Tremendous steps forward in understanding the dark world of the type III factor were made through the work of Tomita, Takesaki and Connes in the late 1960s and early 1970s. Spectral invariants, arising from the modular automorphism group of the Tomita–Takesaki theory, enabled Connes to sub-classify type III factors as type III_λ $(0 \leqq \lambda \leqq 1)$. On the other hand, Takesaki's duality theorem for crossed products, together with the modular theory once again, permitted powerful links to be established between type III algebras and the (better understood) type II algebras. These developments led on to the work of Connes, and finally Haagerup, which showed that for hyperfinite factors (inductive limits of matrix algebras) the classification I_n, I_∞, II_1, II_∞, III_λ is complete up to *-isomorphism.

In contrast with other recent books which offer a more rounded introduction to the theory of both von Neumann algebras and C*-algebras, Sunder's book aims as directly as possible for the advances outlined above (excluding the hyperfinite theory). The emphasis is on the basic ideas rather than complete detail, so that whilst most of the important results are stated in full generality, proofs are often given under certain simplifying assumptions (typically, as in the case of the Tomita–Takesaki theorem, that a particular operator is bounded). Although such assumptions may be very restrictive in practice, it seems a reasonable approach for someone wishing to acquire the flavour of this area "first time around" without being unduly daunted by too many technicalities. Even so, a solid background in analysis is very much a prerequisite. The author is careful to point out the short-cuts that are taken and generally takes care to indicate the sources that the reader should consult for further details.

The first two chapters deal with the pre-Tomita–Takesaki era. Chapter 0 reviews some basic operator theory, topologies for operator algebras and the double commutant theorem, and gives an introduction to the predual which is perhaps a little too heavily biased towards the special case of the trace class. Choosing what to include in such an introductory chapter will always be a rather difficult matter, but the functional calculus could perhaps have been treated more fully here. Chapter 1 is devoted to the basic type I, II, III-classification, with the treatment quickly specializing to the case of factors. In the comparison theory the author chooses to work with subspaces rather than projections, and this leads to some awkward notation with italic and calligraphic *M*'s in close proximity.

The Tomita–Takesaki theory is covered in Chapter 2, with sections on weights, the KMS condition and the Radon–Nikodym theorem of Pedersen and Takesaki. Chapter 3 begins with Connes' unitary cocycle theorem on the essential uniqueness of the modular automorphism group and then proceeds via a discussion of spectral theory to the III_λ classification. Chapter 4 covers discrete and continuous crossed products, Takesaki's duality theorem and the links between type III and type II algebras, and gives constructions for factors of the various types.

A considerable number of results throughout the text are dealt with in the form of exercises. These form a sufficiently important part of the work that the reader is necessarily forced into action. However, the load is lightened by the author's engaging sense of humour which pervades the book. For example, when pointing out that a possibly unbounded operator might turn out to be bounded, he writes "when that happens, the consequent relief would, it is hoped, offset the conflict with our notational convention". Few mortals will disagree! Backed up by other texts and original sources, Sunder's book should provide a good introduction to a difficult and important area of mathematics.

R. J. ARCHBOLD

BURN, R. P., *Groups: a path to geometry* (Cambridge University Press, Cambridge) xii+242 pp., cloth: 0 521 30037 1, 1985, £30, paper: 0 521 34793 9, 1987, £9.95,

Several authors have attempted to write books on group theory providing an alternative approach to a standard course. As well as R. P. Burn's book, another book from the Cambridge

University Press stable providing such an alternative approach springs immediately to mind. This is *Presentations of Groups* by D. L. Johnson, where an understanding of the abstract theory of generators and relations for groups is obtained by "observing and performing easy and concrete calculations". D. L. Johnson's book is, of course, at a more advanced level than R. P. Burn's book. Other books aim to encourage the learning of elementary group theory by giving many group theory problems together with comprehensive solutions. The subject of this review provides yet another approach to group theory. In aiming to be "faithful to the historical origins of the theory" the groups discussed in the book are groups of transformations. Most of the groups described in the book are examples in two- and three-dimensional space. As with the other authors, R. P. Burn feels very strongly that mathematics is best learnt doing examples.

The twenty-three chapters of the book are divided into three main sections. The first five chapters including chapters on the Möbius group and the regular solids are followed by a more abstract introduction to group theory describing topics such as group axioms, cosets, direct products, homomorphisms and conjugacy. The final chapters are concerned with linear fractional groups, affine groups, orthogonal groups and wallpaper groups. The layout of each chapter consists of a particular topic taught through a series of examples with cross-references to other more standard approaches. At the end of each chapter there is a summary of the theory described. Before answers are given to the questions in the chapter there is a historical note and this I found especially interesting. Even in the contents at the beginning of the book there is, with each chapter, a brief description of the main result described in that chapter.

The book is very much at the level of advanced school-pupils (perhaps very advanced as far as the complete book is concerned) and undergraduates in their early years of study. The book is neatly presented and well-produced and the author manages to convey a lively enthusiasm for his subject. I recommend it to all those who have an interest in the examples that form the basis of a beginning course in group theory. Many mathematicians will enjoy its geometrical approach.

C. M. CAMPBELL

McBride, A. C., *Semigroups of linear operators: an introduction* (Pitman Research Notes in Mathematics Series 156, Longman Scientific and Technical, 1987), pp. 134, 0 582 99484 5, £14.

The author has written this introduction to semigroups of linear operators for anyone with an Honours student's knowledge of complex and functional analysis preferably, though not essentially, including the Lebesgue integral. His first chapter explains the notion of a one-parameter semigroup, and discusses four examples which are referred to throughout the text, namely the semigroup of translations, the Gauss–Weierstrass and Poisson semigroups, and the fractional power semigroup. This is followed by chapters on infinitesimal generators and on resolvents and the Hille–Yosida–Phillips theorem. Next come two shorter chapters, on exponential formulae which enable one to recover a semigroup from its generator, and on the generation of one semigroup from another by a relation such as $U(t)=\int_0^\infty p_t(u)T(u)\,du$ where p_t is a probability density function. The final chapter shows how the preceding theory can be applied to the solution of initial-value problems associated with the partial differential equations $u_t=u_x$, $u_t=u_{xx}$ and $u_{tt}=u_{xx}$ while referring the reader to more advanced texts for the study of other abstract Cauchy problems.

Many proofs are given with complete rigour, but the author does not hesitate on appropriate occasions to give just an outline proof or to omit details completely. His informal style and excellent judgment make this a very readable introduction to the subject, for example for someone who might encounter semigroups in p.d.e. theory but would find the specialist literature formidable. The test has been reproduced photographically from a very pleasing typescript, and I found only a small number of misprints.

PHILIP HEYWOOD

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