

# Contents

	<b>Introduction</b>	<i>page</i> 1
<b>1</b>	<b>Preliminaries</b>	3
	1.1 Notation	3
	1.2 Hausdorff Measures	3
	1.3 Lipschitz Maps	5
<b>2</b>	<b>Rectifiable Curves</b>	6
<b>3</b>	<b>One-Dimensional Rectifiable Sets</b>	9
	3.1 Definitions and Tangents	9
	3.2 Densities	11
	3.3 Projections	13
	3.4 Analyst's Travelling Salesman Problem	14
<b>4</b>	<b>Higher-Dimensional Rectifiable Sets</b>	18
	4.1 Definitions and Area and Coarea Formulas	18
	4.2 Tangent Planes	20
	4.3 Tangent Measures	21
	4.4 Densities	23
	4.5 Projections	27
	4.6 Multiscale Approximations	29
	4.7 Reifenberg-Type Results	30
	4.8 Lebesgue Null-Sets and Singular Measures	32
	4.9 Minkowski Content and Discrete Energies	34
<b>5</b>	<b>Uniform Rectifiability</b>	37
	5.1 One-Dimensional Sets	37
	5.2 Lipschitz Maps and Approximation by Planes	38
	5.3 Density Ratios	41
	5.4 Projections	42

5.5	Basic Tools	43
5.6	Parabolic Rectifiability	44
<b>6</b>	<b>Rectifiability of Measures</b>	47
6.1	Some Basic Facts and Examples	47
6.2	Square Functions in General Dimensions	48
6.3	Square Functions and One-Dimensional Measures	49
6.4	Square Functions and Distance of Measures	51
<b>7</b>	<b>Rectifiable Sets in Metric Spaces</b>	52
7.1	Definition and Norm	52
7.2	Densities when $m = 1$	52
7.3	Densities and Area Formula for General $m$	54
7.4	Tangent Planes	56
7.5	Cheeger's Differentiability Spaces and Alberti Representations	56
7.6	Projections as Lipschitz Images	58
7.7	Metric Tangents	60
7.8	Menger Curvature	62
<b>8</b>	<b>Heisenberg and Carnot Groups</b>	63
8.1	The Heisenberg Group $\mathbb{H}^n$	63
8.2	Some Analytic Tools in Heisenberg and Carnot Groups	65
8.3	Definitions of Rectifiability	65
8.4	Rectifiable Sets and Tangent Subgroups	70
8.5	Densities and Tangent Measures	71
8.6	Projections	75
8.7	Uniform Rectifiability	75
<b>9</b>	<b>Bounded Analytic Functions and the Cauchy Transform</b>	78
9.1	Removable Sets and Menger Curvature	78
9.2	Projections	83
9.3	Principal Values	84
9.4	Square Functions	85
9.5	Other Related Kernels	86
<b>10</b>	<b>Singular Integrals</b>	89
10.1	A Few Words in General	89
10.2	$L^2$ -Boundedness and Uniform Rectifiability	90
10.3	Principal Values	94
10.4	Lipschitz Harmonic Functions	95
10.5	Parabolic Singular Integrals	97
10.6	Heisenberg Groups	97

<b>11</b>	<b>Harmonic Measure and Elliptic Measures</b>	99
11.1	Harmonic Measure	99
11.2	Elliptic Measures in Codimension 1	103
11.3	Elliptic Measures in Codimension Bigger Than One	103
<b>12</b>	<b>Sets of Finite Perimeter and Functions of Bounded Variation</b>	107
12.1	Sets of Finite Perimeter	107
12.2	Plateau-Type Problems	110
12.3	Functions of Bounded Variation	111
12.4	Perimeter in Heisenberg and Carnot Groups	114
<b>13</b>	<b>Currents and Varifolds</b>	115
13.1	Currents in Euclidean Spaces	115
13.2	Currents in Metric Spaces	120
13.3	Varifolds	121
<b>14</b>	<b>Minimizers and Quasiminimizers</b>	125
14.1	Quasiminimizers	125
14.2	Mumford–Shah Functional	126
14.3	Some Free Boundary Problems	127
<b>15</b>	<b>Rectifiability of Singularities</b>	129
15.1	Mass Minimizing Currents and Stationary Varifolds	129
15.2	Energy Minimizing Maps	132
15.3	Mean Curvature Flow	135
15.4	Gromov–Hausdorff Limits and Related Matters	137
15.5	Measure Solutions of PDEs	139
15.6	A Free Boundary Problem	142
<b>16</b>	<b>Miscellaneous Topics Related to Rectifiability</b>	143
16.1	Curvature Measures	143
16.2	Dynamical Systems	144
16.3	Higher-Order Rectifiability	146
16.4	Fractal Rectifiability	148
	<i>References</i>	149
	<i>Index</i>	171

