightly degraded plastron (i.e. $L_p = L < L_{ss}$) at speeds up to $U = 5.5 \text{ m s}^{-1}$ but degraded plastron (i.e. $L_p = L_{ss} < L$) at $U = 6.7 \text{ m s}^{-1}$, indicating weakened plastron stability at higher flow speeds, as expected. Incidentally, most of the 60 mm long trenches on RE sample (i.e. RE d153-p75L60 and RE d153-p100L60) were found maintaining a pinned or slightly degraded plastron up to $U=5.5 \text{ m s}^{-1}$ corroborating the existence of plastron reported in Xu et al. (2020b). Figure 6(c)presents 4 side-view pictures of 4 RE + NG samples with 4 different trench depths (i.e. $RE + NG_{d50.6}$, $RE + NG_{d67.5}$, $RE + NG_{d90}$, and $RE + NG_{d153}$) at a high speed $(U = 6.3 - 6.7 \text{ m s}^{-1})$. While the depinning of interfaces by high shear stress was apparent on shallow trenches ($RE + NG_{d50.6}$), the degraded plastron on the front region of the trench was shortened and disappeared with increasing trench depth, as predicted by the theory.

Figure 6(d) presents 3 pairs of pictures taken from 3 samples of different roughness types with $d = 90 \,\mu\text{m}$ at a high speed ($U = 6.4-6.7 \,\text{m s}^{-1}$). On the sample with re-entrance but without nano-grass (e.g. RE_d90), most trenches had regions of no plastron. In comparison, on the samples with nano-grass regardless of re-entrance (e.g. NG_d90 and $RE + NG_{d90}$, nearly all trenches were found to have a pinned or slightly degraded plastron, demonstrating the effectiveness of adding nano-grass to the micro-trench. Incidentally, note the RE sample was populated with no interface and pinned or slightly depinned interface but no depinned interface. The same behaviour was true for all other RE samples, as shown in figures 10–13 of Appendix E. The lack of the depinned interface on RE was likely because once the meniscus is depinned from the top edge, where the re-entrance (on which $\theta_a \sim 180^\circ$, effectively) maximizes the Laplace pressure, the smooth sidewalls (on which $\theta_a \sim 116^\circ$) could not provide the same level of Laplace pressure, letting the contact line slide down quickly to the fully wetted state (i.e. no interface). On the other hand, while the region of no interface was negligible on the NG and RE + NGsamples, depinned interface were found on shallow trenches at high speeds, as shown in figures 12 and 13 of Appendix E. The depinned interface was likely because the rough

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Figure 6. Sample images for key trends. Some regions are colour-outlined to help identify the plastron states, which were determined using the corresponding image pairs in Appendix E. Blue, yellow and red indicate the pinned or slightly depinned interface (i.e. $h/w \le 0.17$), depinned interface (i.e. 0.17 < h/w < d/w) and no interface (i.e. h/w = d/w), respectively. (a) The effect of trench width w shown by the side camera. Narrower trenches maintained the plastron better. (b) The effect of shear stress τ_w shown by the side camera. Slower flows maintained the plastron better. (c) The effect of trench depth d shown by the side camera. Deeper trenches maintained the plastron better. (d) The effect of nano-grass shown by the two cameras. For each pair of images, the top image was taken by the side camera, and the bottom image was taken by the rear camera. While the plastron was lost significantly on RE at this high flow speed ($U = 6.4-6.7 \text{ m s}^{-1}$), a pinned or slightly degraded plastron was found for all trenches on NG and RE + NG, demonstrating the effectiveness of adding nano-grass. (e) Effects of dynamic water pressure and interfacial contamination shown by the rear camera. Regions with trench length L = 2.5 mm, 10 mm, and 60 mm are outlined. The inset picture shows the pinned or slightly depinned interfaces at the front end of the 60 mm trenches.

sidewalls (on which $\theta_a \sim 166^\circ$) provided a similarly large Laplace pressure as the top edge. In other words, the nano-grass, while increasing the plastron stability, especially calls for an appropriate observation method, such as the two-camera system used in this study, to detect the degraded plastron, which may otherwise be interpreted as a pinned or no plastron.

4.2. Deviations from the linear increase of air pressure along a trench

Recall § 2.4, which discussed the additional effects that may cause the plastron morphology to deviate from the trend of linearly decreasing pressure difference along the trench. The magnitude of pressure difference is expected to be smaller at the rear end than at the front end, as depicted in figure 2(c), because $P_{water} > P_{air}$ in the current experimental conditions. First, for an example, as shown on the 60 mm length trenches

in figure 6(e) (i.e. p75L60, p100L60), while a significant portion of the front region had no plastron, the very front end was found to have a plastron. This is a deviation from the linear theory, which predicts the pressure difference increasing toward the front of trench. We believe that this small but interesting deviation from the front wetting can be explained by the pressure of the flowing water decreasing right past the front end, as depicted in figure 2(d) and supported by figure 8(c) of Appendix C. Second, throughout the collected images, including figure 6(e), the plastron was frequently found to be lost at the rear end. This is a deviation from the linear theory, which predicts the pressure difference decreasing toward the rear of trench. We believe this deviation, which we will call 'rear wetting', may be partially explained by the water pressure increasing near the rear end, as explained with figure 2(d) and supported by figure 8(c) of Appendix C. However, the deviation at the rear end was found to be more common and more pronounced than the deviation at the front end. For example, rear wetting was observed on all trenches of all RE samples at $U > 4.6 \text{ m s}^{-1}$ and some trenches on NG and RE + NG samples, as shown in figures 10–13 of Appendix E. The stronger deviation at the rear end may be explained by the pressure increase by the dynamic flow exasperated by the negative effects of interfacial contaminants, as explained in § 2.4. Also, the rear wetting was not directly affected by the trench length, making its wetting effect more significant on shorter trenches. For an example, on the RE_d67.5 sample shown in figure 6(e), the rear wetting had a relatively small effect (<5%) on p75L60, but a large effect ($\sim50\%$) on p75L2.5. In addition, the rear wetting tended to be more significant on deeper trenches, possibly because the trapped air there was more compressible and provided less dynamic resistance against depinning. In any case, the rear wetting was found to be \sim 4 times shorter on NG and RE + NG than on RE, manifesting another significant benefit of nano-grass for future applications.

The mechanism of rear wetting calls for a significant investigation in the future, as it seems inevitable for drag-reducing SHPo surfaces. As discussed in $\S 2.4$, the rear wetting may arise from the increased local water pressure when the boundary condition changes from slip to no slip at the trench end, combined with the stagnant cap formed by the surfactant (or particles) advected to the rear end. While the former would require numerical and experimental studies of hydrodynamic issues involving free surfaces, the latter would further involve diffusion and interfacial phenomena. To the best knowledge of the authors, Landel et al. (2020) was the only study so far that reported a stagnant-cap region on the SHPo trench. In their study, the Péclet number was the main non-dimensionalized parameter, $Pe = LU/D_I > 10^3$, where U is bulk velocity and D_I is the diffusion coefficient of the interfacial surfactant. Assuming a typical environmental surfactant SDS, which has $D_I = 7 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$, for our trenches, i.e. $L = O(10^{-3} - 10^{-2} \text{ m})$, and the maximum speed, i.e. $U = 7.2 \text{ m s}^{-1}$, $Pe = O(10^7 - 10^8)$, which suggests a stagnant-cap region at the rear end. Since the theory of Landel et al. (2020) only fits two regimes where the stagnant cap either covers the entire plastron or does not exist, additional advancement would be desired to estimate the distribution of stagnant cap on a SHPo trench, which is likely affected by the trench dimensions, water speed and surfactant concentration and properties, as discussed in the studies of stagnant cap on arising bubbles (He, Maldarelli & Dagan 1991; Dukhin et al. 2015).

4.3. Comparisons with the theoretically estimated steady-state plastron length

While further advanced analysis is necessary for unifying all the factors for the prediction of the plastron length in turbulent flow, for convenience here we prepare a preliminary estimation of the steady-state plastron length to compare with our experimental conditions,

where the leading end of trench tends to be depinned first. Based on the equilibrium state of the air pressure in static water, (2.4), and linear gradient due to the shear, (2.10), we estimate the air pressure to be in the scale of

$$\bar{P}_{air}(x) \sim sP_{atm} + \frac{2D}{1+2DN} \frac{c_{sl}w^2}{c_p d^3} \tau\left(x - \frac{L}{2}\right), \tag{4.1}$$

which shows the average air pressure increasing linearly in the x-direction. The difference between the water pressure on the trench and the air pressure in the plastron along the trench can be expressed based on (2.12) with the following trend:

$$\Delta P(x) \sim \bar{P}_{water} + P'_{water} - sP_{atm} - \frac{2D}{1+2DN} \frac{c_{sl}w^2}{c_p d^3} \tau \left(x - \frac{L}{2}\right) - P'_{air}, \qquad (4.2)$$

where $P'_{water} - P'_{air} \sim 0$ if the effect of turbulent fluctuation is small. Under the flow conditions of this study, we expect the leading end of trenches reaches the Laplace pressure limitation prior to the trailing end of trenches, leading to $\Delta P(0) = \Delta P_{\sigma,max}$, which leads to an estimated trend of the steady-state plastron length as

$$L_{ss} \sim [\Delta P_{\sigma,max} - \bar{P}_{water} + sP_{atm} - (P'_{water} - P'_{air})] \frac{1+2D}{D} \frac{c_p d^3}{c_{sl} w^2 \tau}, \qquad (4.3)$$

where, again, $P'_{water} - P'_{air} \sim 0$ if the effect of turbulent fluctuation is small. Note the deviation caused by the rear wetting was not included in (4.3). While the deterioration effect of the pressure fluctuation terms remains unclear at this point, the nano-grass coverage would certainly make the plastron more stable on NG and RE + NG, compared with RE used in the previous open-water studies (Xu *et al.* 2020*b*, 2021).

To qualitatively show the effects of nano-grass, trench dimensions and flow conditions (i.e. wall shear stress), the actual plastron lengths L_p on 60 mm long trenches were measured from all the images using ImageJ and plotted in figure 7, and the estimated theoretical plastron lengths L_{ss} from (4.3) were drawn as colour-shaded ranges in the same figure for comparison, accordingly showing similar trends. If $L_{ss} < L_{max} = 60$ mm, the interface at the front of the 60 mm long trench should be depinned, and the plastron length could be observed as $L_p = L_{ss}$. For the calculation of the theoretical estimation, the flow conditions of the experiments were used: air saturation level within s = 100 %–101 %, average water pressure as $P_{water} \sim 1500$ Pa, and the wall shear stress on the SHPo surface τ_w estimated from the boat speeds U measured using the regression equation in Appendix D. Besides, the pressure fluctuation term, i.e. $P'_{water} - P'_{air}$, was intentionally ignored in the estimation range to allow the comparison. By increasing the boat speed beyond the ones used by Xu et al. (2020b), which did not observe any shear-driven wetting, we have observed severely degraded plastron on the same RE sample. In comparison, the NG and RE + NG samples were confirmed to have a clearly improved plastron stability and showed a better matching between the estimated ranges and experimental results. Although the theoretically estimated range of plastron length on RE was similar to those on NG and RE + NG, the metastable state of the re-entrant edge on RE was vulnerable to the many fluctuations in the environmental water and the pressure fluctuation of the highly turbulent flows under the boat.

