12

Regge poles, elementary particles and weak interactions

12.1 Introduction

So far in this book we have been solely concerned with hadronic interactions, which are the principal field in which Regge theory has been used. We have ignored electromagnetic effects in assuming that isospin is an exact symmetry of the scattering processes, and have not needed to mention the weak-interaction properties of the particles such as β-decay, etc. But of course any discussion of the electromagnetic or weak interactions of hadrons necessarily involves consideration of their hadronic properties too, because it is the strong interaction which is mainly responsible for the composite structure of the hadrons. Regge theory has played a small but not insignificant role in the development of theories of these weaker interactions, and clearly if there is to be any chance of unifying all the interactions they must be reconciled with Regge theory. In this chapter we shall look rather briefly at the problems which may arise in so doing.

Basically there are two such problems. First, weak interactions (and from now on we shall usually use the word 'weak' to refer to both electromagnetism and the weak interaction) are generally formulated in terms of a Lagrangian field theory for the interaction of a basic set of elementary particles. These are the leptons, l (i.e. electron e, muon μ, and neutrinos ν_e, ν_μ), photon γ, vector boson W, etc., and elementary hadrons (which at least initially do not lie on Regge trajectories but occur as Kronecker $\delta_{\sigma J}$ terms in the J plane). Alternatively the hadrons may be composed of elementary quarks bound together by the exchange of 'gluon' particles. The question then arises as to whether these elementary particles can be 'Reggeized' as a result of the interaction, i.e. whether they can be made to lie on Regge trajectories. This problem is obviously fundamental to attempts to marry field theory to Regge physics, and we examine it in section 12.3.

Secondly, theories of the coupling of the weak interactions to hadrons are generally used only to first order in the weak coupling constant (e^2 or G) and so the constraints of unitarity are inoperative. This means that fixed poles in the J plane, $\sim (J-J_0)^{-1}$, are not neces-

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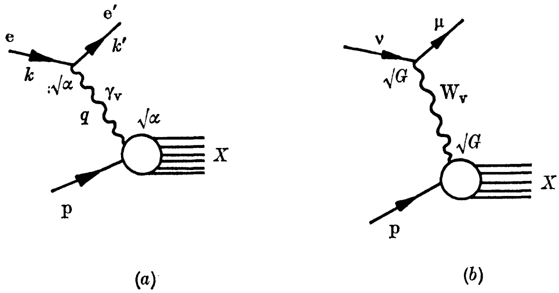

FIG. 12.1 (a) Deep inelastic electron scattering on a proton, $ep \rightarrow eX$, in the one-photon exchange approximation. The coupling is $\sqrt{\alpha} \approx 137^{-\frac{1}{2}}$ at each vertex, and the bottom part of the diagram is the amplitude for $\gamma_v p \rightarrow X$, where γ_v is the 'virtual' photon of 'mass' q^2. (Real photons have $q^2 = 0$ of course.) (b) Deep inelastic neutrino scattering $\nu p \rightarrow \mu X$ in the single virtual vector boson exchange approximation. The Fermi weak-interaction coupling $\sqrt{G}$ appears at each vertex.

sarily forbidden, and some theories such as current algebra actually require them. However, one is then led to wonder what would happen if one tried to work to all orders in the coupling since the results of sections 3.4 and 4.7 suggest that such fixed poles must be Reggeized by unitarity. But if so, what particles lie on the resulting trajectories? These questions will be examined in section 12.4.

But the main significance of Regge theory is that it tells us about the asymptotic behaviour to be expected in scattering amplitudes, and we conclude with a very short review of Regge predictions for weak scattering amplitudes. These include electromagnetic processes like 'deep inelastic' electron scattering, $ep \rightarrow eX$ (fig. 12.1 (a)) which, when the known electron–photon coupling, $\sqrt{\alpha}$, and the photon propagator have been extracted, depends just on the cross-section for the absorption of a virtual photon by a proton, i.e. $\gamma_v p \rightarrow X$. Or we may have neutrino scattering $\nu p \rightarrow lX$ (where $l = e$ or μ depending on whether ν is an electron- or muon-type neutrino) which can be described, at least as a matter of convenience, as $W_v p \rightarrow X$, where $W^{\pm}$ is the hypothetical 'intermediate vector boson' which in some theories is regarded as the mediator of the weak interaction (fig. 12.1 (b)). So γ_v and W_v couple to the electromagnetic and weak 'currents' of the hadrons respectively, and in this chapter we are concerned with such hadronic currents.

We shall not, however, attempt a full introduction to weak interaction theories, and the reader who is unfamiliar with these topics will find the books by Bransden, Evans and Major (1973), Gasiorowicz (1966), Bernstein (1968) and Feynman (1972) very useful, in addition to the references appearing later in the text.

12.2 Photo-production and vector dominance

There has been one exception to our exclusion of non-hadronic interactions from consideration thus far. In table 6.5, and at various points where we have discussed Regge phenomenology, we have included photo-production processes like $\gamma p \to \pi^+ n$ among those to be examined. The reason for this is that at high energies photons seem to behave almost exactly like hadrons, except for their weaker coupling. The explanation for this behaviour seems to be that photons couple to hadrons mainly via the vector mesons, as in fig. 12.2(*a*) (see for example Gilman (1972)).

