CORRECTION TO THE PAPER "ON FUNCTIONS AND EQUATIONS IN DISTRIBUTIVE LATTICES"

by SERGIU RUDEANU

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In the above mentioned paper, published in these *Proceedings*, Vol. **16** (Series II), Part 1, June 1968, pp. 49-54, I have generalized some results obtained by Professor R. L. Goodstein for distributive lattices with 0 and 1, to the case when the lattice L is not required to have least and greatest elements.

The proofs were based on the fact that every lattice function $f(x)$ can be written in the form $f(x) = A \cup Bx$, with $A \leqq B$. However, Professor James C. Abbott has kindly called my attention to the fact that the above property implies the existence of 0 and 1. For, the representation of the identity function $f(x) = x$ yields $a \leqq a \cup bx = x \leqq a \cup b = b$, i.e. a and b are the least and greatest elements, respectively.

However, the results in my paper can be saved by appropriate extensions of the definitions. Namely, we shall consider that the representation $A \cup Bx$, with $A \leqq B$, includes also the functions Bx and $A \cup x$. For the particular case of the former functions, any inequality of the text which is of the form $A \leqq D$ or $CA \leqq D$ will be considered as automatically fulfilled. For the latter functions, any inequality of the form $D \leqq B$ or $D \leqq B \cup C$ will be considered as automatically fulfilled. It is easy to see that continuing in this way we can recapture all the theorems, with suitably extended meanings.

Thus, for instance, such a specialization of Lemma 1 states that the inequality $bx \leqq c \cup dx$ is equivalent to $bx \leqq c \cup d$; also the inequality $a \cup bx \leqq dx$ is possible only if a is the least element of the lattice (from (7) which reads $a \leqq x$) and if this is the case, $a \cup bx \leqq dx$ holds if and only if $bx \leqq d$; etc.

An alternative (but essentially equivalent) way is the following: embed L in a lattice $\bar{L}$ with 0 and 1, associate to each lattice function $f : L^n \rightarrow L$ the lattice function $\bar{f} : \bar{L}^n \rightarrow \bar{L}$ which has the same formal expression, apply Goodstein's theorems to $\bar{f}$ and interpret the results in terms of the function f.

As a matter of fact, even for lattices with 0 and 1, Theorem 4 is more comprehensive than Goodstein's corresponding result. Theorem 5, which has no analogue in Goodstein's paper, refers to biresiduated lattices, which have necessarily 0 and 1 ($a : a = 1$, $a :: a = 0$).

INSTITUTUL DE MATEMATICĂ
CALEA GRIVIȚEI 21
BUCUREȘTI 12
ROMANIA

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