CONNECTION BETWEEN THE HIPPARCOS CATALOGUE AND THE FK5

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ABSTRACT

Two methods are presented which can be applied to determine the rotation of the Hipparcos system of stars with respect to the FK5. These methods are presently tested in numerical simulation runs.

1. INTRODUCTION

After the end of the ESA astrometry mission Hipparcos a catalogue of positions, proper motions and parallaxes of 100 000 stars with an internal accuracy of a few milliarcsec resp. milliarcsec per year will be available. Due to its technical specifications the satellite will not be able to observe directly extragalactic objects and there may be difficulties in observing minor planets. That is why the Hipparcos system of proper motions may contain an unphysical rotation which can be detected by a comparison with the system of the forthcoming FK5.

2. THE FUNDAMENTAL REFERENCE SYSTEM

At present, the FK4 is the best available approximation to an inertial reference frame. Nevertheless it is not free from rotation. As has been shown by Fricke (1977, 1982) the equinox of FK4 is moving at a rate of $\dot{E} = +1''.275 \pm 0''.15$ (m.e.) per century.

A second effect has to be taken into account. The system of FK4 proper motions is based on an assumption on a certain value for the constant of general precession. With the adoption of the IAU (1976) System of Astronomical Constants the error in this constant is reduced to $\pm 0''.15$ per century. With these improvements the uncertainty of the rotation of the new fundamental system, the FK5, will be around $0''.2$ per century, the same value that we expect for the accuracy of the best proper motions for Hipparcos.

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3. ROTATION MATRICES

The classical method to describe a rotation between two coordinate systems consists in determining the elements of the rotation matrix. This method has been described by Froeschlê and Kovalevsky (1982) who applied it to the link of Hipparcos to the extragalactic reference system. In the present case, where the angles of rotation between the Hipparcos system and the FK5 are quite small - much less than a second of arc - it is justified to use the matrix of infinitesimal rotation

$$A = \begin{pmatrix} 1 & a_{12} & a_{13} \\ -a_{12} & 1 & a_{23} \\ -a_{13} & -a_{23} & 1 \end{pmatrix}$$

so that the equations of condition for the positions at time t_0 read

$$(1) \quad \vec{y} - \vec{x} = (A - 1) \cdot \vec{x} \ .$$

1 is the unit matrix and $\vec{y}$ and $\vec{x}$ denote the cartesian coordinates of a star in the two different systems:

$$\vec{x} = \begin{pmatrix} \cos\delta \ \cos\alpha \\ \cos\delta \ \sin\alpha \\ \sin\delta \end{pmatrix} \qquad \vec{y} = \begin{pmatrix} \cos\delta' \ \cos\alpha' \\ \cos\delta' \ \sin\alpha' \\ \sin\delta' \end{pmatrix}$$

The equations of condition for proper motions are

$$(2) \quad \vec{\mu}_y - \vec{\mu}_x = B \cdot \vec{x} \ ,$$

where

$$B = \begin{pmatrix} 0 & b_{12} & b_{13} \\ -b_{12} & 0 & b_{23} \\ -b_{13} & -b_{23} & 0 \end{pmatrix}$$

is the time derivative of the matrix A, so that the transformation matrix for positions from one system to the other at time t is given by $A(t) = A + (t-t_0) \cdot B$. $\vec{\mu}_x$ and $\vec{\mu}_y$ are the cartesian proper motions in the two systems

$$\vec{\mu}_x = \begin{pmatrix} -\mu_\alpha \cos\delta \ \sin\alpha - \mu_\delta \sin\delta \ \cos\alpha \\ \mu_\alpha \cos\delta \ \cos\alpha - \mu_\delta \sin\delta \ \sin\alpha \\ \mu_\delta \cos\delta \end{pmatrix}$$

$\vec{\mu}_y$ is defined analogously using the primed quantities. It should be noted that equations (1) and (2) together supply only four independent equations per star because of the condition $||\vec{x}|| = 1$. It follows from

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the "orthogonality" of A that $||\vec{y}|| = 1$. For proper motions we have e.g. the condition $||\vec{\mu}_x|| = (\mu_\alpha \cos\delta)^2 + \mu_\delta^2$.