The rear wetting made the plastron shorter than the estimation by (4.3) on NG and RE + NG especially for d = 90, 153 µm, but the effect was small (< ~8 %). There was no significant difference between NG and RE + NG, as expected from the theory. It should be noted that, for simplicity, the theoretical wall shear stress on the SHPo surface τ_w was





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estimated from the wall shear stress on a smooth surface τ_{w0} by assuming 30% drag reduction for all the speeds, which was the typical drag-reduction value from the previous works for p100 (Xu *et al.* 2020b, 2021) with gas fraction w/p = 0.9 in turbulent boundary flows for $U > \sim 5 \text{ m s}^{-1}$. Although the theoretical wall shear stress should be 10–20 % larger than the estimated ones used in figure 7 for U < -5 m s⁻¹, this effect was expected to be ignorable because a larger L_{ss} would not change the fact that all the surfaces should have a pinned plastron (i.e. $L_p = L < L_{ss}$) at low speeds (i.e. $U < \sim 5 \text{ m s}^{-1}$) anyway due to the small wall shear stress. Besides, the theoretical shear stress for p75 should be 5 %-10 %larger than that on p100 in the same water flow due to the smaller pitch (Xu et al. 2021), increasing L_{ss} values for p75 surfaces by 5 %–10 %, while the experimental values will still fit the estimated values reasonably well. We note the theoretically estimated steady-state length L_{ss} of (4.3) was for pinned plastron, while the experimentally measured L_p was for both pinned and slightly degraded plastron due to the finite resolution of observation. Unfortunately, the shear-driven drainage model in the current form (Wexler et al. 2015; Liu et al. 2016) does not allow us to quantitatively estimate how a slight degradation of plastron would affect its steady-state length. However, we believe the effect was minor because a slightly degraded plastron is unstable with a very short lifetime in the current experiment, making its population small in the measured data. Most importantly, NG and RE + NGhave been demonstrated to maintain a pinned (including slightly degraded) plastron in the 60 mm long trench in turbulent boundary layer flows up to 7.2 m s^{-1} in accordance with the theoretical estimation, suggesting a direction toward high-performance SHPo surfaces for drag reduction.

5. Conclusion

To evaluate longitudinal micro-trench SHPo surfaces in high-speed flows of open water, which represent the operating conditions of common watercraft, we have studied how the sustainability of pinned plastron is affected by the pressure, air saturation level and wall shear of the water, and how the trends may be distorted by other factors, such as trench boundaries, surfactant and turbulent fluctuation. To model the effect of water pressure, an existing theory was used. To model the effect of wall shear stress of flowing water, another existing theory was used after a scale analysis revealed the diffusion of trapped air by the wall shear is small for the tested flow conditions. Distortions by the dynamic effect of flows were anticipated at the front and rear ends of the trench and corroborated by a numerical simulation. To evaluate the theoretical models and the distorting effects, micro-trench SHPo surfaces with combinatorial variations of trench width, trench depth, trench length and nano-roughness have been prepared and tested underneath a 13 foot motorboat in brackish water at a sea mouth. A unique observation technique using two underwater cameras was employed to differentiate pinned (and slightly degraded) plastrons from degraded (and no) plastrons rather than the common practice of determining whether the plastron is present or depleted. The experimental results corroborated the theoretical estimations reasonably well, considering the many assumptions in the models and the uncertainties inevitable in the field tests. When the trench surfaces were coated with nano-grass, nearly all the trenches tested were confirmed to have a pinned (or slightly degraded) plastron. This work contributed to designing SHPo surfaces geared toward field conditions for drag reductions, anti-biofouling, anti-corrosion, etc.

Supplementary movie. Supplementary movie is available at https://doi.org/10.1017/jfm.2023.184.

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Appendix A. Videos of plastron loss by shear drainage at high-speed flows

Underwater videos of the longitudinal trench SHPo surface used by Xu *et al.* (2020*b*) and tested at two different maximum speeds of boat are shown in Movie S1. The sample was filled with 7 cm long trenches made of re-entrant edges and smooth sidewalls (i.e. type RE by the designation of this report). For these close-up videos, two side cameras were used simultaneously (differently from the use of one side camera in the main study) to cover an entire sample. The top videos confirmed a pinned plastron being maintained at speeds up to 8 knots (4.1 m s^{-1}), which was near the maximum boat speed tested by Xu *et al.* (2020*b*), and the bottom videos showed the plastron being drained out by the shear stress at 10 knots (5.1 m s^{-1}) and completely lost at 13 knots (6.7 m s^{-1}), motivating the current study of developing the nano-grass-covered SHPo surfaces. Following the test procedures by Xu *et al.* (2020*b*) and unlike the current report, the air saturation level was not measured for this visualization.

Appendix B. Scaling comparison of the three air fluxes in a trench

The air diffusion across the air–water interface (meniscus) will lead to a diffusion-driven air flow inside the plastron. Since air diffusion rate across the meniscus varies with the Laplace pressure and meniscus area, the diffusion-driven flow flux scales as $q_d \sim k_p \sigma L$, where k_p is the interfacial mass transfer coefficient, σ is the air–water interfacial tension and L is the trench length. In turbulent boundary layer flows, k_p is defined by 'film theory' (Cussler & Cussler 2009) as

$$k_p = \frac{D_{air}}{\delta_c} \frac{M}{k_H \rho_{air}},\tag{B1}$$

where D_{air} is the diffusion coefficient of air (i.e. $1.75-2.00 \times 10^{-5} \text{ cm}^2 \text{ s}^{-1}$); *M* is the molecular weight of air (i.e. 29 g mol⁻¹); ρ_{air} is the density of air (i.e. 0.0012 g cm⁻³); δ_c is diffusion length, which depends on the flow condition. Henry's constant k_H (i.e. $1.21-1.34 \text{ atm mM}^{-1}$) is irrelevant to the hydrostatic pressure unless the immersion depth is very large (e.g. k_H increases ~14 % at immersion depth ~1000 m Enns, Scholander & Bradstreet 1965). Ling *et al.* (2017) studied the effect of Reynolds number on diffusion length in turbulent boundary layer flow over a SHPo surface and found the relation between the Sherwood number $SH_{\Theta 0} = 0.34Re_{\tau 0}^{0.913}$. Here, $SH_{\Theta 0} = \Theta_0/\delta_c$, where Θ_0 is the momentum boundary layer thickness on a smooth surface and approximated to be $\Theta_0/\hat{x} = 0.01277Re_x^{-0.1341}$ (Nagib, Chauhan & Monkewitz 2007) for the purpose of scaling

$U ({ m m \ s^{-1}})$	$\hat{x}(\mathbf{m})$	Re_x	$\Theta_0 (10^{-3} \text{ m})$	$Re_{\tau 0}$	$\delta_c \ (10^{-6} \text{ m})$
2.0	2.3	$5.17 imes 10^6$	3.70	3255	6.75
7.2	1.5	1.10×10^7	2.21	6549	1.93

 Table 2. Approximation for parameters on boat tests. Here, U and x were measured from experiments, and other parameters were estimated from theoretical equations.

estimation, with the Reynolds number defined as $Re_x = U\hat{x}/v$, where \hat{x} is the streamwise distance from the leading edge of immersed boat hull to the sample surface (\hat{x} is labelled as ~2 m in figure 5*a*). Since the boat is tilted and elevated (by planing) when it speeds up, i.e. \hat{x} decreases slightly with *U*, the tilting angle and waterline of the boat were measured for all the individual experiments to estimate \hat{x} for each test run. For the purpose of estimating friction Reynolds number $Re_{\tau 0} = \delta_0(\tau_{w0}/\rho)^{0.5}/v$, the boundary layer thickness on the smooth surface δ_0 can be approximated to be $\delta_0/\hat{x} = 0.16Re_x^{-1/7}$ (White & Majdalani 2006) for the purpose of scaling estimation, and the shear stresses on smooth surface τ_{w0} were established in Appendix D. In the turbulent boundary layer flow under the boat set-up, using the relations above, we can estimate δ_c at the minimum and maximum Reynolds numbers ($Re_x = 5.17 \times 10^6$ and 1.10×10^7 , corresponding to the minimum and maximum boat speed U = 2.0 m s⁻¹ and 7.2 m s⁻¹) to be 6.75 and 1.93 μ m, respectively. The parameters estimated using the relations above are listed in table 2. By inputting these values in (B1), we obtain $k_p = 0.5 \times 10^{-10} - 1.6 \times 10^{-10}$ m (s Pa)⁻¹.

By considering micro-trenches with aspect ratio d/w = 1 and gas fraction w/p = 0.9, based on (2.6) and (2.7), the shear-driven flow and the pressure-driven flow scale as $q_{sl} \sim 10^{-3} \tau w^3 \mu_{air}^{-1}$ and $q_p \sim 10^{-2} h^3 \sigma \mu_{air}^{-1} L^{-1}$, respectively. Then, we can estimate the magnitude of each flux inside the trench by substituting the exemplary values into the scaling equations. By assuming (i) the trench has width and depth of $w = h = 90 \ \mu m$ and length of $L = 60 \ mm$ and (ii) the shear stress on the SHPo surface is $\tau_w \sim 50 \ Pa$, the scaling equations lead to $q_d/q_{sl} \sim O(10^{-4})$ and $q_d/q_p \sim O(10^{-4})$. Therefore, we conclude that, for the trench geometries and flow conditions relevant to the current study, the diffusion-driven flow is negligibly small compared with the shear-driven flow and pressure-driven flow.

Appendix C. Numerical simulation for water pressure on and air pressure in a trench

To check the quantitative relevance of the deviations of water pressure P_{water} and air pressure P_{air} anticipated in figure 2(*d*), a numerical simulation of a two-phase turbulent boundary layer flow was performed on the three-dimensional modelled domain using the unsteady Reynolds-averaged Navier–Stokes (RANS) technique and volume of fluid (VOF) multiphase model provided by ANSYS Fluent 17.1 (Ansys Inc, PA, USA). The geometric configuration of the numerical simulation is shown in figure 8(*a*) with the boundary conditions. Here, the notation is such that \hat{x} , \hat{y} and \hat{z} denote the streamwise, wall-normal and spanwise coordinates, and corresponding time-averaged velocity components are *u*, *v* and *w*. To trigger a bypass transition from a laminar to turbulent flow, a rectangular rod is introduced near the inlet. The computational domain sizes in the streamwise, wall-normal and spanwise directions were 0.1202 m, 0.02 m and 7.5 × 10⁻⁵ m, respectively, with corresponding grid numbers of 2404, 151 and 30. For a trench that is positioned



Figure 8. Three-dimensional simulation for a turbulent flow of water over a micro-trench filled with air, assuming a flat air-water interface. (a) Schematic of the computational domain and boundary conditions. (b) Velocity vector superimposed with the volume fraction contour (the axes are not scaled). The blue and red contours on the $\hat{x}\hat{y}$ -plane indicate the water and the air phases, respectively. (c) Mean pressure and (d) root mean square of the turbulent pressure fluctuations for the water and air right above and below the interface $(\hat{y} = \hat{y}_{interface} \pm 1.0 \times 10^{-6} \text{ m})$, respectively, or at the wall ($\hat{y} = 0$). The water and air pressure variations along the trench corroborate figure 2(d).

at $\hat{x} = 0.0602$ m away from the inlet, the sizes in the streamwise, wall-normal and spanwise directions were 0.01 m, 5.06×10^{-5} m and 6.75×10^{-5} m, respectively, with corresponding grid numbers of 200, 41 and 27. The governing equations for the unsteady RANS simulations including VOF model were discretized through the finite volume method with second-order central difference scheme and fully implicit second-order temporal discretization, and a pressure-based solver was adopted for an incompressible flow with the $k-\omega$ shear-stress transport turbulence model (Menter 1994).