The photon has $Q = B = S = 0$, $(J^P)C_n = (1^-)-$, but not being a hadron it does not have a definite isospin. It is found to behave like a mixture of $I = 0$ and 1, with no strong evidence for $I > 1$ components. The hadrons which share these properties are the vector mesons, the ρ with $I = 1$, and ω and ϕ with $I = 0$ (together with any daughters these may have). Fig. 12.2(*a*) suggests that one should write

$$A_H(\gamma 2 \to 34) = \sum_V \frac{e}{f_V} A_H(V2 \to 34) \tag{12.2.1}$$

where

$$e = \sqrt{(4\pi\alpha)} \equiv \left(\frac{e^2}{\hbar c}\right)^{\frac{1}{2}} \approx \left(\frac{4\pi}{137}\right)^{\frac{1}{2}} \tag{12.2.2}$$

(since in our units $\hbar = c = 1$). In (12.2.1) f_V is the coupling between vector meson and photon, and $V = \rho, \omega, \phi$, plus any other vector mesons one may care to add. The coupling f_V is directly related to the partial decay width $\Gamma(V \to e^+e^-)$ through fig. 12.2(*b*), which gives

$$\Gamma = \frac{4\pi\alpha m_V}{3f_V^2} \tag{12.2.3}$$

so f_V can be determined independently. Also the electromagnetic form factors which describe the photon coupling to a given hadron can be approximated by vector meson exchange, like fig. 12.2(*c*). Thus the pion's electromagnetic form factor can be written

$$F_\pi(q^2) \approx \frac{e g_{\rho\pi\pi}}{f_\rho} \frac{m_\rho^2}{m_\rho^2 - q^2} \tag{12.2.4}$$

exhibiting the pole at $q^2 = m_\rho^2$.

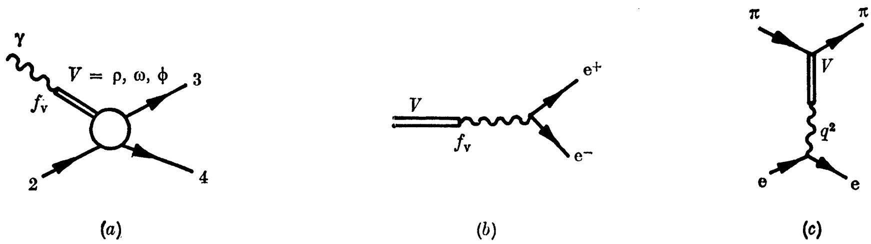

FIG. 12.2 (*a*) Vector dominance hypothesis in $\gamma 2 \to 34$. The photon couples to the hadrons via the vector mesons $V = \rho, \omega, \phi$. (*b*) The decay $V \to e^+e^-$. (*c*) The pion electromagnetic form factor determined in $e\pi \to e\pi$. It is assumed that the pion couples to the virtual photon exchange via V.

The obvious difficulty with (12.2.1) is that the photon, being massless, has helicities $\mu_\gamma = \pm 1$ only, from gauge invariance, whereas the vector mesons have $\mu = 1, 0, -1$, so the relation can only be true for transversely polarized mesons. It is not clear in which Lorentz frame the equality should hold, but it is generally supposed, and seems to be true experimentally, that the relation applies in the s-channel centre-of-mass frame, i.e. the helicity frame.

So we can make Regge hypotheses about photo-production amplitudes simply by treating the photon as a mixture of $I = 0, 1$ vector mesons as in fig. 12.3. This can be tested using for example the relation (Beder 1966, Dar *et al.* 1968)

$$\frac{1}{2}\left[\frac{d\sigma_{\perp,\parallel}}{dt}(\gamma p \to \pi^+ n) + \frac{d\sigma_{\perp,\parallel}}{dt}(\gamma n \to \pi^- p)\right] = \frac{e^2}{f_\rho^2}(\rho_{11} \pm \rho_{1-1})\frac{d\sigma}{dt}(\pi^- p \to \rho^0 n) \quad (12.2.5)$$

where $\perp, \parallel \equiv$ photon polarization perpendicular/parallel to the production plane. It has been assumed that the ω and ϕ contributions can be neglected because of their small couplings, and by taking the sum of π^+ and π^- photo-production the ρ–ω interference term in the square modulus of (12.2.1) is eliminated. The density matrix combination $(\rho_{11} \pm \rho_{1-1})$ for the ρ decay gives the required ρ helicities (see section 4.2). Such relations work rather well in general.