All existing astrometric catalogues show up systematic zonal deflections from an ideal sphere that will influence the results of the determination of the coefficients of rotation. This will be the case for the FK5 and also for Hipparcos. There systematic errors may arise, e.g. as a result of the unidirectional scanning law. Therefore it is necessary to determine the elements of the rotation matrices together with other systematic effects existing in the differences $\vec{x}_i - \vec{y}_i$ and $\vec{\mu}_{x_i} - \vec{\mu}_{y_i}$ (i = 1 .... N, where N is the number of stars common to the two systems). In practice, this can only be done if N is sufficiently large, so that the number of unknowns does not increase linearly with the number of equations. Such a method is described in the next motion.

4. SYSTEMATIC DIFFERENCES OF TWO ASTROMETRIC SYSTEMS DESCRIBED BY ORTHOGONAL FUNCTIONS

The method has been developed by Schwan (1977) and is employed for the determination of astrometric catalogue comparisons as described by Bien et al. (1978). Here only the main features are exposed; for details the reader is referred to the papers cited above.

Let Δ_i denote the difference Hipp - FK5 for the star i (Δ may stand for $\Delta\alpha \cos\delta$, $\Delta\delta$, $\Delta\mu_\alpha \cos\delta$, $\Delta\mu_\delta$), than Δ_i may be described by

$$(3) \quad \Delta_i = \sum_{j=1}^{g} c_j Z_j (\alpha_i, \delta_i, m_i) + \varepsilon_i \quad (i = 1 \; .... \; N).$$

ε_i is the random part in the differences, the other terms form the systematic part. The Z_j represent a set of orthogonal functions on a three dimensional space whose coordinates have the meaning of right ascension, declination and magnitude. (One could imagine that there are different rotation angles between the systems depending on different magnitudes). For Z_j we may use products of Hermite- and Legendre polynomials and Fourier terms. If we choose a strictly orthogonal representation all the coefficients c_j are independent of each other, so that they can be computed separately without solving a whole system of normal equations. In other words we are able to test one function Z_j after another to check if they yield significant contributions to the Δ_i. F-test is used for this purpose. Advantages of this system of functions over spherical harmonics are explained in Bien et al. (1978).

5. RELATION BETWEEN THE "SYSTEMATIC DIFFERENCE METHOD" AND ROTATION MATRICES

Putting $\delta' = \delta + \Delta\delta$, $\alpha' = \alpha + \Delta\alpha$, $\mu'_\alpha = \mu_\alpha + \Delta\mu_\alpha$, $\mu'_\delta = \mu_\delta + \Delta\mu_\delta$, writing down explicitly equations (1) and (2), and treating the quantities Δ as small (neglecting higher order terms) we determine the relation between the Δ_i

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and the matrices A and B. The result reads:

$$(4)\qquad \Delta\delta_i = -a_{13}\cos\alpha_i - a_{23}\sin\alpha_i$$

$$(5)\qquad \Delta\alpha_i\cos\delta_i = -a_{12}\cos\delta_i - a_{13}\sin\alpha_i\sin\delta_i + a_{23}\cos\alpha_i\sin\delta_i$$

$$(6)\qquad \Delta\mu_{\delta_i} = -b_{13}\cos\alpha_i - b_{23}\sin\alpha_i$$

$$(7)\qquad \Delta\mu_{\alpha_i}\cos\delta_i = -b_{12}\cos\delta_i - b_{13}\sin\alpha_i\sin\delta_i + b_{23}\cos\alpha_i\sin\delta_i\ .$$

These equations demonstrate easily the meaning of the elements of the rotation matrices. a_{12} describes a roation around the pole, a_{13} and a_{23} the rotation of the pole in the direction of the equinox resp. perpendicular to that direction; the latter quantities are better determined by equation (4) than by (5). The coefficients a_{12}, a_{13} and a_{23}, as well as b_{12}, b_{13} and b_{23} can thus be found by comparison with the corresponding coefficients c_j in the equations (3).