Assuming $U = 2 \text{ m s}^{-1}$ and $P_{atm} = 101.3$ kPa, figure 8(b) shows the time-averaged velocity vector superimposed with the void fraction contour α on the $\hat{x}\hat{y}$ -plane that are extracted in the middle of the trench ($\hat{z} = 3.75 \times 10^{-5} \text{ m}$). The air–water interface stays almost flat along the trench. Over the trench, a slip of the water flow (here, the spatially averaged slip velocity is estimated to be 1.37 m s^{-1}) is clearly observed, and the backflow is generated in the air within the trench by the slip of the interface. Finally, figure 8(c) shows the mean water and air pressure right above and below the interface, respectively. The decrease and increase of the mean water pressure near the front and rear ends of





Figure 9. Wall shear stress on a smooth surface τ_{w0} at different boat speeds. The experimental data fit the power regression line.

the trench, respectively, and the linear increase of the mean air pressure along the trench confirm the trend anticipated in figure 2(d). The decrease of the mean water pressure at the front end is caused by the flow acceleration as water flows from the no-slip surface to the slip interface. On the other hand, the increase of the mean water pressure at the rear end is caused by the flow deceleration as water flows from the slip interface to the no-slip surface. The increase of the mean air pressure along the trench is caused by the flow deceleration as water flows from the slip interface to the no-slip surface. The increase of the mean air pressure along the trench is caused by the flow impingement on the rear wall. In figure 8(d), small turbulent pressure fluctuations are observed in the water on the interface, whereas no turbulent pressure fluctuation is observed in the air underneath the interface.

Appendix D. Experiments for shear stress vs. boat speed

The shears stress on a smooth surface τ_{w0} underneath the boat has been measured with the custom shear sensor (Xu *et al.* 2020*a*) over the range of boat speeds *U* used in the current study and plotted in figure 9. The power regression line, which fits the experimental data reasonably well (especially considering the varying environmental conditions the field tests are subjected to), is used to estimate the wall shear stress for the plastron observation runs. The power regression also ensures the shear stress is zero when the boat speed is zero. The shear stress on the SHPo surface, τ_w , is estimated as $0.7\tau_{w0}$, which assumes 30 % drag reduction. The estimation for 30 % drag reduction of SHPo surface was confirmed by previous work on the same boat (Xu *et al.* 2020*b*) and in a high-speed towing tank under similar flow condition (Xu *et al.* 2021). The ~30 % drag reduction has also been proven to be consistent by the authors' recent experiments (unpublished), which used the same flow conditions as this study.

Appendix E. Picture pairs of all the 12 samples taken underwater at 4 selected boat speeds



Figure 10. Sample images at speeds in the range $3.3-3.8 \text{ m s}^{-1}$. Blue indicates a pinned or slightly depinned interface (i.e. $h/w \le 0.17$); yellow indicates a depinned interface (i.e. 0.17 < h/w < d/w); and red indicates no interface (i.e. h/w = d/w).



Figure 11. Sample images at speeds in the range 4.4–5 m s⁻¹. Blue indicates a pinned or slightly depinned interface (i.e. $h/w \le 0.17$); yellow indicates a depinned interface (i.e. 0.17 < h/w < d/w); and red indicates no interface (i.e. h/w = d/w).

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Figure 12. Sample images at speeds in the range 5.2–5.7 m s⁻¹. Blue indicates a pinned or slightly depinned interface (i.e. $h/w \le 0.17$); yellow indicates a depinned interface (i.e. 0.17 < h/w < d/w); and red indicates no interface (i.e. h/w = d/w).



Figure 13. Sample images at speeds in the range 6.3–6.7 m s⁻¹. Blue indicates a pinned or slightly depinned interface (i.e. $h/w \le 0.17$); yellow indicates a depinned interface (i.e. 0.17 < h/w < d/w); and red indicates no interface (i.e. h/w = d/w).

Appendix F. Experimental data

The experimental data is shown in table 3. Due to the limitations in the field experiments and as commonly done in the literature (Fukagata *et al.* 2006; Busse & Sandham 2012; Ling *et al.* 2017), we used the estimated parameters based on flow conditions of smooth surface. The wall shear stress τ_{w0} , at different boat speeds of different tests based on figure 9, was used for the calculation of the wall friction velocity $u_{\tau0} = (\tau_{w0}/\rho)^{1/2}$. The non-dimensionalized length scale quantities were normalized using wall unit $\delta_{v0} = v(\tau_{w0}/\rho)^{-1/2}$; for example, $w^{+0} = w/\delta_{v0}$. The wetting length of the boat, which is the distance between the sample and the leading end of the immersed hull, \hat{x} , was estimated based on the boat tilting angle and immersion depth.

0	p100	5469 6187	7557	7934	9415	7375	4828	6724	3318	3929	1122	2080	0	0	7601	7745	9063	9951	9740	6871	8003	3627	5324	2007	3006	1627	4744	5961	7004	7833	8097	8778	
L_p^+	p75	5469 6187	7557	7934	9415	10069	8509	9866	7291	8234	4870	5949	4351	3666	7601	7745	9063	11155	12237	12464	13141	8963	10514	7832	8876	7622	4744	5961	6953	7932	8009	8730	
	ange	18295	13322	12650	10681	8668	8204	7945	7641	7419	7087	6975	6625	6480	22277	21774	18648	15177	13805	12657	12546	11490	11375	11099	10881	10780	59100	46997	39654	34139	32254	30735	
0.	p1001	2080 1849	1515	1438	1215	1023	933	903	869	844	806	793	753	737	2533	2476	2121	1726	1570	1439	1427	1307	1293	1262	1237	1226	6720	5344	4509	3882	3668	3495	
$L_{\rm ss}^+$	ange	33831	24635	23391	19752	16639	15171	14691	14129	13720	13104	12898	12252	11983	40528	39611	33926	27610	25115	23027	22824	20904	20693	20192	19795	19612	104695	83254	70246	60476	57137	54446	
	p75 rä	11800	9747	9255	7815	6583	6002	5812	5590	5428	5185	5103	4847	4741	16035	15672	13423	10924	9937	9110	9030	8270	8187	7989	7832	7759	41422	32939	27793	23927	22606	21541	
	T^{+0}	5469 6187	7557	7934	9415	11175	12237	12630	13156	13513	14189	14385	15176	15496	7601	7745	9063	11155	12237	13369	13484	14686	14866	15202	15509	15693	4744	5961	7041	8190	8676	9076	lext page.
	d^{+0}	4.61 5 22	6.37	6.69	7.94	9.42	10.32	10.65	11.09	11.40	11.97	12.13	12.80	13.07	8.55	8.71	10.20	12.55	13.77	15.04	15.17	16.52	16.72	17.10	17.45	17.65	7.12	8.94	10.56	12.28	13.01	13.61	aption see r
0+	p100	8.20 9.78	11.33	11.90	14.12	16.76	18.36	18.95	19.73	20.27	21.28	21.58	22.76	23.24	11.40	11.62	13.59	16.73	18.35	20.05	20.23	22.03	22.30	22.80	23.26	23.54	7.12	8.94	10.56	12.28	13.01	13.61	3. For c
M	p75	6.15 6 96	8.50	8.93	10.59	12.57	13.77	14.21	14.80	15.20	15.96	16.18	17.07	17.43	8.55	8.71	10.20	12.55	13.77	15.04	15.17	16.52	16.72	17.10	17.45	17.65	5.34	6.71	7.92	9.21	9.76	10.21	Table
0-	p100	9.12 10.31	12.59	13.22	15.69	18.63	20.40	21.05	21.93	22.52	23.65	23.98	25.29	25.83	12.67	12.91	15.10	18.59	20.39	22.28	22.47	24.48	24.78	25.34	25.85	26.16	7.91	9.93	11.74	13.65	14.46	15.13	
P^+	p75	6.84 7 73	9.45	9.92	11.77	13.97	15.30	15.79	16.44	16.89	17.74	17.98	18.97	19.37	9.50	9.68	11.33	13.94	15.30	16.71	16.86	18.36	18.58	19.00	19.39	19.62	5.93	7.45	8.80	10.24	10.85	11.35	
	$We_{\tau 0}(\times 10^{-4})$	12.72 14.30	17.58	18.46	21.90	26.00	28.47	29.38	30.61	31.44	33.01	33.47	35.31	36.05	17.68	18.02	21.08	25.95	28.47	31.10	31.37	34.17	34.58	35.37	36.08	36.51	11.04	13.87	16.38	19.05	20.18	21.12	
	$Re_{ au 0}$	3748 4160	4954	5167	5677	6114	6305	6363	6428	6465	6519	6531	6560	6564	4979	5061	5571	6110	6305	6451	6462	6545	6552	6561	6564	6564	3315	4037	4661	5282	5448	5576	
	$Re_{\delta 0}(\times 10^{5})$	1.04	1.35	1.41	1.53	1.64	1.69	1.70	1.71	1.72	1.73	1.73	1.74	1.74	1.36	1.38	1.51	1.64	1.69	1.72	1.72	1.74	1.74	1.74	1.74	1.74	0.92	1.11	1.27	1.43	1.48	1.51	

