ERRATUM: TWO HYPERBOLIC SCHWARZ LEMMAS

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In [1] there is an error, as pointed out to us by J.M. Isidro. In that paper we consider the set \mathcal{R}_m of all *m*-rotations, that is, $\mathcal{R}_m = \{cz^m : |c| = 1\}$; the set $\operatorname{Aut}(\mathbb{D})$ of the automorphisms of the unit disc \mathbb{D} and the set of the *m*-automorphisms of \mathbb{D} , $\operatorname{Aut}_m(\mathbb{D}) = \{\psi \circ R \circ \varphi : \psi, \varphi \in \operatorname{Aut}(\mathbb{D}), R \in \mathcal{R}_m\}$. We asserted that

(1)
$$\left\{f \in \operatorname{Aut}_m(\mathbb{D}): f(a) = b\right\} = \{\varphi_b \circ R \circ \varphi_a: R \in \mathcal{R}_m\},$$

where $\varphi_a(z) = (a-z)/(1-\overline{a}z)$. Equality (1) is not true. For instance, it suffices to consider $f(z) = \varphi_{1/4} \circ z^2 \circ \varphi_{1/2}$. We have that $f \in \operatorname{Aut}_2(\mathbb{D})$, f(0) = 0 and, after calculations, $f(z) = (7z^2 - 6z)/(6z - 7)$. Then it is evident that will never hold $f(z) = \varphi_0 \circ R \circ \varphi_0$ with $R \in \mathcal{R}_2$, because φ_0 is equal to identity.

However it is easy to check that the first set in (1) contains the second one. This lets us save Lemma 2.1 [1] as follows.

LEMMA 2.1. Assume that $m \in \mathbb{N}$, $a \in \mathbb{D}$, $f \in H(\mathbb{D})$, |f| < 1 on \mathbb{D} and $\mu(f, a) \ge m$. Then we have

(2)
$$\left|\frac{f(z) - f(a)}{1 - \overline{f(a)}f(z)}\right| \leq \left|\frac{z - a}{1 - \overline{a}z}\right|^m \quad (z \in \mathbb{D})$$

and $|f^{[m]}(a)| \leq m!$.

Further, if either equality holds in (2) for some $z \neq a$ or $|f^{[m]}(a)| = m!$ then $f \in \operatorname{Aut}_m(\mathbb{D})$.

With this new auxiliary result all the other statements in [1] can be established with the same proofs.

References

 L. Bernal-González and M.C. Calderón-Moreno, 'Two hyperbolic Schwarz lemmas', Bull. Austral. Math. Soc. 66 (2002), 17-24.

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