Another interesting consequence of (12.2.1) is that

$$A_H(\gamma 2 \to V2) = \sum_V \frac{e}{f_V} A_H(V2 \to V2) \quad (12.2.6)$$

so, neglecting the spin dependence, and the possibility of transitions

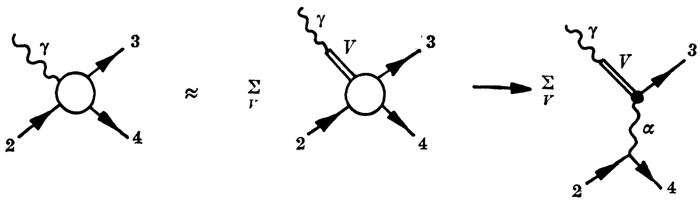

FIG. 12.2 Regge pole approximation to photo-production using vector dominance.

like $V_i 2 \to V_j 2$, $i \neq j$, we have

$$\frac{d\sigma}{dt}(\gamma 2 \to V2) = \frac{e^2}{f_V^2}\frac{d\sigma}{dt}(V2 \to V2) \tag{12.2.7}$$

and for $t = 0$, using the optical theorem (1.9.6),

$$\frac{d\sigma}{dt}(\gamma 2 \to V2)_{t=0} = \frac{e^2}{f_V^2}\frac{1}{16\pi}(\sigma_{V2}^{\text{tot}})^2 \tag{12.2.8}$$

assuming (for simplicity) that at high energies $A(V2 \to V2)$ is pure imaginary due to P exchange. So the differential cross-section for photo-producing vector mesons on protons, say, gives the ρp total cross-section. A further step is to take

$$A_H(\gamma 2 \to \gamma 2) = \sum_V \frac{e}{f_V} A_H(\gamma p \to Vp) \tag{12.2.9}$$

which, again neglecting the spin dependence and real parts for simplicity, gives

$$\frac{d\sigma}{dt}(\gamma p \to \gamma p) = e^2\left\{\sum_V \frac{1}{f_V}\left[\frac{d\sigma}{dt}(\gamma p \to Vp)\right]^{\frac{1}{2}}\right\}^2 \tag{12.2.10}$$

(though this relation does not seem to work so well).

The success of the vector-dominance hypothesis allows us to treat high-energy photo-production processes just like ordinary hadronic processes.

12.3 The Reggeization of elementary particles*

In a Lagrangian field theory the contribution of an elementary particle propagator for a particle of mass m and spin σ takes the form (cf. (2.3.1), and Appendix B of Bjorken and Drell (1965))

$$\frac{(2\sigma+1)g^2 P_\sigma(z_s)}{s-m^2} \tag{12.3.1}$$

* This section may be omitted at first reading.

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(where g is the coupling constant) and so from (2.2.1) and (A.20) contributes only to the $J = \sigma$ s-channel partial wave. Hence its contribution is not analytically continuable in the J plane and we must regard it as a Kronecker $\delta_{J\sigma}$ term

$$A_J(s) = \delta_{J\sigma} \frac{g^2}{16\pi(s-m^2)} \tag{12.3.2}$$

We have found that there is no evidence for such terms in hadronic physics, which suggests that Lagrangian field theories are inapplicable to strong interactions.

This conclusion may be too hasty, however, because (12.3.1) is only the Born approximation, the first term in a perturbation expansion of the theory, and it is possible that other terms might appear to cancel the $\delta_{J\sigma}$ and replace it by a moving Regge pole

$$A_J(s) \approx \frac{\beta(s)}{J-\alpha(s)}, \quad \alpha(m^2) = \sigma \tag{12.3.3}$$

instead, in which case the input elementary particle would be 'Reggeized' by unitarization of the field theory. For this to happen the theory must be able to generate a Kronecker δ to cancel the input, and in fact such $\delta_{J\sigma}$ terms may well arise at nonsense points (Gell-Mann and Goldberger 1962, Gell-Mann *et al.* 1962, 1964).

In section 4.8 we found that at right-signature sense–nonsense (sn) points J_0, since $e^J_{\mu\mu'} \sim (J-J_0)^{-\frac{1}{2}}$ we need a SCR to cancel the infinity, which would be incompatible with unitarity, giving $A_J \sim (J-J_0)^{\frac{1}{2}}$. This causes sense–nonsense decoupling as described in section 6.3. However, suppose we consider just the left-hand cut of the partial-wave amplitude, $A^{\mathrm{L}}_{HJ}(s)$ (cf. (3.5.1)), which stems from the crossed t- and u-channel singularities, and may be regarded as the input 'potential' for the N/D method of calculating partial-wave amplitudes (section 3.5). $A^{\mathrm{L}}_{HJ}(s)$ is not restricted by unitarity and so from the Froissart–Gribov projection (4.5.7) we can expect

$$\left.\begin{aligned} \langle s|\, A^{\mathrm{L}}_J \,|s\rangle &\sim \text{const nt} \\ \langle s|\, A^{\mathrm{L}}_J \,|n\rangle &\sim (J-J_0)^{-\frac{1}{2}} \\ \langle n|\, A^{\mathrm{L}}_J \,|n\rangle &\sim (J-J_0)^{-1} \end{aligned}\right\} \tag{12.3.4}$$

for $J \to J_0$, where $|s\rangle$ and $|n\rangle$ are respectively sense and nonsense helicity states for $J = J_0$.