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0-	p100	10852	10828	9935	9591	10049	10884	11977	5786	4768	4060	8111	6287	7251	7988	4816	4856	6785	7024	8055	8333	8961	10063	10475	10009	10390	11338	11249	
L_p^+	p75	11423	12131	12201	12468	12899	12078	12542	12118	12197	12491	12856	12807	12253	13972	4816	4856	6785	7024	8263	8357	9195	10290	10778	10884	11692	12489	13174	
	range	24065	22124	21641	20144	19648	19414	18734	18197	18024	17810	17506	17362	17229	17046	135507	135370	96324	93349	79420	78558	71183	63729	60445	59641	53116	49601	46068	
-0	p1001	2736	2516	2461	2291	2234	2208	2130	2069	2049	2025	1991	1974	1959	1938	15409	15393	10953	10615	9031	8933	8094	7247	6873	6782	6040	5640	5238	
$L_{s,}^+$	ange	42631	39193	38337	35685	34807	34391	33188	32237	31929	31550	31011	30757	30521	30196	224592	224364	159649	154718	131632	130204	117981	105625	100182	98850	88036	82209	76354	
	p75 r	16867	15507	15168	14119	13771	13607	13131	12754	12632	12483	12270	12169	12075	11947	88859	88769	63164	61214	52080	51515	46679	41790	39637	39110	34831	32526	30209	
	L^{+0}	11605	12637	12916	13875	14228	14385	14896	15350	15484	15675	15968	16072	16230	16386	4816	4856	6785	7024	8263	8357	9195	10290	10844	10976	12326	13227	14228	next page.
	d^{+0}	17.41	18.96	19.37	20.81	21.34	21.58	22.34	23.03	23.23	23.51	23.95	24.11	24.35	24.58	12.28	12.38	17.30	17.91	21.07	21.31	23.45	26.24	27.65	27.99	31.43	33.73	36.28	aption see
0+	p100	17.41	18.96	19.37	20.81	21.34	21.58	22.34	23.03	23.23	23.51	23.95	24.11	24.35	24.58	7.22	7.28	10.18	10.54	12.40	12.53	13.79	15.43	16.27	16.46	18.49	19.84	21.34	3. For ca
M	p75	13.06	14.22	14.53	15.61	16.01	16.18	16.76	17.27	17.42	17.63	17.96	18.08	18.26	18.43	5.42	5.46	7.63	7.90	9.30	9.40	10.34	11.58	12.20	12.35	13.87	14.88	16.01	Table
0+	p100	19.34	21.06	21.53	23.12	23.71	23.97	24.83	25.58	25.81	26.13	26.61	26.79	27.05	27.31	8.03	8.09	11.31	11.71	13.77	13.93	15.32	17.15	18.07	18.29	20.54	22.04	23.71	
P	p75	14.51	15.80	16.14	17.34	17.79	17.98	18.62	19.19	19.36	19.59	19.96	20.09	20.29	20.48	6.02	6.07	8.48	8.78	10.33	10.45	11.49	12.86	13.55	13.72	15.41	16.53	17.79	
	$We_{\tau 0}(\times 10^{-4})$	27.00	29.40	30.05	32.28	33.10	33.47	34.66	35.71	36.02	36.47	37.15	37.39	37.76	38.12	11.20	11.30	15.78	16.34	19.22	19.44	21.39	23.94	25.23	25.53	28.68	30.77	33.10	
	$Re_{ au 0}$	6198	6364	6400	6497	6522	6531	6553	6563	6564	6564	6561	6559	6554	6549	3359	3383	4514	4651	5308	5340	5612	5913	6043	6072	6319	6436	6522	
	$Re_{\delta 0}(\times 10^5)$	1.66	1.70	1.71	1.73	1.73	1.73	1.74	1.74	1.74	1.74	1.73	1.73	1.73	1.73	0.93	0.94	1.24	1.27	1.44	1.45	1.52	1.59	1.62	1.63	1.69	1.71	1.73	

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0-	p100	10526	7722	2751	9451	5900	6816	8237	8329	9914	8013	8217	8283	7705	6909	6708	6910	5071	5291	7037	7332	8665	9774	11181	11188	10509	11065	
L_p^+	p75	13210	12938	11646	13652	12724	6816	8237	8329	10209	11160	11380	11066	10632	9118	9712	9492	5071	5291	7037	7332	8665	9774	11181	11188	12066	13062	
	range	44470	43082	41864	41750	41369	12668	10492	10376	8447	7497	7399	1669	6555	6161	6105	6078	28630	27389	20607	19771	16750	14859	12976	12983	12027	11138	
s -0	p100	5057	4899	4760	4747	4704	1063	880	870	60L	629	621	586	550	517	512	510	2401	2297	1728	1658	1405	1246	1088	1089	1009	934	
$L_{ m s}^+$	ange	73705	71405	69385	69198	68565	21298	17640	17445	14202	12605	12439	11753	11021	10359	10264	10220	47356	45303	34085	32702	27706	24578	21463	21475	19893	18422	
	p75 r	29161	28251	27452	27378	27127	9746	8073	7983	6499	5768	5692	5378	5043	4740	4697	4677	21671	20732	15598	14965	12679	11247	9822	9827	9103	8430	
	L^{+0}	14750	15216	15668	15672	15857	6816	8237	8329	10209	11513	11685	12355	13169	14013	14166	14198	5071	5291	7037	7332	8665	9774	11181	11188	12066	13062	next page.
	d^{+0}	37.61	38.80	39.95	39.96	40.44	5.71	6.91	6.98	8.56	9.65	9.80	10.36	11.04	11.75	11.88	11.90	5.70	5.95	7.92	8.25	9.75	11.00	12.58	12.59	13.57	14.70	aption see
-0	p100	22.13	22.82	23.50	23.51	23.79	10.22	12.36	12.49	15.31	17.27	17.53	18.53	19.75	21.02	21.25	21.30	7.61	7.94	10.56	11.00	13.00	14.66	16.77	16.78	18.10	19.59	3. For ca
Å	p75	16.59	17.12	17.63	17.63	17.84	7.67	9.27	9.37	11.49	12.95	13.15	13.90	14.82	15.77	15.94	15.97	5.70	5.95	7.92	8.25	9.75	11.00	12.58	12.59	13.57	14.70	Table
-0	p100	24.58	25.36	26.11	26.12	26.43	11.36	13.73	13.88	17.02	19.19	19.48	20.59	21.95	23.36	23.61	23.66	8.45	8.82	11.73	12.22	14.44	16.29	18.64	18.65	20.11	21.77	
P^{-}	p75	18.44	19.02	19.58	19.59	19.82	8.52	10.30	10.41	12.76	14.39	14.61	15.44	16.46	17.52	17.71	17.75	6.34	6.61	8.80	9.16	10.83	12.22	13.98	13.98	15.08	16.33	
	$We_{\tau0}(\times 10^{-4})$	34.31	35.40	36.45	36.46	36.89	15.86	19.16	19.38	23.75	26.78	27.18	28.74	30.64	32.60	32.95	33.03	11.80	12.31	16.37	17.06	20.16	22.74	26.01	26.03	28.07	30.39	
	$Re_{ au 0}$	6548	6561	6564	6564	6562	4532	5298	5331	5893	6181	6213	6323	6430	6507	6518	6520	3511	3643	4659	4827	5444	5779	6115	6117	6278	6417	
	$Re_{\delta 0}(\times 10^5)$	1.74	1.74	1.74	1.74	1.74	1.24	1.44	1.45	1.59	1.66	1.66	1.69	1.71	1.73	1.73	1.73	0.97	1.01	1.27	1.32	1.48	1.56	1.64	1.64	1.68	1.71	

Sustainability of the plastron

			P^{\perp}	0+	2	0+				L_{ss}^+	0-0		Γ_{b}^{+}	0
$Re_{\delta 0}(\times 10^5)$	$Re_{ au 0}$	$We_{\tau0}(\times10^{-4})$	p75	p100	p75	p100	d^{+0}	L^{+0}	p75 1	range	p1001	range	p75	p100
1.73	6515	32.87	17.66	23.55	15.90	21.20	15.90	14131	7783	17007	862	10282	12205	5643
1.73	6561	37.15	19.96	26.61	17.96	23.95	17.96	15968	6890	15056	763	9103	12226	3519
1.73	6560	37.18	19.98	26.64	17.98	23.97	17.98	15981	6876	15025	762	9084	12867	4547
0.96	3463	11.61	6.24	8.32	5.61	7.48	7.48	4989	35596	77784	4051	48295	4989	4989
1.11	4030	13.84	7.44	9.91	69.9	8.92	8.92	5948	29730	64967	3383	40337	5948	5948
1.14	4135	14.26	7.66	10.22	6.90	9.19	9.19	6129	28851	63046	3283	39144	6129	6129
1.33	4874	17.25	9.27	12.36	8.34	11.12	11.12	7414	23925	52282	2723	32461	7414	7414
1.40	5154	18.40	9.89	13.18	8.90	11.87	11.87	7910	22381	48908	2547	30366	7910	7910
1.44	5303	19.19	10.31	13.75	9.28	12.38	12.38	8250	21439	46849	2440	29088	8250	8250
1.58	5883	23.66	12.71	16.95	11.44	15.25	15.25	10170	17419	38065	1982	23634	10170	10170
1.63	6072	25.53	13.72	18.29	12.35	16.46	16.46	10976	16125	35237	1835	21878	10366	10235
1.63	6078	25.60	13.75	18.34	12.38	16.50	16.50	11002	16096	35174	1832	21838	11002	10340
1.65	6164	26.58	14.28	19.04	12.85	17.14	17.14	11427	15515	33903	1766	21050	10917	10785
1.68	6303	28.44	15.28	20.37	13.75	18.33	18.33	12223	14515	31718	1652	19693	11773	11009
1.69	6321	28.70	15.42	20.56	13.88	18.51	18.51	12339	14376	31415	1636	19505	11717	10787
1.72	6479	31.79	17.08	22.77	15.37	20.49	20.49	13663	12966	28333	1475	17591	13209	12907
1.73	6497	32.28	17.35	23.13	15.61	20.81	20.81	13877	12771	27908	1453	17328	11407	9688
1.73	6518	32.97	17.71	23.62	15.94	21.26	21.26	14171	12487	27288	1421	16942	13463	12706
1.74	6547	34.24	18.40	24.53	16.56	22.08	22.08	14720	12037	26304	1370	16331	13810	12580
1.10	3986	13.66	7.34	9.79	6.61	8.81	14.98	5873	66394	145085	8075	96280	5873	5873
1.11	4045	13.90	7.47	9.96	6.72	8.96	15.23	5974	65006	142052	7907	94267	5974	5974
1.42	5205	18.62	10.00	13.34	9.00	12.00	20.40	8002	48534	106057	5903	70380	8002	8002
1.44	5312	19.25	10.35	13.79	9.31	12.41	21.10	8276	46918	102526	5707	68037	8276	8276
1.62	6043	25.22	13.55	18.07	12.20	16.26	27.65	10841	35843	78324	4360	51977	10841	10841
1.68	6272	27.98	15.04	20.05	13.53	18.04	30.67	12028	32312	70609	3930	46857	12028	11762
1.69	6341	29.02	15.59	20.79	14.03	18.71	31.81	12473	31169	68111	3791	45199	12386	12171
1.71	6427	30.59	16.44	21.91	14.79	19.72	33.53	13149	29558	64592	3595	42864	13039	12619
1.73	6507	32.59	17.51	23.35	15.76	21.02	35.73	14010	27721	60577	3372	40199	13585	12859
1.73	6515	32.87	17.66	23.55	15.90	21.20	36.03	14131	27497	60088	3344	39875	13955	13182
					Table	3 For ca	ntion see 1	lext naice						
					TUDI	J. 1 UI VI	huvu ave	ILAI Puëv.						

