

THE GLACIOLOGICAL STUDIES OF THE BAFFIN ISLAND EXPEDITION, 1950

Part V: ON THE VARIATION OF THE SHEAR STRESS ON THE BED OF AN ICE CAP

By SVENN ORVIG

(Arctic Institute of North America)

ABSTRACT. Some recent work by Dr. J. F. Nye on the calculation of the thickness of ice sheets has prompted the author to apply Nye's formula to the Barnes Ice Cap in Baffin Island. A gravimetric survey of the southern lobe was carried out in 1950, and the data from four of the traverses have been used to calculate the variation of the shear stress on the bed. Some of the values obtained are exceptionally low, possibly explained by the fact that the lines of travel over the surface do not necessarily correspond to the lines of greatest slope or lines of flow. Only in one direction are the values relatively high, and it is concluded that there is a considerably greater movement of the ice in this direction. This conclusion is in agreement with observations on the ground.

RÉSUMÉ. Des investigations récentes du docteur J. F. Nye à l'égard de la méthode de calculer l'épaisseur de l'Inlandsis ont suggéré à l'auteur l'idée d'appliquer la formule Nye à la calotte de glace de Barnes à Baffin Island. Un relevé gravimétrique du lobe sud a été fait en 1950, et on en a employé les données de quatre des parcours afin de calculer les variations des tensions de cisaillement au lit du glacier. Un certain nombre des valeurs ainsi obtenues se montrent exceptionnellement faibles, ce qui s'explique peut-être par le fait que les lignes de parcours ne correspondent pas nécessairement aux lignes de pente maximum ou aux lignes d'écoulement. Ces valeurs ne sont relativement fortes que dans une seule direction, et on en a conclu qu'il existe un mouvement de la glace beaucoup plus rapide de ce côté. Cette conclusion est d'accord avec les observations faites sur le terrain.

In a recent paper by Dr. J. F. Nye of the Cavendish Laboratory, Cambridge, a method is presented by which the thickness of ice sheets can be calculated.¹ The tendency of the ice to spread laterally and to flow downhill is balanced by the inward shear force exerted by the rock floor. By Nye's method it is possible to make an approximate calculation of the variation of the shear stress on the bed of an ice cap if the bed and ice surface profiles are known.

For a sheet of ice resting on a bed the slope of which changes both in magnitude and direction, the shear stress on the bed is calculated by Nye to be approximately $\tau = \rho gh \sin \alpha$, provided that the local values of the thickness of ice (h) and surface slope (α) are used, and that these values do not change much in distances of order h . ρ is the density of the ice, and g is the gravitational acceleration.

Calculations on alpine valley glaciers show that the shear stresses on their beds are between 0.5 and 1.5 bars.² (1 bar = 10^6 gm./cm. sec.² = 10^6 dynes/cm.²). Nye assumes for a first calculation that τ is constant ~ 1 bar over the floor of a moving ice sheet, "— as shear stresses much smaller than 1 bar produce extremely small rates of strain, while shear stresses much greater than 1 bar produce very much larger rates of deformation than those existing in glaciers and ice-sheets." In the case of an almost stationary mass of ice, a considerably smaller shear stress can exist on the bed.

With a constant value of τ (shear stress), and if the height of the bed is known, the absolute height of a (former) ice surface can be calculated. Another type of problem that can be solved by Nye's method is the calculation of the thickness of ice of a glacier, when the heights and slopes of the ice surface are known. One needs to assume a value for τ in this case also.

Nye found that $\tau = 0.88$ bar gave the best fit when he compared his calculated heights of the Greenland Ice Cap with the observed surface as reported by the French Greenland Expedition of 1948–51. He therefore assumed τ to be 0.88 bar everywhere in Greenland, and calculated the height of the bed of the Greenland Ice Cap by using surface contours from a survey map of scale 1 : 5,000,000. The values for τ and α were uncertain, but Nye found some significant features. Even with the value of τ reduced to 0.5 bar, the floor of the ice cap appears to be as much as 1200 m. below sea level, at its lowest point.