For example in spins $1+\frac{1}{2} \to 1+\frac{1}{2}$ (fig. 12.4(a)) the u-channel spin $= \frac{1}{2}$ exchange Born term (fig. 12.4(c)) $\sim s^{-\frac{1}{2}}$, and gives a fixed

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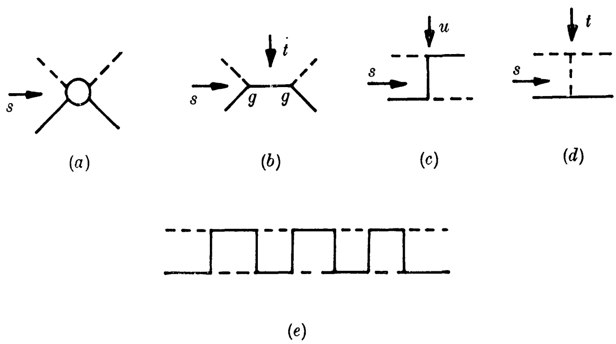

FIG. 12.4 (*a*) The amplitude for spins $1+\frac{1}{2}\rightarrow 1+\frac{1}{2}$ in the s channel (— spin $=\frac{1}{2}$, --- spin $=1$ particles). (*b*) The s-channel Born term. (*c*) The u-channel Born term. (*d*) The t-channel Born term. (*e*) Unitarization of the u-channel Born term.

singularity like (12.3.4) at $J=\frac{1}{2}$, which is a sense–nonsense point for the helicity $1+\frac{1}{2}\rightarrow 1+\frac{1}{2}$ amplitude. With composite particles one would expect this singularity to be cancelled by other contributions to give the SCR of (4.8.3), but with elementary particles there is no need for this to happen. In fact as $g^2\rightarrow 0$ the Born terms must be dominant.

Then if we treat fig. 12.4(*c*) as the first term of a perturbation expansion in g^2, with higher order terms like fig. 12.4(*e*), we can write the full solution in the form (Calogero *et al.* 1963*a*)

$$\langle s|\,A_J\,|s\rangle = \langle s|\,A_J^{\mathrm{nn}}\,|s\rangle\,[1+\sum_n \langle s|\,A_J^{\mathrm{L}}\,|n\rangle\langle n|\,A_J\,|s\rangle] \qquad (12.3.5)$$

where A_J^{nn} is the amplitude obtained when nonsense intermediate states are excluded from the perturbation series, while $\sum_n$ is over nonsense states only. Now $\langle s|\,A_J^{\mathrm{L}}\,|n\rangle \sim (J-J_0)^{-\frac{1}{2}}$ from (12.3.4) but unitary requires $\langle n|\,A_J\,|s\rangle \sim (J-J_0)^{\frac{1}{2}}$ so the second term in the bracket is finite but non-zero. So for elementary-particle theories the nonsense states give a finite contribution to the analytically continued (in J) ss partial-wave amplitude $\langle s|\,A_J\,|s\rangle$ which makes it different from the physical partial-wave amplitude, which is just $\langle s|\,A_J^{\mathrm{nn}}\,|s\rangle$.

In some circumstances this difference may be exactly equal to the elementary-particle $\delta_{J\sigma}$ term (12.3.2), $\sigma=\frac{1}{2}$, from the s-channel pole, fig. 12.4(*b*), so that

$$\langle s|\,A_J^{\mathrm{nn}}\,|s\rangle + \delta_{J\sigma}\langle s|\,A_\sigma\,|s\rangle = \langle s|\,A_J\,|s\rangle \qquad (12.3.6)$$

Then the physical amplitude is after all equal to the analytically continued amplitude, and the solution will exhibit Regge behaviour.

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This will clearly not happen in general, but it may in particular theories.

Though A_J^{nn} and A_J both have the same left-hand cuts they need not be identical because of the CDD ambiguity of section 3.5, and in fact A_J can be regarded as the solution to the N/D equations with the nonsense states excluded from the unitarity relation, but with a CDD pole for $J = J_0 = \sigma$ corresponding to the elementary-particle exchange, fig. 12.4 (*b*). It is a general feature of N/D solutions that poles which arise as bound or resonant states of a given channel appear as CDD poles in channels which are coupled thereto (see for example Squires (1964), Atkinson, Dietz and Morgan (1966), Jones and Hartle (1965)). The number of CDD poles needed is equal to the number of independent helicity amplitudes for which $J = J_0$ is a nonsense value. However, partial-wave amplitudes must also have the correct threshold behaviour (determined by l rather than J, see (4.7.6)) and Mandelstam (1965) pointed out that in some situations there is a unique amplitude containing no more than one CDD pole which satisfies these threshold conditions, in which case (12.3.6) must be satisfied.

This is in fact true of the spins $= 1 + \frac{1}{2}$ example mentioned above, though it is not true for the elementary vector meson in the t channel (fig. 12.4 (*d*)) which does not Reggeize in this way. Abers and Teplitz (1967) (see also Abers, Keller and Teplitz (1970)) have analysed the general spin problem, and find that the cases $(\sigma_1, \sigma_2) = (0, 0)$, $(0, \frac{1}{2})$ and $(\frac{1}{2}, \frac{1}{2})$ do not work, but that higher spins, like for example $(\frac{1}{2}, \sigma)$, $\sigma \geqslant 1$, which have suitable nonsense states, often will obey (12.3.6). But as such high-spin field theories are generally un-renormalizable it is not clear whether these results are useful.

Of course even if (12.3.6) is not automatically satisfied by the CDD solution it may actually be satisfied when the masses and couplings take on particular values so as to make the SCR hold, giving a bootstrap type of solution, but this cannot happen for weak coupling theories.

