The prickly genius – Colin MacLaurin (1698–1746)

IAN TWEDDLE

1. *An outline of MacLaurin's life and career.*

Colin MacLaurin was born in February 1698 at Kilmodan, Glendaruel, Argyllshire, where his father was a minister. Much of MacLaurin's upbringing was the responsibility of his uncle, a minister in Kilfinan, Argyllshire: MacLaurin's father died some six weeks after his son's birth and his mother died in 1707.

In 1709 at the tender age of eleven MacLaurin entered Glasgow University to study for the ministry, although he soon became interested in geometry and was perhaps influenced by Robert Simson, who was certainly in Glasgow during part of MacLaurin's student days and who became Professor of Mathematics at Glasgow in 1711. MacLaurin graduated M.A. in 1713, for which he produced a short philosophical dissertation (in Latin) 'On gravity and other forces'.

Following his graduation MacLaurin returned to Kilfinan, where he spent several years in private study of mathematics and divinity, before being appointed in 1717 Professor of Mathematics at Marischal College, Aberdeen. It is interesting to note that in connection with this appointment MacLaurin was described as a student of divinity. However, from then on he was very much a mathematician, his main interest at that time being Newton's 'organic geometry', in which curves are defined in terms of fixed points and systems of lines through them which move in prescribed ways. In 1719 MacLaurin spent several months in London, taking with him the manuscript of his forthcoming book *Geometria Organica* (1720). He met Newton, who, as subsequent events confirmed, was most impressed by the young Scottish mathematician, and was elected a Fellow of the Royal Society in November, one of his proposers being Halley.

The early 1720s were very productive years for MacLaurin. During this period his 'organic' ideas developed into the beginnings of projective geometry. He was also concerned with more applied topics and in 1725 was awarded a prize by the Académie Royale des Sciences, Paris, for his dissertation on the 'Percussion of bodies'. During 1722–4 he was in fact on continental Europe, acting as tutor to the son of Lord Polwarth. It appears that MacLaurin took up this appointment without waiting for final approval from the authorities at Aberdeen or making provision for his classes during his absence. On his return he was called to account for his actions, but it seems that MacLaurin's standing was now such that his wayward behaviour could be overlooked and he was allowed to resume his position at Aberdeen. However, a short time later MacLaurin quarrelled with the authorities over voting rights in rectorial elections and in 1725 they learned through the press that MacLaurin had been appointed to assist the ageing James

https://doi.org/10.2307/3619883 Published online by Cambridge University Press

Gregory* at Edinburgh. Apparently, strong support from Newton had been a major factor in securing this appointment for MacLaurin, who remained Professor of Mathematics at Edinburgh for the rest of his life. In 1733 he married Anne Stewart, daughter of the Solicitor-General for Scotland; they had seven children.

MacLaurin continued to be active in research during his tenure at Edinburgh. In 1737 he was instrumental in forming the *Edinburgh Philosophical Society*, which was an extension of an existing medical society and developed into the *Royal Society of Edinburgh*. In 1740 he was again awarded a prize by the Académie Royale des Sciences, this time for a Latin dissertation 'On the physical cause of the flow and ebb of the sea'.† From at least the mid-1730s MacLaurin had been working on a treatise which was partly aimed at answering criticisms of Newtonian calculus. This important and very influential work was published in 1742 as the *Treatise of Fluxions*. In it MacLaurin often gave two treatments of a topic, one using geometrical methods and aimed at those who doubted the validity of calculus, the other using 'modern' analytical methods. Of particular interest among its contents is his theory of the figure of the earth and its gravitational forces, which is also contained in part in his 1740 dissertation. Other mathematicians were working on this topic, notably James Stirling, Thomas Simpson and the French mathematician Clairaut, but MacLaurin was probably the first to give a treatment which was extensive and sound both mathematically and physically.

Two other books were published posthumously in 1748: *An Account of Sir Isaac Newton's Philosophical Discoveries* and *Treatise of Algebra*, which became a popular textbook. Some of the material in these dates from the 1720s. MacLaurin published twelve papers in the Royal Society's *Philosophical Transactions* on topics both mathematical and general‡ – two intriguing titles: 'An account of a monstrous double birth in Lorraine' and 'On the bases of the cells wherein the bees deposit their honey'. Two papers were published posthumously (1754) in the *Edinburgh Philosophical Essays*; these were presented to the Edinburgh Philosophical Society in 1740.

Unfortunately in the late 1720s and early 1730s MacLaurin became involved in two very public controversies, which perhaps did some harm to his reputation. The first of these was with George Campbell over priority in the discovery of certain results on complex roots; to some extent MacLaurin's dispute was with the Royal Society, since he objected to their having accepted a paper on this topic by Campbell when MacLaurin had already published one related paper and had indicated that a sequel was imminent. The second was with William Braikenridge, who published in

---

* This was a nephew of the famous James Gregory (1638–75).