It is now possible to apply Nye's formula in another area. The southern lobe of the Barnes Ice Cap in Baffin Island is well known as a result of the observations in 1950 by members of the Arctic Institute of North America expedition.³

A gravimetrist from the Dominion Observatory in Ottawa took part in the expedition, and the results of his work on the ice cap have now been published.⁴ Littlewood's work aimed at making a survey of the southern lobe of the Barnes Ice Cap; this lobe is roughly circular with a radius of approximately 10 miles (16 km.) (see Fig. 1, p. 244). Three long and a number of shorter traverses were made, establishing 155 gravity stations. The objects of the survey were to attempt to determine the thickness of the ice and to outline the topographical features of the bed. Littlewood gives for all the stations: latitude, longitude, distance from a reference station, ice-surface elevation, rock elevation, ice thickness, and observed gravity. In his calculations he uses an ice density of 0.91 gm./cm.³. This is the value found by the glaciologists at Camp A, near the centre of the lobe,⁵ and has been used in the present calculations. The observed gravity on the ice cap varied between 982.4742 cm./sec.² (near the edge on the north side) and 982.3674 cm./sec.² (at Camp A). The difference is too small to influence the calculated value of τ in any more than the third decimal. The value for g as observed at Camp A is therefore used at all stations in calculating τ from Nye's formula.

The slope has been calculated from the distance and the difference in elevation between neighbouring stations. Littlewood notes in his paper that the absolute elevation for each station may be in error by as much as 10 per cent, since all elevations are relative to the elevation of Camp A, which was determined by a number of aircraft altimeter readings. However, this error in no way influences the calculations, as only differences in elevation are used in the two cases. Assuming that the combined errors in the computed ice thickness is within ± 35 feet (10.7 m.) (Littlewood, p. 121), the error in the calculated value of τ will only be present in the third decimal.

Fig. 1 shows the southern lobe of the Barnes Ice Cap with the lines of the gravity traverses. Fig. 2 (p. 245) shows the cross-sections on these traverses. The whole ice cap is shown in Fig. 2, p. 3, *Journal of Glaciology*, Vol. 2, No. 11, March 1952.

The tables on page 246, numbered I to IV, give the calculated values for τ (the shear stress) for a number of stations, using Nye's formula: $\tau = \rho gh \sin \alpha$. The traverses A-B and A-C slope from Camp A down to the edge of the ice cap. Camp A was not located on the highest point of the southern lobe, however, and the traverse A-D is therefore treated in two separate parts, the first is the slope from station A-142 (the highest point) to Camp A, and the second is the north slope from A-142 to D.

The traverse L-K showed that the ice surface between these two stations forms a wave. Only seven stations sloping down eastwards (towards the edge) have been used.

These values for the shear stress on the bed have been calculated along the traverses which do not necessarily correspond to the lines of greatest slope or lines of flow.

The values of τ along the traverses do not differ greatly, except along A-142 to D where the values are quite high, and where they are caused by a combination of steep slope and relatively thick ice. The average slopes along the traverses are: Traverse A-B, $1^\circ 23'$; Traverse A-C, $1^\circ 07'$; Traverse A-D (part 1), $0^\circ 43'$; Traverse A-D (part 2), $2^\circ 27'$; Traverse L-K, $0^\circ 44'$.

It is reasonable to conclude, therefore, that there is a considerably greater movement of ice in the direction A-142 to D, *i.e.* towards the north-east side of the southern lobe, than in the other directions, especially as Glen has shown that the rate of strain in ice is proportional to a high power of the stress.⁶ This conclusion is in agreement with the observations of Goldthwait, who found that there was a general recession of the ice on the south-west and advance on the north-east. The net effect is a very slow shift north-eastwards of this end of the ice cap.⁷

The mean value of τ along the other traverses is around 0.40 bar, which is below the lower limit cited by Nye from alpine valley glaciers (0.5-1.5 bars), and should produce small rates of strain in the ice, particularly because the temperature of the ice is thought to be below the pressure melting point throughout.⁸ The investigations on the Barnes Ice Cap showed that it is nearly

stationary, and all signs indicated only small rates of deformation where slow retreat was in progress.⁸

It is probable that 0.40 bar is a reasonable magnitude of the shear force exerted by the rock floor under an ice cap in a topographical setting similar to that of the Barnes Ice Cap, with higher values to be expected in directions where the activity of the ice is greater, as is the case in a northerly direction from the neighbourhood of station A-142.