0	p100	12763	13402	5819	7323	8059	8822	9033	8357	7572	7555	7392	6007	6934	4850	5591	7302	7891	8768	10429	11442	12028	12232	12483	11634	12209	11283	
L_p^+	p75	14128	14169	5819	7323	8858	10130	10981	11115	10130	10133	9939	9083	9703	4850	5591	7302	7891	8768	10429	11845	12716	13494	13503	14239	15110	15166	
	range	39314	38733	15206	12094	9992	8541	7902	7608	6849	6299	6492	6462	5883	30711	26616	20420	18865	16996	14283	12572	11682	11020	11028	9919	9491	9312	
-0	p1001	3297	3249	1963	1561	1290	1102	1020	982	884	852	838	834	759	3964	3436	2636	2435	2194	1844	1623	1508	1423	1424	1280	1225	1202	
L_{ss}^+	ange	59243	58367	25566	20333	16800	14359	13286	12791	11515	11095	10915	10864	9891	50797	44023	33775	31203	28112	23625	20794	19323	18228	18240	16406	15699	15403	
	p75 r	27111	26710	12630	10045	8299	7094	6563	6319	5689	5481	5392	5367	4886	25094	21748	16686	15415	13888	11671	10273	9546	9005	9011	8105	7755	609L	
	L^{+0}	14351	14559	5819	7323	8858	10354	11188	11639	12916	13409	13609	13686	15015	4850	5591	7302	7891	8768	10429	11845	12716	13494	13503	14997	15686	15997	next page.
	d^{+0}	36.60	37.13	4.88	6.14	7.43	8.68	9.38	9.76	10.83	11.24	11.41	11.47	12.59	5.46	6.29	8.22	8.88	9.86	11.73	13.33	14.31	15.18	15.19	16.87	17.65	18.00	ption see
0+	p100	21.53	21.84	8.73	10.98	13.29	15.53	16.78	17.46	19.37	20.11	20.41	20.53	22.52	7.28	8.39	10.95	11.84	13.15	15.64	17.77	19.07	20.24	20.25	22.50	23.53	24.00	3. For ca
т, л	p75	16.15	16.38	6.55	8.24	96.6	11.65	12.59	13.09	14.53	15.09	15.31	15.40	16.89	5.46	6.29	8.22	8.88	9.86	11.73	13.33	14.31	15.18	15.19	16.87	17.65	18.00	Table
0+	p100	23.92	24.27	9.70	12.21	14.76	17.26	18.65	19.40	21.53	22.35	22.68	22.81	25.02	8.08	9.32	12.17	13.15	14.61	17.38	19.74	21.19	22.49	22.50	24.99	26.14	26.66	
P^{-}	p75	17.94	18.20	7.27	9.15	11.07	12.94	13.98	14.55	16.14	16.76	17.01	17.11	18.77	6.06	6.99	9.13	9.86	10.96	13.04	14.81	15.89	16.87	16.88	18.75	19.61	20.00	
	$We_{\tau 0}(\times 10^{-4})$	33.39	33.87	13.54	17.04	20.61	24.09	26.03	27.08	30.05	31.20	31.66	31.84	34.93	11.28	13.01	16.99	18.36	20.40	24.26	27.56	29.58	31.39	31.41	34.89	36.49	37.22	
	$Re_{\tau 0}$	6529	6540	3954	4822	5507	5929	6117	6204	6400	6455	6474	6481	6556	3379	3820	4810	5143	5478	5947	6241	6374	6463	6464	6556	6564	6560	
	$Re_{\delta 0}(\times 10^5)$	1.73	1.74	1.09	1.32	1.49	1.60	1.64	1.66	1.71	1.72	1.72	1.72	1.74	0.94	1.05	1.31	1.40	1.48	1.60	1.67	1.70	1.72	1.72	1.74	1.74	1.73	