† The prize was shared with Euler and Daniel Bernoulli.

‡ Fellows were encouraged to report curiosities to the Royal Society; this could earn some remission of subscription for remotely located Fellows who could not avail themselves of the Society's regular meetings.

https://doi.org/10.2307/3619883 Published online by Cambridge University Press

1733 a little book on geometry and the description of curves; MacLaurin claimed that he had shown Braikenridge some of the results which appeared in this book in the 1720s and accused Braikenridge of passing them off as his own.

People of genius usually make an impact on a broad front and MacLaurin was no exception. He applied his mathematics to help customs officials at Glasgow determine the amounts contained in molasses barrels so that appropriate duties could be levied. A pension scheme for widows of ministers and professors, although not devised by him, did benefit from his advice and fine-tuning; MacLaurin actually went to London to support the Bill for its establishment through Parliament.

MacLaurin's last years coincided with the *Second Jacobite Rebellion*, when Bonnie Prince Charlie attempted to regain the British throne for the deposed House of Stewart. In 1745 MacLaurin became actively involved in the defence of Edinburgh against the encroaching Jacobite forces. When it became clear that the city would fall, MacLaurin fled to York. He returned to Edinburgh the following year in poor health, from which he did not recover. He died there on 14 June 1746.*

2. *Some of MacLaurin's mathematics.*

Unfortunately much of MacLaurin's large output is forgotten, either because the material is unfashionable (for example, his geometry) or because his methods have been superseded (for example, his theory of the figure of the earth). However, it is still possible to find his name linked with various results in text books. Here are some examples.

(i) *MacLaurin's Theorem.*

This is the expansion

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

MacLaurin discussed this result in the *Treatise of Fluxions*, applying it in particular to obtain the usual series expansions and to discuss maxima and minima problems (second derivative test). MacLaurin's result is a special case ($a = 0$) of *Taylor's Theorem*

$$f(x) = f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3 + \dots$$

and generations of textbooks have confused the issue by asserting that MacLaurin was unaware of Taylor's general result. However, MacLaurin stated quite clearly: 'This result was given by Dr. Taylor, *method. increm.*'† and moreover, it is clear from the manner in which MacLaurin applied his version that he realised that the general result simply corresponds to a

* MacLaurin is buried in Greyfriars Churchyard, Edinburgh. His memorial stone, which is set high on the exterior south wall of the church, is easily located.

† This refers to a paper, Methodus incrementorum directa et inversa, published by Brook Taylor (1685–1731) in the *Philosophical Transactions* (1715).

change of origin – so in a sense the simple case is the general case. The result has probably become known as 'MacLaurin's Theorem' because MacLaurin demonstrated very clearly how it could be used.

(ii) *The Euler-MacLaurin Summation Formula.*

Starting with 'MacLaurin's Theorem' MacLaurin derived the following important results in the *Treatise of Fluxions*:

$$f(0) + f(1) + f(2) + \ldots + f(k-1)$$
$$= \int_0^k f(t)\,dt - \frac{1}{2}\{f(k) - f(0)\} + \frac{1}{12}\left\{f'(k) - f'(0)\right\} - \frac{1}{720}\left\{f'''(k) - f'''(0)\right\} + \ldots$$

and

$$f(\tfrac{1}{2}) + f(\tfrac{3}{2}) + f(\tfrac{5}{2}) + \ldots + f(k - \tfrac{1}{2})$$
$$= \int_0^k f(t)\,dt - \frac{1}{24}\{f'(k) - f'(0)\} + \frac{7}{5760}\{f'''(k) - f'''(0)\} - \ldots$$

These are usually called respectively the first and second forms of the Euler-MacLaurin summation formula. They give a means of analysing/estimating certain sums by means of integrals, and the values of functions and their odd-order derivatives at end points; the coefficients can be expressed in terms of Bernoulli numbers, factorials and powers of 2. It is known that MacLaurin had these results in the mid-1730s. However, Euler had already presented a version of the first form at St Petersburg in the early 1730s, so that he was almost certainly its original discoverer, while credit for the second form can go to MacLaurin. Again MacLaurin illustrated his results with applications. He also cleverly combined the two forms to deal with alternating sums

$$a_1 - a_2 + a_3 - \ldots + a_{2n-1} - a_{2n},$$

using the first form to deal with $a_1 + a_3 + \ldots + a_{2n-1}$ and the second form to deal with $a_2 + a_4 + \ldots + a_{2n}$. (Note that the integral terms will cancel out on subtraction.)

(iii) *The MacLaurin-Cauchy Integral Test.*

The Integral Test is commonly given in elementary courses on analysis:

Let f be a non-negative, decreasing function defined on $[N, \infty)$. Then $\sum_{n=N}^{\infty} f(n)$ converges if and only if $\int_N^{\infty} f(x)\,dx$ converges.