So only in certain field theories is Reggeization of the input elementary particles likely. However, it has been pointed out by Grisaru, Schnitzer and Tsao (1973) that these Reggeization rules may be applicable in re-normalizable unified gauge theories of strong, electromagnetic and weak interactions (see Iliopoulos (1974) for a review and references) in which the hadrons are viewed as composed initially of elementary spin $= \frac{1}{2}$ quarks bound together by elementary spin $= 1$

vector gluons. In gauge theories both the spin $= \frac{1}{2}$ and spin $= 1$ particles may be Reggeized though their interactions, unlike the case considered above. So the fact that only Reggeons are observed in hadronic physics does not necessarily preclude the existence of an elementary-particle sub-structure.

12.4 Fixed poles*

The diagrams of figs. 12.1 and 12.3 differ from hadronic scattering amplitudes in that though the blobs are assumed to contain the full set of hadronic singularities required by unitarity, the weak coupling constant $e^2/4\pi \equiv \alpha \approx \frac{1}{137}$ or $G \approx 1 \times 10^{-5} m_N{}^{-2}$ (the Fermi weak interaction coupling constant) appears explicitly only to the first order. The only γ (or W) to appear is an external particle to this blob. So for example the unitarity equation for the amplitude $\gamma + 2 \to 3 + 4$ is (fig. 12.5) usually taken to be

$$\operatorname{Im}\{A(\gamma+2\to 3+4)\} = \sum_n A(\gamma+2\to n)\, A^*(n\to 3+4) \quad (12.4.1)$$

with a sum over hadronic intermediate states only. Were we to include photon intermediate states as well, as in the other terms on the right-hand side of fig. 12.5, to give

$$\operatorname{Im}\{A(\gamma+2\to 3+4)\} = \sum_n A(\gamma+2\to n)\, A^*(n\to 3+4) + \sum_n A(\gamma+2\to \gamma+n)\, A^*(\gamma+n\to 3+4) + \ldots \quad (12.4.2)$$

the terms in the second summation would be smaller than those in the first by another factor α (or G for weak interactions), which is why they are generally neglected. So if we specialise to the two-body intermediate state $3+4$ by remaining below the inelastic threshold (fig. 12.6) (e.g. $\gamma p \to \pi^+ n$ below the $\pi\pi n$ threshold), and project into partial waves, we have, instead of (4.7.4),

$$A_{HJ}(s) - (A_{HJ}(s))^* = 2i\rho_H(s)\, A_{HJ}(s)\, (A^{el}_{HJ*}(s))^* \quad (12.4.3)$$

where $A^{el} \equiv A(34 \to 34)$. A similar equation holds in the t channel for $\gamma\overline{3} \to \overline{2}4$.

The fact that $A_{HJ}(s, \gamma 2 \to 34)$ appears only linearly on the right-hand side of (12.4.3) means that the theorems we enunciated in section 4.7 on the impossibility of real-axis fixed poles (except those at wrong-signature nonsense points which are shielded by cuts) do not

* This section may be omitted at first reading.

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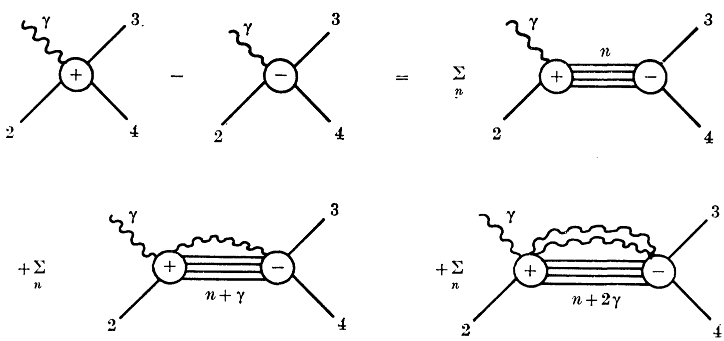

FIG. 12.5 The unitarity equation for $A(\gamma 2 \rightarrow 34)$ including higher order terms in the weak coupling.

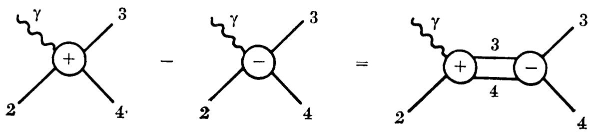

FIG. 12.6 The two-body unitarity equation for $\gamma 2 \rightarrow 34$ valid below the inelastic threshold.

apply to these weak amplitudes. So fixed poles might occur at right-signature points, in which case they would contribute to the asymptotic behaviour. In other words the SCR which must hold to prevent such fixed poles in hadronic amplitudes may not be satisfied in weak amplitudes.