MS. received 17 November 1952

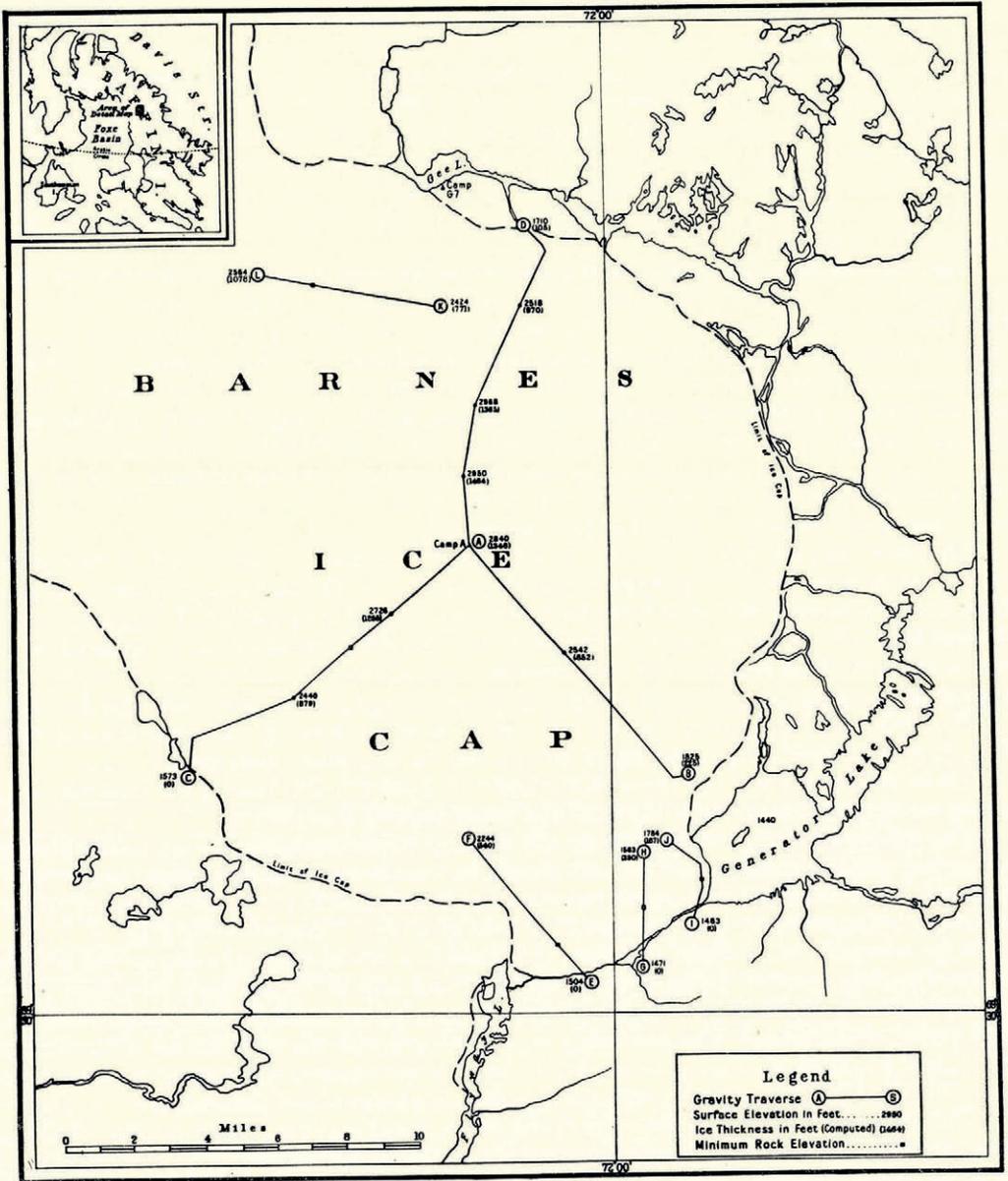


Fig. 1. The south-eastern lobe of the Barnes Ice Cap showing the lines of the gravity traverses (Reproduced by courtesy of the Editor of Arctic)