In the *Treatise of Fluxions* MacLaurin developed the ideas of this result in connection with the convergence of series whose terms are defined by rational functions. He summed up with the remark:

> 'When the area *APNF* has a limit, we not only conclude from this, that the sum of the progression represented by the ordinates has a limit; but when the former limit is known, we may by it approximate to the value of the latter ....'

Here the area *APNF* is just $\int_N^{N+n} f(x)\,dx$. With, for example, $f(x) = x^{-2}$

we see that $\sum_N^\infty n^{-2}$ is approximately $\int_N^\infty x^{-2}dx = N^{-1}$, which explains why it is impossible to get a good approximation to $\sum_1^\infty n^{-2} = \pi^2/6$ by just adding up even a fairly large number of initial terms. The contribution from Cauchy (1789–1857) is of course much later.

(iv) *MacLaurin's method of drawing conics.*

A conic is uniquely determined if we know five distinct points on it. But how can we construct the conic from five given points? MacLaurin had the following solution to this problem in the early 1720s.

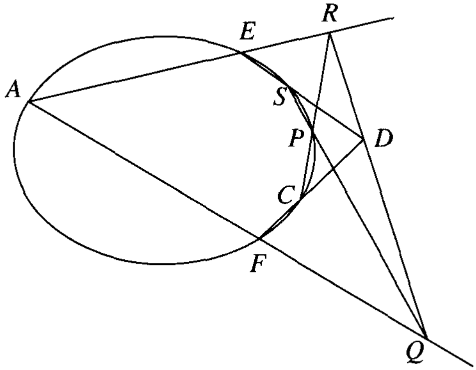

FIGURE 1

Let A, F, C, S, E be the five given points. Join AF, FC, AE, ES and let FC, ES meet at D. Take any line through D and let it meet AF in Q and AE in R. Then P, the point of intersection of QS and RC, lies on the conic. The whole conic may be generated by varying the line through D.

This construction was also given by Braikenridge in 1733 and was one of the disputed results in the controversy mentioned above. It is in fact the converse of Pappus's Theorem for a conic.

(v) *MacLaurin's ellipsoids.*

In MacLaurin's theory the earth is considered as a fluid sphere made up of particles which attract each other according to the inverse square law of attraction; when rotated this sphere takes on a new form in which the particles are in relative equilibrium. MacLaurin showed that an ellipsoid formed by rotating an ellipse about its minor axis is a possible form for the rotating earth. In honour of this achievement such equilibrium forms for rotating fluids are called 'MacLaurin's ellipsoids'.

Concluding remarks.

MacLaurin was certainly one of the greatest of the historically significant Scottish mathematicians. He was by no means unique in his time, being the youngest of three distinguished Scots who were born in the late 17th century and who all produced mathematical work that is still relevant today. The others are Robert Simson (1687–1768) and James Stirling

(1692–1770). They knew each other of course and there is evidence that there was a certain amount of mutually beneficial exchange of ideas.

Much insight into MacLaurin's life and work can be gleaned from his published correspondence [1]. To mark the 250th anniversary of MacLaurin's death the Edinburgh Mathematical Society published [2], which is concerned with MacLaurin's work, mentioned earlier, aimed at levying appropriate duties on imported molasses. Two much older references are still worth consulting [3, 4 (Ch. 9)].

References

1. Stella Mills, *The collected letters of Colin MacLaurin*, Shiva (1982).
2. Judith V. Grabiner, A mathematician among the molasses barrels: MacLaurin's unpublished memoir on volumes, *Proc. Edinburgh Math. Soc.* **39** (1996) pp. 193–240.
3. Charles Tweedie, A study of the life and writings of Colin MacLaurin, *Proc. Edinburgh Math. Soc.* **8** (1915) pp. 132–151.
4. I. Todhunter, *A history of the mathematical theories of attraction and the figure of the earth*, Macmillan (1873) reprinted by Dover (1962).

IAN TWEDDLE

Department of Mathematics, University of Strathclyde, Glasgow G1 1XH

A theological proof

$$\text{GO}\varnothing\text{D} \times \frac{1}{\varnothing} = \text{GOD} \Rightarrow \text{God is infinite good.}$$

Sent in by a reader who would prefer to remain anonymous.

More theology

Our anonymous theologian writes;

Chapter 3 of the Book of Ecclesiastes begins with a poem, 'For everything there is a season: a time to be born and a time to die;' and concludes 'a time of war and a time of peace'.

This is very thought-provoking and clear, but verse 5 is not; 'A time to scatter stones and a time to gather them'.

After puzzling over this I saw that 'calculus' is Latin for stones and things became clear. The second half of the verse must mean the collecting of small stones into a heap. Since integration is the summation of many small elements, the second half can be translated 'there is a time for integral calculus', so the first half must mean 'there is a time for differential calculus'.

Ecclesiastes was a teacher of mathematics!