It was pointed out by Bronzan *et al.* (1967) and Singh (1967) that current algebra predicts the occurrence of such fixed poles. (For an introduction to current algebra see Renner (1968), or Adler and Dashen (1968).) This is because current algebra theory relates the magnitude of the single-current coupling to that of the two-current amplitude, and in particular for $\gamma_{\mathrm{v}}+2 \rightarrow \gamma_{\mathrm{v}}+2$ (fig. 12.7) it gives (Fubini 1966, Dashen and Gell-Mann 1966)

$$\frac{1}{\pi}\int_{s_{\mathrm{T}}}^{\infty} \mathrm{d}z_t\, D_{sH}^{I_t=1}(s,t,q^2,q'^2) = F(t) \tag{12.4.4}$$

where γ_{v} is an iso-vector photon and 2 is a spinless particle (or we can regard (12.4.4) as a spin-averaged equation). $F(t)$ is the iso-vector form factor of particle 2, and $D_{sH}^{I_t=1}$ is the odd-signature discontinuity in s $(= D_s - D_u)$ of the reduced t-channel helicity amplitude $\hat{A}_{H_t}(s,t,q^2,q'^2)$ for the process $\gamma(q^2)+\gamma'(q'^2) \rightarrow 2\overline{2}'$. This is the helicity amplitude with

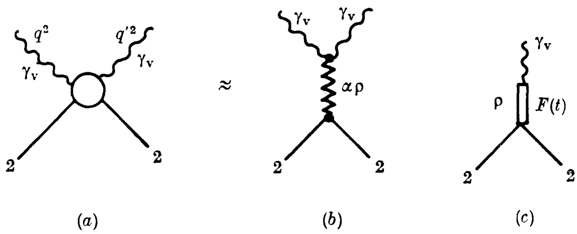

FIG. 12.7 (a) Virtual-photon Compton scattering $\gamma_v + 2 \to \gamma_v + 2$. (b) The ρ-exchange approximation to this amplitude. (c) The ρ pole in the electromagnetic form factor of particle 2.

the half-angle factor

$$\xi_{\lambda\lambda'}(z_t) \equiv \left(\frac{1+z_t}{2}\right)^{\frac{1}{2}|\lambda+\lambda'|}\left(\frac{1-z_t}{2}\right)^{\frac{1}{2}|\lambda-\lambda'|} = [\tfrac{1}{4}(1-z_t^2)]^{\frac{1}{2}\lambda} \quad (12.4.5)$$

extracted (see (4.4.1), with $\lambda_\gamma = \pm 1$, $\lambda_2 = 0$ so $\lambda = 0$ or 2), and with $I = 1$ in the t channel. We expect the dominant $I_t = 1$ exchange to be the ρ trajectory. This is because we are dealing with isovector (charged) photons: for real photons ρ exchange is forbidden by charge conjugation. So for $\lambda_\gamma = -\lambda_{\gamma'} = 1$, i.e. $\lambda = 2$, we expect $D_{sH} \sim s^{\alpha_\rho(t)-2}$ in (12.4.4) giving

$$F(t) \sim \frac{1}{\alpha_\rho(t)-1} \sim \frac{1}{\alpha'(t-m_\rho^2)} \quad \text{for} \quad t \to m_\rho^2, \alpha_\rho \to 1 \quad (12.4.6)$$

so the form factor has the ρ pole as anticipated in fig. 12.7(c). However, the point $J = 1$ is a sense–nonsense point for this amplitude ($\lambda \equiv |\lambda_\gamma - \lambda_{\gamma'}| = 2$, $\lambda' = |\lambda_2 - \lambda_{2'}| = 0$) so we would expect a superconvergence relation to hold. Indeed if 2, 2′ are replaced by protons, and we fix on $t = 0$ where D_s can be replaced by $\sigma_{\gamma\rho}^{\text{tot}}$ using the optical theorem (1.9.6), then the SCR becomes

$$\int_0^\infty \frac{d\nu}{\nu}(\sigma_P(\nu) - \sigma_A(\nu)) = \frac{2\pi^2 e^2}{m_p^2}\mu_p' \quad (12.4.7)$$

where σ_P and σ_A are the total γp cross-sections with spins parallel and anti-parallel (respectively) and μ_p' is the proton's anomalous magnetic moment (i.e. the form factor at $t = 0$). Equation (12.4.7) is the well-known Drell–Hearn (1966) sum rule, which certainly seems to hold experimentally, and it gives the residue of the ρ pole a nonsense factor $\alpha_\rho(t) - 1$ to cancel (12.4.6).

However the left-hand side of (12.4.4) is the residue of a fixed pole at the sn point $J = 1$ so, since the form factor $F(t)$ is certainly nonvanishing, current algebra predicts that there will be a right-signature fixed pole which contributes to the asymptotic behaviour

$$\mathrm{Re}\{\hat{A}(\gamma 2 \to \gamma 2)\} \sim -\frac{F(t)}{z_t} \tag{12.4.8}$$

and so the full odd-signature amplitude behaves like

$$\hat{A}(\gamma\mathrm{p} \to \gamma\mathrm{p}) = \frac{1}{\pi}\int^{\infty} \frac{\mathrm{d}z'_t D_{sH}(s',t)}{z'_t - z_t} = -\frac{1}{z_t}\frac{1}{\pi}\int^{\infty} \mathrm{d}z'_t D_{sH}(s',t)$$
$$+\frac{1}{z_t}\frac{1}{\pi}\int^{\infty} \frac{\mathrm{d}z'_t z'_t}{z'_t - z_t} D_{sH}(s',t) \to -\frac{F(t)}{z_t} + \frac{G_\rho(t)\,(z_t)^{\alpha_\rho(t)-2}}{\sin \pi\alpha_\rho(t)} \tag{12.4.9}$$

So there is a moving Regge pole $\alpha_\rho(t)$, and a fixed pole at $J = 1$, but no singularity at $t = m_\rho^2$, $\alpha_\rho = 1$ because at this point the two terms cancel, with $F(t)$ behaving as in (12.4.6).