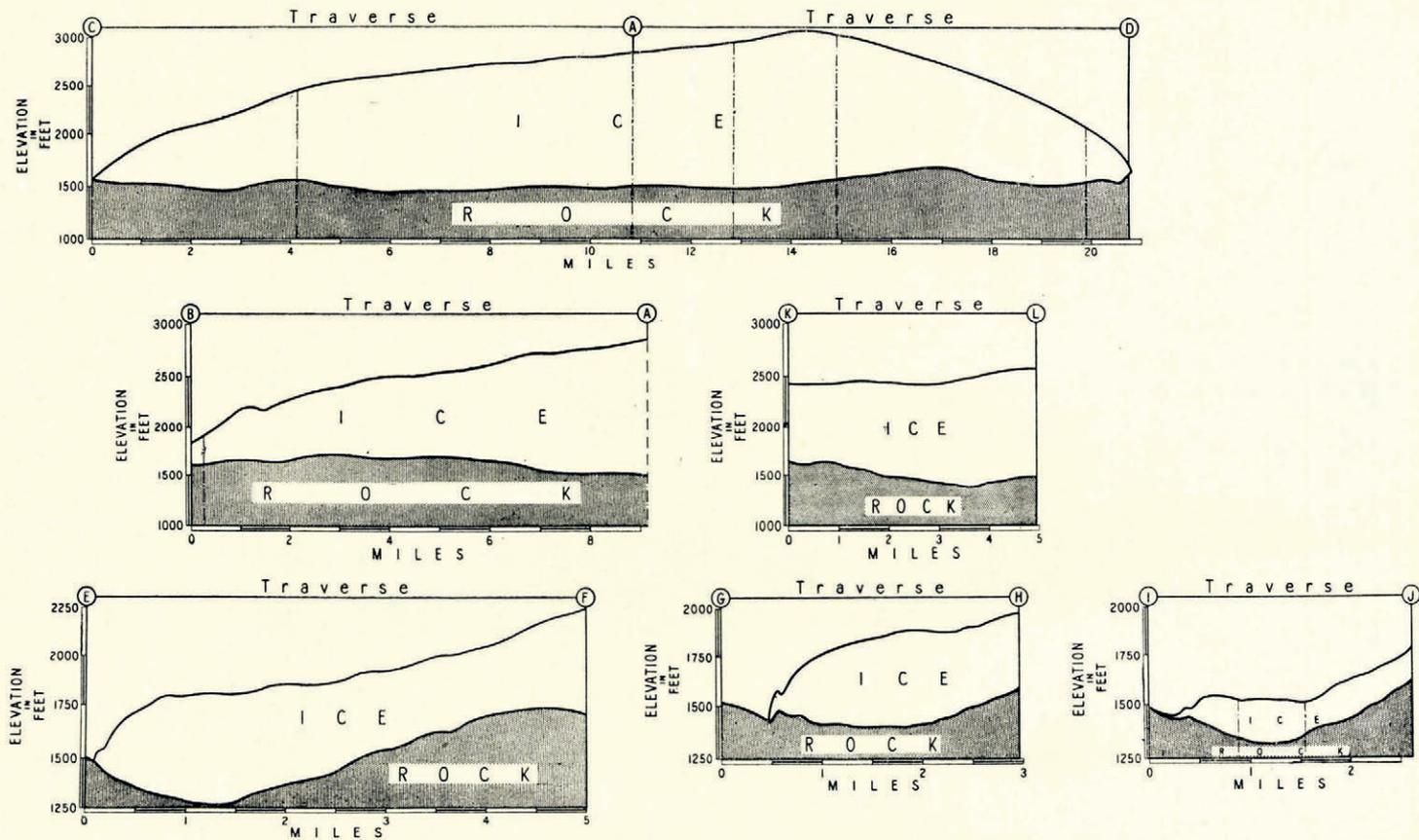


Fig. 2. Cross-sections of the Barnes Ice Cap along the lines of the gravity survey
(Reproduced by courtesy of the Editor of Arctic)

