On a theorem in Determinants.

By Sir William Thomson.

The Ancient Methods for the Duplication of the Cube.

By J. S. Mackay, M.A., F.R.S.E.

The object of the present paper is merely to exhibit the methods employed by the ancient Greek geometers in their solution of this celebrated problem. A critical discussion of these methods, of the origin of the problem, and perhaps also an account of more recent researches and a notice of the literature connected therewith may form the subject of a subsequent paper.

The mythical origin of the problem is told by Eratosthenes in his letter to King Ptolemy III. (Euergetes), and mention is there made of the form into which Hippocrates of Chios (about 444 B.C.) recast the problem. Hippocrates's contribution may be set out thus:—

If AB, CD, EF, GH be four straight lines,

then $$AB : GH = \left\{ \begin{array}{c} AB : CD \\ CD : EF \\ EF : GH \end{array} \right\}.$$

Now if $$AB : CD = CD : EF = EF : GH,$$

then $$AB : GH = \text{triplicate of } AB : CD,$$
$$= AB^3 : CD^3.$$

Hence if $$GH = 2\,AB, \quad CD^3 = 2\,AB^3.$$

If, therefore, AB^3 is a given cube, and it is required to double it, take a straight line GH equal to 2AB, and between AB and GH insert two mean proportionals CD, EF. CD^3 will be the cube required.

The solutions which follow are translated from the commentary of Eutocius of Ascalon (about 555 A.D.) on Archimedes's treatise *Of the Sphere and Cylinder*. See Torelli's *Archimedis Quae Supersunt Omnia* (Oxonii, 1792), pp. 135-149, or Heiberg's *Archimedis Opera Omnia* (Lipsiae, 1881), pp. 67-127. I have retained the order or disorder in which Eutocius gives the solutions, but have affixed approximate dates to their authors. The solution of Pappus, which is also given by Eutocius, I have translated from Pappus himself. See Commandine's *Pappi Alexandrini Mathematicae Collectiones*, or

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