Sustainability of the plastron

Transmetric (1.4) Disc Processing Procesing Processing Proces	metrol (N) page	Ğ		- <u>P</u>	+0 100	W Show	64	0+1	1+0	. 27.4	L_s^+	+0 s	00 40 H	L_p^+	-0
38.83 20.86 27.82 18.78 25.04 8.78 55.04 8.78 6600 7280 14.73 1150 8910 133.83 9302 13.57 7.23 8.46 1.23 8.46 1.23 8.46 1.23 8.46 1.240 5395 30247 8146 8146 8146 8146 8146 8146 8146 8146 8146 8146 8146 8146 8146 8146 8146 8146 8146 8146 9106 1222 11.23 11240 11241 15.79 10556 10518 10526	38.83 20.86 27.82 87.83 25.94 18.75 58.32 33.406 57.31 57.33 57.31 57.33 57.31 57.33 57.31 57.33 57.31 57.33 57.31 57.31 57.31 57.31 57.31 57.31 57.31 57.31 57.31 57.31 57.31 57.33 57.31 57.33 57.31 57.33 57.31 57.33 57.31 57.33 <	0	$e_{\tau 0}(\times 10^{-4})$	cjd	p100	c/d	p100	$q_{\perp 0}$	$\Gamma_{\pm 0}$		range	p100	range	c/d	p100
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	113.7 7.29 9.72 6.56 8.75 8.17 5332 53400 67791 5433 23606 7519 7511 7174 11276 112	9	38.83	20.86	27.82	18.78	25.04	18.78	16690	7280	14737	1150	8910	13848	9482
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	1749 9.40 12.53 8.46 11.28 7519 2605 3266 7519 7519 18.95 10.18 13.53 9.16 12.22 814.6 814.7 9055 9057 3055 30247 8146 8145 22.46 12.07 16.09 10.86 14.48 19.57 11.22 18.33 3054 9157 3153 9555 9555 9555 9555 9555 9555 9555 9555 9555 11712 11716 1176 11716 11716 1176 11716 11716 11716 11716 11716 11712 11712 11712 11712 11712 11712 11712 11712 11712 11712 11712 11716 11736	5	13.57	7.29	9.72	6.56	8.75	8.75	5832	33490	67791	5433	42090	5832	5832
18.95 10.18 13.58 9.16 1.2.22 1.2.26 1.2.27 1.2.27 1.2.26	18.95 10.18 13.58 9.16 12.22 12.22 8146 9557 9555 9555 9555 9555 9555 9555 9555 9555 9555 9555 9555 9555 9555 9555 9555 9555 9555 9555 9556 11726 11726 11726 11726 11726 11726 11726 11726 11726 11726 11726 11726 11726 11726 11726 11726 11726 11737 11712 11712 11712 11712 11712 11712 11712 11712 11712 11712 11713 11713 11713 11713 11713 11713 11713 11713 11713 11713 11713 <t< td=""><td>Э</td><td>17.49</td><td>9.40</td><td>12.53</td><td>8.46</td><td>11.28</td><td>11.28</td><td>7519</td><td>26016</td><td>52662</td><td>4221</td><td>32696</td><td>7519</td><td>7519</td></t<>	Э	17.49	9.40	12.53	8.46	11.28	11.28	7519	26016	52662	4221	32696	7519	7519
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	22.46 12.07 16.09 10.86 14.48 9655 20261 41014 3287 23.465 9655	9	18.95	10.18	13.58	9.16	12.22	12.22	8146	24067	48717	3905	30247	8146	8146
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	23.23 12.48 16.64 11.23 14.98 19.85 196/7 37791 31.83 2.4555 99.85 99.85 196/7 37791 30.72 2.34.56 10.756 10.756 10.756 10.756 10.757 11.712 11.725 11.712 11.713 11.716 11.7176 11.717 11.717	9	22.46	12.07	16.09	10.86	14.48	14.48	9655	20261	41014	3287	25464	9655	9655
24.49 13.16 77.54 11.84 15.79 15.79 10526 10526 10526 10526 10526 26.23 14.10 18.79 12.64 112.76 11276 11276 11276 11276 27.25 14.95 19.94 13.46 71.57 77.57 77.57 17.57 17.57 11712 27.25 14.95 19.94 13.66 21.15 21.16 21.15 21.15 21.15 21.15 21.15 21.15 21.15 27.35 17.62 23.50 15.86 21.15 21.16 11.961 13872 28080 2251 14.097 13591 35.51 19.62 26.15 77.65 23.54 15.692 13562 13562 13563 36.51 19.72 26.13 17.76 23.56 23.54 15.692 13766 15627 36.73 19.72 26.16 77.65 23.54 15.692 12497 25796 2027 14418 13778 36.73 19.72 26.91 16.47 7.07 9.43 16.47 7.07 9.43 16.47 7.14 9.5561 15692 13356 37.66 13.72 161.43 14874 151562 12984 1007 15692 13756 37.60 17.65 23.54 14.81 13772 14616 1452 5794 5794 5794 5794 17.52 91.41 12.52 <t< td=""><td>24.49 13.16 17.54 11.87 15.79 15.79 15.79 15.79 15.79 15.79 15.71 17.27 17.27 11.276 11.26 23.56 13.591 27.61 27.66 13.591 13.76 13.591 13.76 13.571 13.76 13.571 13.771 13.793 13.763 13.763 13.763 13.763 13.763 13.763 13.763 13.763 13.763 13.763 13.764 13.764 13.763 13.763 13.763 13.763 13.764 13.763 13</td><td>2</td><td>23.23</td><td>12.48</td><td>16.64</td><td>11.23</td><td>14.98</td><td>14.98</td><td>9985</td><td>19617</td><td>39709</td><td>3183</td><td>24655</td><td>9985</td><td>9985</td></t<>	24.49 13.16 17.54 11.87 15.79 15.79 15.79 15.79 15.79 15.79 15.71 17.27 17.27 11.276 11.26 23.56 13.591 27.61 27.66 13.591 13.76 13.591 13.76 13.571 13.76 13.571 13.771 13.793 13.763 13.763 13.763 13.763 13.763 13.763 13.763 13.763 13.763 13.763 13.764 13.764 13.763 13.763 13.763 13.763 13.764 13.763 13	2	23.23	12.48	16.64	11.23	14.98	14.98	9985	19617	39709	3183	24655	9985	9985
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	26.23 14.10 18.79 12.69 16.91 11.276 12.690 18.772 13.872 28.080 25.51 13.572 13.572 13.576 15.692 13.376 36.51 19.62 2.61 11.63 12.416 12.440 15.752 15.642 13.356 14.67 13.562 15.692 13.356 14.523 14.562 15.66 15.692 13.356 14.562 14.562 15.692	<u> </u>	24.49	13.16	17.54	11.84	15.79	15.79	10526	18639	37731	3024	23426	10526	10526
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	27.25 14.64 19.52 13.18 17.57 17.57 117.12 16753 33316 2.658 2.0559 11961 11961 27.83 14.95 19.94 13.46 17.94 11961 16384 333165 2.658 2.0592 11961 11961 33.54 18.02 2.4.03 15.28 2.1.15 2.1.15 2.1.15 2.1.15 2.1.15 1.1.401 13591 13591 33.54 18.02 2.4.03 15.2.8 2.1.65 17.65 2.3.54 15.692 13597 1643 14875 35.73 19.73 2.6.31 17.76 2.3.68 2.3.68 15.786 14.525 14875 14875 37.55 19.11 9.77 9.43 15.162 15.716 5732	5	26.23	14.10	18.79	12.69	16.91	16.91	11276	17349	35118	2815	21804	11276	11276
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	27.83 14.95 19.94 13.46 17.94 17.94 17.94 11961 16384 33166 2658 20592 11961 11391 32.80 17.62 24.03 16.22 21.15 21.15 14097 13872 28080 2251 17434 44097 13391 35.51 19.62 26.31 17.65 23.54 23.56 12497 13594 27516 15706 15492 13391 36.53 19.62 26.31 17.76 23.58 23.56 12497 13594 2015 1643 14875 37.55 20.18 26.43 8.57 14.38 12116 24525 1966 15527 16143 14875 13.30 71.44 9.53 56.34 7163 112.58 56.44 2057 56.44 2057 56.44 5716 5716 5732 7532 7532 7532 7532 7532 7532 7532 7532 7532 7532	2	27.25	14.64	19.52	13.18	17.57	17.57	11712	16753	33912	2718	21055	11712	11712
32.80 17.62 23.50 15.86 21.15 21.15 14097 13872 28080 2251 17434 14097 137591 33.54 18.02 24.03 16.22 21.63 21.63 14418 13594 27517 2205 17085 14418 13758 36.51 19.62 26.15 71.65 23.54 23.54 53.54 23.54 23.54 15692 13356 36.73 19.62 26.15 71.65 23.54 23.54 15692 123766 15692 13356 37.55 20.18 26.90 18.176 24.21 24.21 24.16 12412 24222 257144 20157 5116 5716 37.55 20.14 7.07 9.43 6.23 43.8 5716 14772 2916 15502 15786 14875 14.62 7.86 10.47 7.07 9.43 6.284 68171 137994 11821 91574 6284 6284 17.52 9.41 12.55 8.47 11.30 92.21 7532 77153 8080 8080 24.61 13.22 11.90 15.87 26.98 10976 16677 6794 7532 7532 18.80 10.10 13.47 9.09 12.12 20.50 8080 8080 70379 7079 70779 25.53 19.76 19.779 1076 16.779 1076 1076 1076 <td>32.80 17.62 23.50 15.86 21.15 21.15 14097 13872 28080 2251 17434 14097 13591 3358 33.54 18.02 24.03 16.22 21.63 14418 13584 27517 2205 17085 14418 13758 35.75 19.62 13562 13356 35.75 19.72 25.91 17.76 23.68 15786 15592 13356 15512 15706 15592 13356 15.52 20.18 26.90 18.16 23.58 5.716 3716 2442 15156 24525 1966 15527 16143 14875 13.30 7.14 9.53 6.43 8.571 4.58 12116 24525 1966 15527 15618 14572 13.30 7.14 9.53 6.43 8.571 4.58 5716 7483 12116 24525 1966 15527 15706 15692 13356 15.562 13.30 7.14 9.53 6.43 8.571 4.58 5716 7483 14875 13.30 7.14 9.53 6.43 8.571 4.58 5716 7483 12116 24525 1944 151562 12984 100578 5716 5716 14.62 7.10 9.21 753 5.576 107724 9.13 7794 1573 8080 8080 24.61 13.22 17.63 11.90 15.87 20.60 8080 53267 107824 9237 71553 8080 8080 25.55 13.72 18.29 12.35 16.46 27.99 10579 40556 82095 7033 54479 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10579 49156 7033 54479 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10576 49565 82095 7033 54479 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10576 49565 82095 7033 54479 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10576 49565 82095 7033 55477 1059 7033 554779 10579 25.53 13.72 13.22 17.51 13.57 18.10 30.76 12.064 35554 71969 6165 47760 12037 11938 23.53 11773 11750 23.00 1751 13577 138 23.11 77.4 20.99 14.17 18.89 32.12 12.596 3415 7963 5923 44583 11773 11750 23.30 1757 11588 23.316 17.38 23.17 15.64 20.89 10579 49563 5992 46418 12.315 12315 23139 23.32 1758 23.317 155.8 20.69 13.97 13663 23.303 62.6103 5535 47760 12037 11938 23.33 23.34 177 23.33 23.34 177 2556 3330 62.6103 5555 11773 11750 23.33 23.317 1556 23.303 62.010 5095 5353 41549 15578 13270 23.33 23.3175 23.33 23.31</td> <td>-</td> <td>27.83</td> <td>14.95</td> <td>19.94</td> <td>13.46</td> <td>17.94</td> <td>17.94</td> <td>11961</td> <td>16384</td> <td>33166</td> <td>2658</td> <td>20592</td> <td>11961</td> <td>11961</td>	32.80 17.62 23.50 15.86 21.15 21.15 14097 13872 28080 2251 17434 14097 13591 3358 33.54 18.02 24.03 16.22 21.63 14418 13584 27517 2205 17085 14418 13758 35.75 19.62 13562 13356 35.75 19.72 25.91 17.76 23.68 15786 15592 13356 15512 15706 15592 13356 15.52 20.18 26.90 18.16 23.58 5.716 3716 2442 15156 24525 1966 15527 16143 14875 13.30 7.14 9.53 6.43 8.571 4.58 12116 24525 1966 15527 15618 14572 13.30 7.14 9.53 6.43 8.571 4.58 5716 7483 12116 24525 1966 15527 15706 15692 13356 15.562 13.30 7.14 9.53 6.43 8.571 4.58 5716 7483 14875 13.30 7.14 9.53 6.43 8.571 4.58 5716 7483 12116 24525 1944 151562 12984 100578 5716 5716 14.62 7.10 9.21 753 5.576 107724 9.13 7794 1573 8080 8080 24.61 13.22 17.63 11.90 15.87 20.60 8080 53267 107824 9237 71553 8080 8080 25.55 13.72 18.29 12.35 16.46 27.99 10579 40556 82095 7033 54479 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10579 49156 7033 54479 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10576 49565 82095 7033 54479 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10576 49565 82095 7033 54479 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10576 49565 82095 7033 55477 1059 7033 554779 10579 25.53 13.72 13.22 17.51 13.57 18.10 30.76 12.064 35554 71969 6165 47760 12037 11938 23.53 11773 11750 23.00 1751 13577 138 23.11 77.4 20.99 14.17 18.89 32.12 12.596 3415 7963 5923 44583 11773 11750 23.30 1757 11588 23.316 17.38 23.17 15.64 20.89 10579 49563 5992 46418 12.315 12315 23139 23.32 1758 23.317 155.8 20.69 13.97 13663 23.303 62.6103 5535 47760 12037 11938 23.33 23.34 177 23.33 23.34 177 2556 3330 62.6103 5555 11773 11750 23.33 23.317 1556 23.303 62.010 5095 5353 41549 15578 13270 23.33 23.3175 23.33 23.31	-	27.83	14.95	19.94	13.46	17.94	17.94	11961	16384	33166	2658	20592	11961	11961
33.54 18.02 24.03 16.22 21.63 21.63 14418 13594 27517 2205 17085 14418 137535 36.73 19.62 26.15 17.65 23.54 23.68 15692 123356 123736 15692 13356 37.55 20.18 26.90 18.16 24.21 24.21 16.422 25144 2015 15692 15892 13356 37.55 20.18 26.90 18.16 24.21 124.22 25144 2015 16433 14875 13.30 7114 9.53 6.43 8.57 14.58 5716 7163 14875 17.62 9.41 12.55 8.47 11.30 92.21 7532 57168 115721 9913 76794 7532 7532 11.50 11.30 19.211 20.60 8080 53267 107824 9237 71553 8080 2553 13.72 11.90 15.87 26.98 10776 10976 7033 54479 10579 2553 13.72 18.20 18.10 30.76 12826 30307 73663 6510 7736 7732 2563 13.774 30.16 11829 35554 71969 6165 47760 12076 2563 13.774 30.76 12829 36390 73663 6310 7883 11773 25.31 15.74 20.99 13.774 30.76	33.54 18.02 24.03 16.22 21.63 21.63 14418 13594 27517 2205 17085 14418 13356 36.51 19.62 26.15 77.65 23.58 23.56 12497 25296 2027 15612 15786 14522 36.51 19.62 26.15 17.76 23.68 15582 12497 25296 2027 15612 15786 14522 36.73 19.62 7.14 9.53 6.43 8.571 14.58 12122 2054 6133 5716 5716 5762 5732 7532 <t< td=""><td>З</td><td>32.80</td><td>17.62</td><td>23.50</td><td>15.86</td><td>21.15</td><td>21.15</td><td>14097</td><td>13872</td><td>28080</td><td>2251</td><td>17434</td><td>14097</td><td>13591</td></t<>	З	32.80	17.62	23.50	15.86	21.15	21.15	14097	13872	28080	2251	17434	14097	13591