TABLE I. TRAVERSE A-B

Station	Dist. between stations, ft.	Surf. elevation, ft.	Diff. in elevation	sin α	Slope, α	Ice thickness, ft.	τ (bar)
A		2840					
B-15	3221	2791	49	0.0152	52'	1274	0.53
B-16	3327	2751	40	0.0120	41'	1235	0.40
B-17	3908	2711	40	0.0102	35'	1170	0.33
B-18	3749	2658	53	0.0141	49'	1039	0.40
B-19	3062	2591	67	0.0219	1° 15'	932	0.56
B-20	3960	2542	49	0.0124	43'	852	0.29
B-21	3855	2506	36	0.0093	32'	815	0.21
B-22	4541	2469	37	0.0081	28'	772	0.17
B-23	3115	2391	78	0.0250	1° 26'	673	0.46
B-24	3643	2318	73	0.0200	1° 09'	634	0.35
B-25	4013	2192	126	0.0314	1° 48'	548	0.47
B-26	3115	2151	41	0.0132	46'	498	0.18
B-27	1584	2026	125	0.0789	4° 31'	385	0.83
B-28	3115	1825	201	0.0645	3° 42'	225	0.40

Number of stations: 14
Arithmetic Mean Value of τ : 0.40 bar

TABLE II. TRAVERSE A-C

A		2840					
C-29	4805	2801	39	0.0081	28'	1322	0.29
C-30	3696	2776	25	0.0068	23'	1291	0.24
C-31	2904	2742	34	0.0117	40'	1246	0.40
C-32	4171	2726	16	0.0038	13'	1255	0.13
C-33	4382	2676	50	0.0114	39'	1219	0.38
C-34	4276	2630	46	0.0108	37'	1175	0.35
C-35	4171	2578	52	0.0125	43'	1101	0.37
C-36	4013	2513	65	0.0162	56'	993	0.44
C-37	3062	2446	67	0.0219	1° 15'	879	0.52
C-38	3802	2316	130	0.0342	1° 58'	777	0.72
C-39	5702	2118	198	0.0347	1° 59'	646	0.61
C-40	1478	2086	32	0.0217	1° 14'	596	0.35
C-41	3696	1977	109	0.0295	1° 41'	455	0.37
C-42	3854	1778	199	0.0516	2° 58'	247	0.35

Number of stations: 14
Arithmetic Mean Value of τ : 0.39 bar

TABLE III. TRAVERSE A-D
PART 1: FROM STATION A-142 TO CAMP A

Station	Dist. between stations, ft.	Surf. elevation, ft.	Diff. in elevation	sin α	Slope, α	Ice thickness, ft.	τ (bar)
A-142	2798	3050	48				
A-141	3485	3002	52	0.0172	59'	1520	0.71
A-140	3221	2950	33	0.0149	51'	1464	0.59
A-139	3802	2917	37	0.0102	35'	1430	0.40
A-138	3696	2880	40	0.0097	33'	1384	0.37
A		2840		0.0108	37'	1344	0.40

Number of stations: 5
Arithmetic Mean Value of τ : 0.49 bar

PART 2: FROM STATION A-142 TO D-153

A-142	2851	3050	26				
A-143	3749	3024	56	0.0091	31'	1479	0.37
A-144	4013	2968	92	0.0149	51'	1365	0.55
A-145	3908	2876	107	0.0204	1° 10'	1230	0.68
A-146	3855	2769	144	0.0274	1° 34'	1091	0.81
A-147	3168	2625	107	0.0374	2° 09'	1012	1.03
A-148	3591	2518	155	0.0338	1° 56'	970	0.89
A-149	4013	2363	202	0.0432	2° 29'	840	0.99
A-150	2851	2161	159	0.0503	2° 53'	637	0.87
A-151	1901	2002	122	0.0558	3° 12'	441	0.67
A-152	1478	1880	170	0.0642	3° 41'	335	0.59
D-153		1710		0.1150	6° 36'	105	0.33

Number of stations: 11
Arithmetic Mean Value of τ : 0.71 bar

TABLE IV. TRAVERSE L-K

L-171	1848	2564	10				
K-170	1637	2554	15	0.0054	19'	1090	0.16
K-169	1531	2539	19	0.0091	31'	1105	0.27
K-168	1690	2520	33	0.0124	43'	1111	0.38
K-167	1478	2487	25	0.0195	1° 07'	1102	0.59
K-166	1690	2462	29	0.0169	58'	1069	0.49
K-165	1637	2433	14	0.0172	59'	1022	0.48
K-164		2419		0.0086	30'	985	0.23

