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Addendum to "Nearly Countable Dense Homogeneous Spaces"

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Abstract. This paper provides an addendum to M. Hrušák and J. van Mill "Nearly Countable Dense Homogeneous Spaces." Canad. J. Math. **66** (2014), 743–758.
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It was brought to our attention by Su Gao that the proof of Theorem 5.2 in our paper is incomplete. We are indebted to him for this observation. The aim of this note is to correct this.

Theorem 5.2 *Let G be a closed subgroup of S_∞ and let κ be the number of orbits for the canonical action $G \times 2^{\mathbb{N}} \to 2^{\mathbb{N}}$. Then there is an action of a Polish group H on $X = \mathbb{N} \times [0,1)$ such that X has κ H-types of countable dense sets.*

Proof Let G act on X in the following natural way: $(g,(n,t)) \longmapsto (g(n),t)$ for $g \in G, n \in \mathbb{N}, t \in [0,1)$. Put

$$F = \left\{ f \in \mathscr{H}(X) : (\forall\, n \in \mathbb{N})(f(n,0) = (n,0)) \right\}.$$

Then F is a closed normal subgroup of $\mathscr{H}(X)$ and hence is Polish. Moreover, for any two countable dense subsets D and E of $\mathbb{N} \times (0,1)$ there exists $f \in F$ such that $f(D) = E$. Treat G also as subgroup of $\mathscr{H}(X)$. The Polish semi-direct product group $H = G \rtimes F$ acts on X as follows: $((g,f),x) \mapsto (f \circ g)(x)$ for $f \in F,\ g \in G,\ x \in X$. Note that topologically, $H = G \rtimes F$ is $G \times F$, but its group operation $*$ is given by

$$(g_1, f_1) * (g_2, f_2) = (g_1 g_2, f_1 g_1 f_2 g_1^{-1}).$$

A typical countable dense subset of X has the form $D \cup A$, where D is a countable dense subset of $\mathbb{N} \times (0,1)$, and $A \subseteq \mathbb{N} \times \{0\}$. By identifying $\mathscr{P}(\mathbb{N} \times \{0\})$ and $2^{\mathbb{N}}$ in the standard way, it is clear that we get what we want. ■

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