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BOOK REVIEWS

FERRANDO, J. C., LÓPEZ PELLICER, M. and SÁNCHEZ RUIZ, L. M. *Metrizable barrelled spaces* (Pitman Research Notes in Mathematics Series Vol. 332, Longman, Harlow, 1995), 238 pp., 0 582 28703 0, £30.

Barrelled spaces have been important in the theory and applications of topological vector spaces since the early days of the subject. One reason for this is that they form the domain spaces for a natural generalization of the familiar closed graph theorem of elementary functional analysis, but in more recent times they have been extensively studied in their own right. As witness to this we may note that, whereas Köthe's classic *Topologische lineare Räume* I (Springer-Verlag, 1960) has only a twelve page section in its final chapter on barrelled spaces plus a few other references to barrelled spaces, Pérez Carreras and Bonet were able to publish in 1987 a work of about 500 pages devoted almost exclusively to this topic (*Barrelled locally convex spaces* (North-Holland, Elsevier)). The present book is a worthy addition to the literature on barrelled spaces and a further testimony to the success of the school of Spanish mathematicians which has grown up under the influence of Manuel Valdivia. The book is largely concerned with the authors' own research but it also includes important results of other leading authorities on barrelled spaces such as S.A. Saxon and M. Valdivia.

A *barrel* in a real or complex topological vector space is a closed absolutely convex absorbent set. Every locally convex space has a base of neighbourhoods of the origin consisting of barrels and if every barrel in such a space is a neighbourhood of the origin the space is said to be *barrelled*. As a result of the Baire Category Theorem all Banach spaces and all Fréchet spaces are barrelled, but these form only a small sample of the class of barrelled spaces and completeness, even in the metrizable case, is not a necessary condition for barrelledness. Abstractions of certain category ideas lead to various "strong barrelledness" concepts, which form much of the subject matter of the book.

Chapter 1 "Some facts about barrelled spaces" deals with the basic properties of barrelled spaces and the smaller classes of Baire-like spaces, unordered Baire-like spaces, totally barrelled spaces and ultrabornological spaces; it also discusses barrelledness of tensor products. A well-known result of Mazur asserts that the least infinite-dimensionality for a Fréchet space is $\mathfrak{c}$, the cardinal number of the real numbers. But what if completeness is replaced by barrelledness? This is discussed in Chapter 2 "Dimensionality and cardinality", the answer being provided by the bounding cardinal $\mathfrak{b}$. Chapters 3 and 4 are devoted to spaces with certain strong barrelledness properties – barrelled spaces of class n and barrelled spaces of class $\aleph_0$. If Σ is an algebra of sets, $\ell_0^\infty(\Sigma)$ denotes the span of the characteristic functions of the elements of Σ with the supremum norm. Chapters 5 and 6 are concerned with barrelledness questions for $\ell_0^\infty(\Sigma)$; it is shown for example that $\ell_0^\infty(\Sigma)$ is barrelled of class $\aleph_0$ if Σ is a σ-algebra.

Some applications to vector-valued and scalar-valued measures are made in Chapter 7. The Bochner integral and barrelledness questions for the related L^p spaces of functions with values in normed spaces form the subject matter of Chapter 8, further vector-valued function spaces, including ℓ_0^∞-type spaces, are examined in Chapter 9 and Chapter 10 deals with "the Pettis integrable function space". The book ends with a list of open problems and an extensive list of references. Each of Chapters 1–10 is supplemented by a section of Exercises and a section of "Notes and Remarks", which outline the origins of many of the results discussed.

Much of the material is very technical, so that a careful reading of the text is no easy undertaking. However the authors have presented their subject well and the English is generally quite acceptable. There are of course a few minor language discrepancies and some typographical errors, but none of these should cause the reader any problems. It is unfortunate that this compendium of high-level research contains the following erroneous assertion on its page 2: ". . . a subset V of a space E is a barrel if and only if V° is bounded in $\sigma(E', E)$ . . . "; it may be corrected as follows: ". . . a subset V of a space E is a barrel if and only if it is the polar of a $\sigma(E', E)$-bounded set . . . '..It is also unfortunate that the authors have not provided an index – with such technical material it is easy to forget definitions but not always easy to locate them again. These are all minor reservations and I can certainly recommend the book to anyone with a research interest in barrelled spaces.

I. TWEDDLE

HERNÁNDEZ, E. and WEISS, G. *A first course on wavelets* (CRC Press, Boca Raton–New York–London–Tokyo, 1996), 512pp., 0 8493 8274 2, $64.95.

This is a very nice book! The content is very interesting and a little out of the ordinary for a book on wavelets. I found it to be written in a style which is easy to read. The authors provide plenty of encouragement to dig deep into the book and seem able to maintain an exposition which continually stirs the reader's interest. I would think any analyst would find things of interest in this book, but researchers in wavelets, harmonic analysis, functional analysis, approximation theory, and signal processing will certainly enjoy large sections of the book.

The authors deal exclusively with wavelets on the real line rather than the more general multivariate wavelets. To my mind this is a wise choice. The overheads involved in dealing with the multivariate case are worth while mainly from the point of view of applications. Otherwise, the added complexity simply serves to obscure the beautiful principles which underlie the theory. In the univariate context an orthonormal wavelet on $\mathbb{R}$ is a function ψ in $L^2(\mathbb{R})$ such that $\{\psi_{j,k} : j, k \in \mathbb{Z}\}$ is an orthonormal basis of $L^2(\mathbb{R})$, where

$$\psi_{j,k}(x) = 2^{(j/2)}\psi(2^j x - k), \qquad j, k \in \mathbb{Z}, x \in \mathbb{R}.$$

In 1910 A. Haar discovered what is now known as the Haar wavelet. It is the function which has value +1 on the interval $[0, 0.5)$, -1 on the interval $[0.5, 1)$ and is zero elsewhere. One of the first uses one might think of for a wavelet would be in making an L^2-decomposition of a function or signal into its wavelet series $\sum_{k,j\in\mathbb{Z}} c_{j,k}\psi_{j,k}$. Here each $c_{j,k}$ is the inner product of the function with $\psi_{j,k}$. At this point the fact that the Haar wavelet has compact support is seen to be a very nice feature—it greatly simplifies the computation of these inner products. A nasty feature (at least to a signal processing person) is that ψ is discontinuous. The construction of compactly supported smooth wavelets was a long time coming and represented a major contribution to the field. It is now common to use the multiresolution analysis idea of Mallat to carry out this construction. The details of this programme are the meat of the first four chapters of the book.

The book then moves on to consider some applications of wavelets. These are applications in