Number of stations: 7
Arithmetic Mean Value of τ : 0.37 bar

REFERENCES

1. Nye, J. F. A method of calculating the thicknesses of the ice-sheets. *Nature*, Vol. 169, No. 4300, 1952, p. 520-30.
2. ——— The mechanics of glacier flow. *Journal of Glaciology*, Vol. 2, No. 12, 1952, p. 82-93. (Reference on p. 86.)
3. Baird, P. D., and others. Baffin Island Expedition, 1950: A preliminary report. *Arctic*, Vol. 3, No. 3, 1950, p. 131-49.
4. Littlewood, C. A. Gravity measurements on the Barnes Ice Cap, Baffin Island. *Arctic*, Vol. 5, No. 2, 1952, p. 118-24.
5. Baird, P. D., Ward, W. H., and Orvig, S. The glaciological studies of the Baffin Island Expedition, 1950, Parts I and II. *Journal of Glaciology*, Vol. 2, No. 11, 1952, p. 6.
6. Glen, J. W. Experiments on the deformation of ice. *Journal of Glaciology*, Vol. 2, No. 12, 1952, p. 111-14.
7. Goldthwait, R. P. Development of end moraines in East-Central Baffin Island. *Journal of Geology*, Vol. 59, No. 6, 1951, p. 569.
8. Baird, P. D., Ward, W. H., and Orvig, S. *Op. cit.*, p. 11.

COMMENTS ON DR. ORVIG'S PAPER

By W. H. WARD

THE existence of relatively high values of the calculated bed shear stress on the traverse A-142 to D-153, and of the greater rates of flow implied, prompts me to suggest the possible causes of this condition.

Most of the exceptionally low values of τ on the other traverses appear to be associated with the line of the traverse being appreciably different from the line of greatest slope, but even neglecting these low values the A-142 to D-153 values are relatively high. Some variation in τ on any radial traverse might be expected. Sliding on the bed is not likely near the centre of an ice cap, but it must occur towards the edge since it is the only way in which rock debris can be brought to the surface of the ice under these conditions. Thus one might anticipate some increase in τ from the centre towards the zone where shearing on the bed has involved the entrainment of rock debris, because the shearing resistance of rock debris on rock is pressure-conditioned. Close to the edge, where the flow is compressional, and overthrusting occurs over stagnant ice, the value of τ might decrease. The values of τ in traverses A-B and A-C suggest this type of variation.

The shape of the whole ice cap has one particular feature that is perhaps significant in respect of the high values of τ along traverse A-142 to D-153. The ice cap is highest along a ridge that lies roughly parallel to, but curiously closer to, its north-east side (see Fig. 2 in Part I of this series of papers¹), and in the south-eastern lobe at least, the bed profile does not account for this displaced ridge.

The causes may be directly climatological, but I am inclined to discount this view. No large variations in the quite small total accumulation were found, and the current marginal accretion by perennial snowdrifts mentioned by Goldthwait,² particularly along the north-east side, would tend to flatten the slope and decrease τ . It has been shown³ that much of the ablation energy is discharged to the lakes adjoining the ice cap. At present, however, only a small part of the north-east side forms cliffs fringing these lakes. This is unlikely to cause a significantly greater discharge of ice, except in the immediate vicinity of the lakes where underwater ablation might make sufficient difference. There is plenty of evidence, however, that the lakes on the north-east side have been much deeper and more extensive in recent times. A noticeable strandline, some 200 ft. (61 m.) above the present level of Generator Lake, is visible about one-third of the way up the hillside in the background of Fig. 3 in Part II of this series of papers.⁴ Goldthwait has mentioned other strandlines to me, and a study of the R.C.A.F. aerial photographs of the larger lakes near the north end of the ice cap reveals the existence of scores of miles of high strandlines surrounding the present lake limits. In the past, almost the whole of the north-east side of the ice cap must have impounded water with a line of fringing ice cliffs. The effect of a general lowering of the water level of these formerly extensive lakes is to remove a large stabilizing force from the north-east side of the ice cap. This would cause a noticeable increase in the rate of flow of the ice towards the lakes that is likely to persist for a long time. The action is similar to the incidence of landslips following the lowering of lake levels.