Thus current algebra predicts that there will be fixed poles at right-signature nonsense points which do not contribute to D_{sH} and hence do not affect the total cross-section, but do contribute to the asymptotic behaviour of the real part of the amplitude. This is hardly surprising in that fig. 12.7 (*c*) has coupled the ρ to a fixed-spin current.

But the question then arises as to what would happen if we were to work not just to first order in e^2, but included all orders, in the (t-channel) unitarity equation, like (12.4.2). Our experience with weak-coupling field-theory solutions like (3.4.17) suggests that the fixed pole at $J = 1$ would turn into a moving Regge pole which $\to 1$ as $e^2 \to 0$, i.e.

$$\alpha(t) = 1 + e^2 f(t) \tag{12.4.10}$$

where $f(t)$ is some function of t like (3.4.19), of order 1. So we anticipate a trajectory with a slope $\alpha' = O(e^2) = O(\frac{1}{137})$. Such a trajectory is not seen in the asymptotic behaviour, nor has it manifested itself as high-mass particles. If, alternatively, the pole remained fixed it would produce Kronecker δ_{JJ_0} terms in the ss amplitudes as described in the previous section. All this suggests that current algebra may itself be wrong, though of course we do not really know how to deal properly with the zero-mass photon as an intermediate state.

Fixed poles do occur at wrong-signature points, and indeed a $J = 1$ fixed pole may be essential to the asymptotic behaviour of the even-signature Compton scattering amplitude (Abarbanel *et al.* 1967). For $\gamma + 2 \to \gamma + 2$ (with 2 spinless) there are just two independent s-channel helicity amplitudes, $\langle 1, 0| A^s |1, 0\rangle$ and $\langle 1, 0| A^s | -1, 0\rangle$. The helicity crossing matrix like (4.3.4) (see Ader, Capdeville and Navelet 1968), relating these amplitudes to those for the t-channel process $\gamma\gamma \to 2\bar{2}$, simplifies at $t = 0$ to

$$\left.\begin{aligned} \langle 1,0|\, A^s\,|1,0\rangle &= \langle 1,-1|\,A^t\,|0,0\rangle, \quad \lambda = 2 \\ \langle 1,0|\, A^s\,|-1,0\rangle &= \langle 1,1|\,A^t\,|0,0\rangle, \quad \lambda = 0 \end{aligned}\right\} \tag{12.4.11}$$

and, since the helicity-flip amplitude $\langle 1,0|\,A^s\,|-1,0\rangle$ must vanish at $t = 0$ by angular-momentum conservation, only $\langle 1,-1|\,A^t|0,0\rangle$ survives. Now $J_t = 1$ is a sn point for this amplitude. The dominant even-signature exchange for this elastic amplitude will be the Pomeron, so if $\alpha_{\mathrm{P}}(0) = 1$, and the P residue has a nonsense decoupling factor at $\alpha_{\mathrm{P}}(t) = 1$, then the P-exchange contribution to $\langle 1,0|\,A^s\,|1,0\rangle$ will vanish at $t = 0$. The optical theorem gives

$$\sigma^{\mathrm{tot}}_{\gamma 2} = 1/s\,\mathrm{Im}\,\{\langle 1,0|\,A^s\,|1,0\rangle\} \tag{12.4.12}$$

so $\sigma^{\mathrm{tot}}_{\gamma 2} \to 0$ as $s \to \infty$ if the P decouples, unlike other total cross-sections.

But this completely contradicts our observation in section 12.2 that the photon behaves like a hadron at high energies, and the observed approximate constancy of $\sigma^{\mathrm{tot}}_{\gamma\mathrm{p}}(s)$ at large s. So either there is a Gribov–Pomeranchuk fixed pole at $J = 1$ which removes the decoupling factor (see table 6.2), in which case $\sigma^{\mathrm{tot}}_{\gamma 2}(s)$ is controlled only by the third double-spectral function, which seems rather odd, or the P residue is singular at $t = 0$. In fact the residue of the $J = 1$ fixed pole in the Froissart–Gribov projection of the $\lambda = 2$ amplitude can be expressed as

$$G_1(t) = \frac{1}{\pi}\int_{m_2^2}^{\infty} \mathrm{d}z_t' D_{sH}(s',t) = \frac{Ke^2}{t} + \int_{s_{\mathrm{I}}}^{\infty} \mathrm{d}z_t'\, D_{sH}(s,t) \tag{12.4.13}$$

where K is a constant and s_{I} is the inelastic threshold. The kinematical t^{-1} factor from the Born diagram in fig. 12.8 stems from the kinematics of the massless photon ($q^2_{t13} = t/4$ from (1.7.15)). So if $G_1(t)$ vanishes the residue of the P pole in D_{sH} must behave like t^{-1} so that the right-hand side can vanish. Clearly $t = 0$ is a special point because it coincides with the initial-state threshold of $\gamma\gamma \to 2\bar{2}$, and photon partial-wave amplitudes may well have an unusual threshold behaviour (see Collins and Gault (1972) and below). For a more complete discussion see Landshoff and Polkinghorne (1972).