6. NUMERICAL TESTING OF THE METHODS

In order to test the quality of the methods described above and of the resp. numerical algorithms several simulation catalogues have been created in a way described below. This work is still in progress, so only an overview of the present situation will be given. The algorithms are tested in a case, where two catalogues of positions only are compared.

The calculation of the three elements of the rotation matrix (method 1) is performed by a least-squares method based on the singular value decomposition of the matrix, whose rows are given by the right hand sides of equation (1). The selected algorithm is described in Lawson and Hanson (1974), whereas for the determination of systematic differences (method 2) the procedures are given in the above mentioned paper by Bien et al. (1978).

All simulation catalogues are based on FK4. In order to simulate the real situation, individual improvements of the FK4 positions have been added to the FK4 coordinates in a random way such that the averaged individual difference between a position in a simulation catalogue and in FK4 are

$$\overline{\Delta_{\alpha\cos\delta}} = 5.5 \quad \text{milliseconds}$$

$$\overline{\Delta\delta} = 0.065 \text{ seconds of arc}$$

The data in the simulation catalogues are, as in FK4, rounded to 1 millisec in right ascension and 0"01 in declination. As shown in table 1 only rotations around two angles a_{12} and a_{23} are simulated up to now. It turns out that both methods work satisfactorily if one takes into account the smallness of the rotation angles compared to the individual differences and the effect of rounding. The slightly superior behaviour of method 1 over method 2 cannot easily be explained at present; further

Table 1. Present results of the numerical simulations. All angles are given in units of 0.01 seconds of arc.

		Input	Method 1	Method 2
Simulation 1	a_{12}	3.0	2.84 ± 0.17	2.43 ± 0.56
	a_{23}	1.0	0.65 ± 0.17	0.64 ± 0.24
Simulation 2	a_{12}	30.0	30.19 ± 0.17	31.22 ± 1.11
	a_{23}	10.0	9.62 ± 0.17	9.64 ± 0.23

simulations should clearify the situation. In these simulations zonal distortions of the sphere will be introduced in order to investigate its influences on the determination of rotation. Special efforts will be made to study the situation at the (equatorial) polar caps. There the differences between Hipparcos and FK5 should be clearly revealed. The reason for this is the fact that the poles are regular points in the Hipparcos observations, whereas in FK5, because it originates from meridian observations on ground, the poles are singularities.

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Discussion:

HEMENWAY: Two obvious reasons for carrying out the reduction of HIPPARCOS to the System of the FK5 are:

1. If the Space Telescope and other techniques work we will have a direct measure of the residual motion of the FK5 system, and
2. If these techniques fail to reach their expected accuracies, then the FK5 will provide the best available reference frame and coordinate system.

RÖSER: I agree.

EICHHORN: I am somewhat concerned about the following. The FK5 is supposed to be a self-consistent system, which means that there are no systematic errors that could not be modeled by a rotation on some axis. If the FK5 and the HIPPARCOS catalogue are compared and one uses a model for the systematic differences which is a sum of cylindrical harmonics, you are admitting that at least one of these is not self-consistent. Where are the weights coming from? Which will you then regard as standard?

RÖSER: A comparison can only yield differences, represented by a model. The origin of the residuals cannot be found by such a procedure.

EICHHORN: You are then not constructing an average system of FK5-HIPPARCOS?

RÖSER: I don't think this should be done. All one can do is subject the HIPPARCOS system to a rotation to make it agree as far as possible with FK5.

CORBIN: Are you going to use the entire FK5 for the comparison with HIPPARCOS?

RÖSER: Yes.

CORBIN: Then I would suggest that you also make separate comparisons using the bright FK5 (FK4) stars and the extension stars namely the FK4 supplement and faint fundamental stars. This will give a good opportunity to see how well the extension stars represent the FK5 system using HIPPARCOS as an intermediary.

RÖSER: This is true; also, one might make a comparison avoiding the polar zones to assess the peculiarities of the data concerning stars near the pole in ground-based catalogues.