36.51 19.62 26.15 17.65 23.54 23.54 15692 15796 15706 15692 13356 36.73 19.73 26.31 17.76 23.68 15786 1422 25144 2015 15612 15786 14752 37.55 20.18 26.90 18.16 24.21 24.21 16143 12116 24525 1966 15227 16143 14752 37.55 20.18 26.90 18.16 24.21 24.21 24.61 2116 24525 1966 15227 16143 14752 13.30 7.14 9.53 643 8.57 14.58 5716 7184 10578 5716 5732 14.62 7.86 10.47 7.07 9.43 16.03 6284 68171 137994 11821 91574 6284 6284 17.52 9.41 12.56 8.47 11.30 19.21 7532 76794 7532 7532 17.52 11.90 17.63 11.90 12.12 20.60 19766 10579 10579 10579 25.53 11.763 11.763 30.76 12.72 36107 70363 63107 76363 1076 27.52 14.79 13.72 31.60 15.74 5284 7033 54479 10579 10976 27.52 13.72 11760 1076 12.12 20.60 11829 36.90 73663 6166 <t< td=""><td>36.51 19.62 26.15 17.65 23.54 23.54 15692 12497 25296 2027 15706 15692 13356 35.73 19.73 26.31 17.76 23.68 23.68 15786 12422 25144 2015 15612 15786 14522 3716 1437 14875 3755 20.18 26.90 18.16 24.21 24.21 16143 14875 12116 24525 1966 15227 16143 14875 13.30 7.14 9.53 6.43 8.57 14.58 5716 7152 12954 10578 5716 5716 5716 5716 1552 9.41 12.55 9.47 11.30 19.21 7532 57168 115721 9913 76794 10578 5716 5732 7532 1553 11.64 11.52 19.12 20.60 8080 55267 107824 9237 71553 8080 8080 80579 24.61 13.22 17.63 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10579 2553 13.77 18.29 12.35 16.46 27.99 10976 39110 79167 6782 5536 10976 10976 25.53 13.77 18.29 13.77 11750 77150 12037 11938 25.53 13.77 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 25.53 13.72 18.29 12.357 16.6 12044 35555 69948 5592 46418 12315 12319 22.533 13.77 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 25.53 13.72 18.20 13.07 3 11750 12319 22.533 13.77 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 25.53 13.72 18.29 12.34 36.27 1424 3555 69948 5592 46418 12315 12319 22.93 3033 15.74 20.09 14.17 18.89 35.11 17.4 30.16 11829 3639 1307 619142 5523 4383 11773 11750 25.33 13.77 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 447760 12037 11938 23.23 23.33 15.74 20.09 14.17 18.89 35.24 71969 6165 44760 12037 11938 23.30 15.74 20.09 14.17 18.89 35.24 12266 3465 6692 4465 553 4459 16579 10579 29.33 33.29 17.38 23.71 15.66 20.60 21.34 36.53 13206 55610 5563 4459 14081 13663 33.24 13563 33.29 14271 30101 60932 5520 40455 13942 12387 33.20 17.88 23.71 16.00 21.34 36.39 14271 30101 60932 5220 40455 13942 13663 13576 33.23 750 20.69 14081 13665 33.24 12269 33.24 12269 33.24 12269 33.27 14224 30201 61133 52.37 40569 14081 13663 33.20 13672 33.23 7303 30930 15.66 12440 3565 4408 13663 13660 12234 12670 12237 33.20 12270 40455 13942 12367 33.20 12270 40455 13942 12367 33.20 12270 40455 13942 12367 33.20 12270 40455 13942 12367 12240 122078 30010 11233 5237 40569 14081 13665 33.20 1406 100972</td><td>З</td><td>33.54</td><td>18.02</td><td>24.03</td><td>16.22</td><td>21.63</td><td>21.63</td><td>14418</td><td>13594</td><td>27517</td><td>2205</td><td>17085</td><td>14418</td><td>13758</td></t<>	36.51 19.62 26.15 17.65 23.54 23.54 15692 12497 25296 2027 15706 15692 13356 35.73 19.73 26.31 17.76 23.68 23.68 15786 12422 25144 2015 15612 15786 14522 3716 1437 14875 3755 20.18 26.90 18.16 24.21 24.21 16143 14875 12116 24525 1966 15227 16143 14875 13.30 7.14 9.53 6.43 8.57 14.58 5716 7152 12954 10578 5716 5716 5716 5716 1552 9.41 12.55 9.47 11.30 19.21 7532 57168 115721 9913 76794 10578 5716 5732 7532 1553 11.64 11.52 19.12 20.60 8080 55267 107824 9237 71553 8080 8080 80579 24.61 13.22 17.63 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10579 2553 13.77 18.29 12.35 16.46 27.99 10976 39110 79167 6782 5536 10976 10976 25.53 13.77 18.29 13.77 11750 77150 12037 11938 25.53 13.77 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 25.53 13.72 18.29 12.357 16.6 12044 35555 69948 5592 46418 12315 12319 22.533 13.77 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 25.53 13.72 18.20 13.07 3 11750 12319 22.533 13.77 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 25.53 13.72 18.29 12.34 36.27 1424 3555 69948 5592 46418 12315 12319 22.93 3033 15.74 20.09 14.17 18.89 35.11 17.4 30.16 11829 3639 1307 619142 5523 4383 11773 11750 25.33 13.77 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 447760 12037 11938 23.23 23.33 15.74 20.09 14.17 18.89 35.24 71969 6165 44760 12037 11938 23.30 15.74 20.09 14.17 18.89 35.24 12266 3465 6692 4465 553 4459 16579 10579 29.33 33.29 17.38 23.71 15.66 20.60 21.34 36.53 13206 55610 5563 4459 14081 13663 33.24 13563 33.29 14271 30101 60932 5520 40455 13942 12387 33.20 17.88 23.71 16.00 21.34 36.39 14271 30101 60932 5220 40455 13942 13663 13576 33.23 750 20.69 14081 13665 33.24 12269 33.24 12269 33.24 12269 33.27 14224 30201 61133 52.37 40569 14081 13663 33.20 13672 33.23 7303 30930 15.66 12440 3565 4408 13663 13660 12234 12670 12237 33.20 12270 40455 13942 12367 33.20 12270 40455 13942 12367 33.20 12270 40455 13942 12367 33.20 12270 40455 13942 12367 12240 122078 30010 11233 5237 40569 14081 13665 33.20 1406 100972	З	33.54	18.02	24.03	16.22	21.63	21.63	14418	13594	27517	2205	17085	14418	13758
36.73 19.73 26.31 17.76 23.68 15786 15786 12422 25144 2015 15612 15786 14875 37.55 20.18 26.90 18.16 24.21 16143 12116 24525 1966 15227 16143 14875 13.30 7.14 9.53 6.43 8.57 14.58 5716 14872 25164 15227 16143 14875 14.62 7.86 10.47 7.07 9.43 16.03 6284 68171 137944 11821 91574 6284 6284 1752 9.41 12.55 8.47 11.30 19.21 7532 57168 11821 913 76794 7532 7532 18.80 10.10 13.47 9.09 12.12 20.60 8080 53267 107824 9237 71553 8080 8080 24.61 13.22 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10779 24.61 13.22 11.90 15.87 26.98 10576 36307 73663 63107 7470 1753 211753 24.752 13.231 1774 30.16 11829 36390 73663 63107 8783 11773 27.52 14.79 19.77 33.16 12.12 27.99 3076 2794 5782 25336 10976 27.53 15.74	36.73 19.73 26.31 17.76 23.68 15786 12422 25144 2015 15612 15786 14522 3716 3715 13.30 7.14 9.53 6.43 8.57 14.58 5716 74874 151562 12984 100578 5716 5716 5715 13.30 7.14 9.53 6.43 8.57 14.58 5716 74874 151562 12984 100578 5716 5715 14.62 8 14.62 7.33 14.62 7.86 10.47 7.07 9.43 16.03 6.284 6.8171 137994 11821 91574 6.284 6.284 6.847 17.52 9.41 12.55 8.47 11.30 19.21 7532 57168 115721 9913 76794 7532 7533 7532 17532 1753 11.50 13.47 9.09 12.12 20.60 8080 53267 107824 9237 71553 8080 8080 25.53 13.72 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10579 25.53 13.72 18.29 12.35 16.44 30.16 11829 36390 77966 6782 52536 10976 10976 25.53 13.72 18.20 12.776 33.00 77616 7783 5544 71960 7579 25.53 13.72 13.31 177.74 30.16 11829 36390 73663 6310 48883 11773 11936 25.53 13.77 18.0 30.76 12064 35554 7196 6678 5703 20017 10579 10579 26.91 13.22 14.79 19.72 13.31 177.74 30.16 11829 36390 73663 6310 48883 11773 11936 23.63 13.77 13.57 18.10 30.76 12014 3555 69948 5992 46418 12315 12319 28.88 15.52 20.69 13.97 18.60 3.212 12596 34157 69142 5923 45883 11773 11936 23.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 11773 11936 23.33 10 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 12316 12470 3237 1333 23.33 177 33.309 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 4583 12510 12470 32.34 1753 33.309 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 4583 12510 12470 32.34 1753 33.309 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 4583 12510 12470 32.34 1753 33.309 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 4583 12510 12470 32.34 13562 33.309 15.74 20.99 14.17 18.89 32.12 12596 34157 6918 49.46 18 12315 12319 29.33 13562 33.309 15.74 20.99 14.71 18.89 32.12 12596 34157 6918 49.45 5923 4583 12510 12470 32.34 1556 33.309 15.74 20.99 14.71 18.89 32.12 12596 34157 69142 5923 45883 12510 12470 33.3309 15.74 20.99 14.71 18.89 32.12 12596 34157 6918 49.45 5923 45883 12510 12470 33.329 33.309 17.78 23.71 10559 13903 30930 5610 5363 41549 13663 13366 33.3265 33.3297 40569 14081 13663	4	36.51	19.62	26.15	17.65	23.54	23.54	15692	12497	25296	2027	15706	15692	13356
37.55 20.18 26.90 18.16 24.21 24.21 $16 43$ 12116 24525 1966 15227 $16 43$ 14875 13.30 7.14 9.53 6.43 8.57 14.58 5716 74874 151562 12984 100578 5716 5716 14.62 7.86 10.47 7.07 9.43 16.03 6284 68171 137994 11821 91574 6284 6284 17.52 9.41 12.55 8.47 11.30 19.21 7532 57168 115721 9913 76794 7532 7532 18.80 10.010 13.47 9.09 12.12 20.60 8080 53267 107824 9237 71553 8080 8080 24.61 13.22 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10976 24.61 13.22 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10976 25.53 13.72 18.29 12.12 20.60 8080 53267 107824 9237 7173 11733 25.63 14.79 19.72 18.10 30.76 11829 36390 73663 6310 47490 12037 27.52 16.46 27.99 10976 31164 34555 69948 5992 46418 12713 28.88 15.74	37.55 20.18 26.90 18.16 24.21 24.21 16143 12116 24525 1966 15227 16143 14875 13.30 7.14 9.53 6.43 8.57 14.58 5716 74874 151562 12984 100578 5716 5716 5716 14.62 7.86 10.47 7.07 9.43 16.03 6284 68171 137994 11821 91574 6284 6284 17.52 9.41 12.55 8.47 11.30 19.21 7532 57168 115721 9913 76794 7532 7532 7532 17552 18.80 13.47 9.09 12.12 20.60 8080 53267 107824 9237 71553 8080 8080 24.61 13.22 17.63 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10579 10579 25.53 13.72 18.29 19.72 13.31 17.74 30.16 12829 36390 73663 6310 48883 11773 11750 28.07 24.61 13.57 18.10 30.76 12064 35554 71969 6165 47760 12077 11938 28.87 15.08 20.11 13.57 18.10 30.76 12414 34555 69948 5992 46418 12315 12319 29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5523 41549 12375 11230 29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5523 41549 12373 11750 32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.309 17.78 23.71 15.69 23.3157 12319 29.300 62610 5353 41549 13663 13276 33.309 17.78 23.71 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.309 17.78 23.71 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.309 17.78 23.71 15.64 20.86 35.45 13903 30930 62610 5353 41549 13663 13276 33.309 17.78 23.71 15.64 20.86 35.45 13903 30930 62610 5353 41549 13663 13276 33.309 17.78 23.77 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13266 33.309 17.78 23.77 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13266 33.330 17.78 23.77 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13266 33.330 17.78 23.77 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13476 1244 30201 61133 52.27 40569 14081 13663 33.27 1356 33.309 17.78 23.77 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 133662 33.320 13766 13942 13664 13866 12440 13664 18886 1244 188 12315 12319 12470 33.309 17.78 23.74 10509 14081 13663 13266 33.236 10766 133 55.27 140569 14081 13663 13266 33.236 10766 1244 30011 60932 55.20 40435 13942 13664 13664 106 121.44 3669 1408 103666 1368 6666 6666 6666 6666 6666 66666	3	36.73	19.73	26.31	17.76	23.68	23.68	15786	12422	25144	2015	15612	15786	14522
13.30 7.14 9.53 6.43 8.57 14.58 5716 74874 151562 12984 100578 5716 5716 14.62 7.86 10.47 7.07 9.43 16.03 6284 68171 137994 11821 91574 6284 6284 17.52 9.41 12.55 8.47 11.30 19.21 7532 57168 115721 9913 76794 7532 7532 18.80 10.10 13.47 9.09 12.12 20.60 8080 53267 107824 9237 71553 8080 8080 24.61 13.22 17.63 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10579 24.61 13.72 18.29 12.35 16.46 27.99 10976 39110 79167 6782 52536 10976 10976 25.53 13.72 18.10 30.76 11829 36390 73663 6310 4883 11773 11750 28.07 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 28.07 15.74 20.99 13.77 11829 36.393 36.3610 5363 41549 13663 12315 12319 28.07 15.74 20.99 14.77 8.88 32.17 18.89 32.12 12214 34	13.30 7.14 9.53 6.43 8.57 14.58 5716 7716 5732 5716 5123 7532	~	37.55	20.18	26.90	18.16	24.21	24.21	16143	12116	24525	1966	15227	16143	14875
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	14.62 7.86 10.47 7.07 9.43 16.03 6.284 68171 137994 11821 91574 6.284 6.284 17.52 9.41 12.55 8.47 11.30 19.21 7532 57168 115721 9913 76794 7532 7532 18.80 10.10 13.47 9.09 12.12 20.60 8080 53267 107824 9237 71553 8080 8080 24.61 13.22 17.63 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10579 25.53 13.72 18.29 12.36 19264 35554 71969 6165 47760 12037 11938 27.52 14.79 19.77 18.10 30.76 12414 34555 69948 5992 46418 12315 12319 28.07 15.68 23.17 18.62 32.12 12566 34157 69142 5923	4	13.30	7.14	9.53	6.43	8.57	14.58	5716	74874	151562	12984	100578	5716	5716
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	17.52 9.41 12.55 8.47 11.30 19.21 7532 57168 115721 9913 76794 7532 7532 18.80 10.10 13.47 9.09 12.12 20.60 8080 53267 107824 9237 71553 8080 8080 24.61 13.22 17.63 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10976 39110 79167 6782 52536 10976 10976 10976 27.52 14.79 19.72 13.31 17.74 30.16 11829 36390 73663 6310 4883 11773 11750 28.07 15.68 20.91 13.67 12.414 34555 69948 5992 46418 12315 12319 28.07 15.62 23.03 6510 5753 69142 5523 10976 12932 28.08 15.74 20.99 13.66 12414 </td <td>9</td> <td>14.62</td> <td>7.86</td> <td>10.47</td> <td>7.07</td> <td>9.43</td> <td>16.03</td> <td>6284</td> <td>68171</td> <td>137994</td> <td>11821</td> <td>91574</td> <td>6284</td> <td>6284</td>	9	14.62	7.86	10.47	7.07	9.43	16.03	6284	68171	137994	11821	91574	6284	6284
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	18.80 10.10 13.47 9.09 12.12 20.60 8080 53267 107824 9237 71553 8080 8080 24.61 13.22 17.63 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10976 39110 79167 6782 52536 10976 <td>0</td> <td>17.52</td> <td>9.41</td> <td>12.55</td> <td>8.47</td> <td>11.30</td> <td>19.21</td> <td>7532</td> <td>57168</td> <td>115721</td> <td>9913</td> <td>76794</td> <td>7532</td> <td>7532</td>	0	17.52	9.41	12.55	8.47	11.30	19.21	7532	57168	115721	9913	76794	7532	7532
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	24.61 13.22 17.63 11.90 15.87 26.98 10579 40556 82095 7033 54479 10579 10579 10579 25.53 13.72 18.29 12.35 16.46 27.99 10976 39110 79167 6782 52536 10976 10976 10976 25.53 13.72 18.29 12.35 16.46 27.99 10976 39110 79167 6782 52536 10976 10976 10976 27.52 14.79 19.72 13.31 17.74 30.16 11829 36390 73663 6310 4883 11773 11750 28.07 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 28.88 15.52 20.69 13.97 18.62 31.66 12414 34555 69948 5992 46418 12315 12319 29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 12510 12470 32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.30 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.78 23.77 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.20 17.78 23.77 15.64 20.86 35.45 13903 30001 60932 5220 40435 11362 12887 and the reacter the Reynolds number based on boundary layer thickness is $R_{800} = U\delta_0/\nu$. The Weber number is defined as $We_{10} = \mu_{water}\mu_{10}/\sigma$. The here M_{10} is reacted from the reacted of the reacted	\mathfrak{c}	18.80	10.10	13.47	9.09	12.12	20.60	8080	53267	107824	9237	71553	8080	8080
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	25.53 13.72 18.29 12.35 16.46 27.99 10976 39110 79167 6782 52536 10976 10976 10976 27.52 14.79 19.72 13.31 17.74 30.16 11829 36390 73663 6310 4883 11773 11750 28.07 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 28.88 15.52 20.69 13.97 18.62 31.66 12414 34555 69948 5992 46418 12315 12319 29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5922 4583 12510 12470 32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.30 17.78 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 parameters. The Reynolds number based on boundary layer thickness is $R_{80} = U\delta_0/\nu$. The Weber number is defined as $W_{e10} = \mu_{water} \mu_T 0^{\circ}$. The averal based on boundary layer thickness is $R_{80} = U\delta_0/\nu$. The weber number is defined as $W_{e10} = \mu_{water} \mu_T 0^{\circ}$. The averal from the rearches T and T the rearch of an the rearches T and T	З	24.61	13.22	17.63	11.90	15.87	26.98	10579	40556	82095	7033	54479	10579	10579
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	27.52 14.79 19.72 13.31 17.74 30.16 11829 36390 73663 6310 4883 11773 11750 28.07 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 28.08 15.52 20.69 13.97 18.62 31.66 12414 34555 69948 5992 46418 12315 12319 29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 12510 12470 32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5353 41549 13663 13276 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.78 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 12867 here is a first for the remoted on boundary layer thickness is $R_{80} = U \delta_0 / v$. The Weber number is defined as $W_{e10} = \mu_{wate} \mu_{e1} / \sigma$. The here is $D_{a12} = \dots \times \mathbb{R}^3 / v$. The remoted from the remoted of the remoted of the remoted from the remoted of the remoted of the remoted from the remoted of the remoted of the remoted from the remoted of the remoted of the remoted from the remoted of the remoted of the remoted of the remoted of the remoted from the remoted of the remoted of the remoted from the remoted of the remoted	\mathbf{C}	25.53	13.72	18.29	12.35	16.46	27.99	10976	39110	79167	6782	52536	10976	10976
28.07 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 28.08 15.52 20.69 13.97 18.62 31.66 12414 34555 69948 5992 46418 12315 12319 29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 12510 12470 29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 12510 12470 32.34 17.38 23.17 15.64 20.86 35.45 13903 62610 5363 41549 13663 13276 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5227 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 <td>28.07 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 28.88 15.52 20.69 13.97 18.62 31.66 12414 34555 69948 5992 46418 12315 12319 2930 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 12510 12470 32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 12867 12887 13662 $33.20 - 17.38 - 23.79 - 16.06 - 21.41 - 36.39 - 14271 - 30101 60932 5220 40435 13942 12887 12867 12867 12887 12887 12887 12887 12887 12887 13662 - 12887 13662 - 12887 12887 13662 - 12887 12887 12887 12887 12887 12887 13662 - 12887 12887 12887 12887 12887 13600 - 12888 - 12887 12887 - 12887 12887 - 128800000000000000000000000000000000000$</td> <td>$\infty$</td> <td>27.52</td> <td>14.79</td> <td>19.72</td> <td>13.31</td> <td>17.74</td> <td>30.16</td> <td>11829</td> <td>36390</td> <td>73663</td> <td>6310</td> <td>48883</td> <td>11773</td> <td>11750</td>	28.07 15.08 20.11 13.57 18.10 30.76 12064 35554 71969 6165 47760 12037 11938 28.88 15.52 20.69 13.97 18.62 31.66 12414 34555 69948 5992 46418 12315 12319 2930 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 12510 12470 32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 12867 12887 13662 $33.20 - 17.38 - 23.79 - 16.06 - 21.41 - 36.39 - 14271 - 30101 60932 5220 40435 13942 12887 12867 12867 12887 12887 12887 12887 12887 12887 13662 - 12887 13662 - 12887 12887 13662 - 12887 12887 12887 12887 12887 12887 13662 - 12887 12887 12887 12887 12887 13600 - 12888 - 12887 12887 - 12887 12887 - 128800000000000000000000000000000000000$	∞	27.52	14.79	19.72	13.31	17.74	30.16	11829	36390	73663	6310	48883	11773	11750
28.8 15.52 20.69 13.97 18.62 31.66 12414 34555 69948 5992 46418 12315 12319 29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45833 12510 12470 29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45833 12510 12470 32.34 17.38 23.17 15.64 20.86 35.45 13903 62610 5363 41549 13663 13276 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887	28.8 15.52 20.69 13.97 18.62 31.66 12414 34555 69948 5992 46418 12315 12319 29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 12510 12470 32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 tparameters. The Reynolds number based on boundary layer thickness is $R_{8,0} = U\delta_0/\nu$. The Weber number is defined as $W_{e,0} = \mu_{water} u_r_0/\sigma$. The heric $D_{a,2} = u_{3,2}/\mu_1$ The Acber number is defined as $W_{e,0} = \mu_{water} u_r_0/\sigma$. The heric $D_{a,2} = u_{3,2}/\mu_1$ The remove of fire theorem of the removes of air sequence of air sequences.	∞	28.07	15.08	20.11	13.57	18.10	30.76	12064	35554	71969	6165	47760	12037	11938
29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 12510 12470 32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 1376 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887	29.30 15.74 20.99 14.17 18.89 32.12 12596 34157 69142 5923 45883 12510 12470 32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 her is <i>P</i> _{a} = <i>v</i> _{a} =	\sim	28.88	15.52	20.69	13.97	18.62	31.66	12414	34555	69948	5992	46418	12315	12319
32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 1376 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887	32.34 17.38 23.17 15.64 20.86 35.45 13903 30930 62610 5363 41549 13663 13276 33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 parameters. The Reynolds number based on boundary layer thickness is $Re_{80} = U\delta_0/\nu$. The Weber number is defined as $We_{40} = \mu_{water} u_{\tau_0}/\sigma$. The base is $Re_{80} = U\delta_0/\nu$. The Properties of the reaction of the reactio	∞	29.30	15.74	20.99	14.17	18.89	32.12	12596	34157	69142	5923	45883	12510	12470
33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887	33.09 17.78 23.71 16.00 21.34 36.27 14224 30201 61133 5237 40569 14081 13662 33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 parameters. The Reynolds number based on boundary layer thickness is $Re_{30} = U\delta_0/\nu$. The Weber number is defined as $We_{10} = \mu_{water} u_{10}/\sigma$. The base is $D_{2,2} = -\frac{1}{2} \delta_{2,1}$. The meter of the renumber is defined from the renuce of air solution of I to be a solution of the renuce of air solution.	6	32.34	17.38	23.17	15.64	20.86	35.45	13903	30930	62610	5363	41549	13663	13276
33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887	33.20 17.84 23.79 16.06 21.41 36.39 14271 30101 60932 5220 40435 13942 12887 l parameters. The Reynolds number based on boundary layer thickness is $R_{eb0} = U\delta_0/\nu$. The Weber number is defined as $W_{et0} = \mu_{water} u_{t0}/\sigma$. The base is $R_{eb0} = -U\delta_0/\nu$. The model is defined as $W_{et0} = \mu_{water} u_{t0}/\sigma$. The base is $R_{eb0} = -U\delta_0/\nu$. The model is defined as $W_{et0} = \mu_{water} u_{t0}/\sigma$. The base is $R_{eb0} = 0.5 \circ M_{eb0}$ is determined from the range of site solution of the range of site solution of the range of the range of the range of site solution of s		33.09	17.78	23.71	16.00	21.34	36.27	14224	30201	61133	5237	40569	14081	13662
	parameters. The Reynolds number based on boundary layer thickness is $Re_{30} = U\delta_0/\nu$. The Weber number is defined as $We_{\tau0} = \mu_{water}\mu_{\tau0}/\sigma$. The second second parameters is $Re_{30} = L^{-1}$ is determined from the reason of sir contrastion	4	33.20	17.84	23.79	16.06	21.41	36.39	14271	30101	60932	5220	40435	13942	12887
It is $Ke_{\tau0} = u_{\tau0}w_{00}/v$. The trench length $L^{-\infty}$ is based on the longest trenches. The range of $L_{s0}^{+} = L^{+0}$ if $L_{s0}^{+} = L^{+0}$.		-	/				-	с , Р	66	65	h	5			

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