It has also been suggested that there might be a fixed pole at the sn point $J_t = 0$ in this amplitude (Damashek and Gilman 1970). Since $J = 0$ is a right-signature point this would give a real constant contribution to the Compton scattering amplitude

$$A(\gamma 2 \to \gamma 2, s, t) = \sum_i A^{\mathrm{R}_i}(s,t) + G_0(t) \tag{12.4.14}$$

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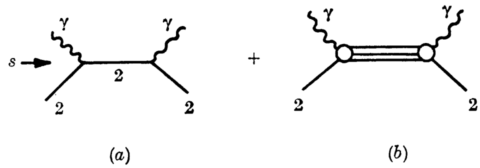


FIG. 12.8 (a) The s-channel Born term in $\gamma 2 \to \gamma 2$. (b) Other s-channel intermediate states which contribute to D_s for $s > s_{\mathrm{I}}$ the inelastic threshold.

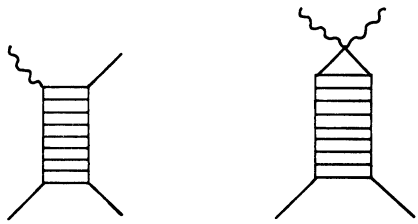
where $A^{\mathrm{R}_i}(s,t)$ are the usual Reggeon exchange amplitudes, P and A_2 and $G_0(t)$ is the fixed-pole contribution, which is independent of s for all t. Gilman and co-workers have attempted fits of forward $\mathrm{d}\sigma/\mathrm{d}t\,(\gamma\mathrm{p}\to\gamma\mathrm{p})$ and $\sigma^{\mathrm{tot}}_{\gamma\mathrm{p}}$ with $\alpha_{\mathrm{P}}(0) = 1$, $\alpha_{\mathrm{A}_2}(0) = \frac{1}{2}$, and find that such a real part is needed. In fact they identify

$$G_0(t) = -\frac{e^2}{m_{\mathrm{p}}} \tag{12.4.15}$$

which is the Thompson amplitude for Compton scattering off a proton at zero photon energy, when the proton structure is not penetrated. However, adjusting the values of the trajectory intercepts seems to make this extra term unnecessary (Close and Gunion 1971), so there is no convincing evidence for this fixed pole. See also Brodsky, Close and Gunion (1972) and Landshoff and Polkinghorne (1972).

Fixed poles have also been searched for in photo-production processes like $\gamma\mathrm{p}\to\pi^+\mathrm{n}$. In the backward direction one might have elementary nucleon exchange at $J_u = \frac{1}{2}$, giving $\mathrm{d}\sigma/\mathrm{d}u \sim 1/s$ at fixed u, but in fact $\mathrm{d}\sigma/\mathrm{d}u \sim s^{-2.6}$ at $u \approx 0$ corresponding to $\alpha_{\mathrm{N}}(0) \approx -0.3$.

The forward direction is particularly interesting because, as we have discussed in sections 6.8j and 8.7f, this process is controlled by evasive π exchange together with a self-conspiring $\pi \otimes \mathrm{P}$ cut. The rapid variation of $\mathrm{d}\sigma/\mathrm{d}t$ near $t = 0$ demands the presence of the pion pole term (see (8.7.5)). However the right-signature point $\alpha_\pi(t) = 0$ is a nonsense point for all $\gamma\pi\to\bar{\mathrm{p}}\mathrm{n}$ t-channel amplitudes since $\lambda_\gamma - \lambda_\pi = 1$, so normally one would expect a nonsense factor and no pion pole. At one time it was though that a fixed pole must be present to remove the need for a nonsense factor (as described above for Compton scattering), but since $J = 0$ is a right-signature point such a fixed pole should be seen in the asymptotic behaviour, which it is not. So again it would seem that the photon coupling is unusual. Now $t = m_\pi^2$ is one of the thresholds of $\gamma\pi\to\bar{\mathrm{p}}\mathrm{n}$, so as with Compton scattering it looks as if the blame can be placed on an unusual threshold behaviour (see Collins and Gault (1972) for references).



(a) (b)

FIG. 12.9 (a) Field-theory model for the coupling of a photon to a composite Reggeon exchange. (b) The two-photon coupling to a ladder which gives rise to a $J = 1$ fixed pole (to first order in e^2).

To summarize then, we have found no very strong evidence for unusual fixed poles in weak amplitudes (despite current algebra) and some evidence against them. Theoretically (Rubinstein, Veneziano and Virasoro 1968, Dosch 1968, Landshoff and Polkinghorne 1972) there is reason to suppose that when currents couple to composite particles, for example particles built from ladders like figs. 12.9(a), (b), the only fixed poles occur at the nonsense points $J = \sigma_\gamma - n$ and $J = \sigma_{\gamma 1} + \sigma_{\gamma 2} - n$, $n = 1, 2, \ldots$ respectively. The latter seem to be closely related to the scaling behaviour seen in deep inelastic electron scattering (section 12.5). But all the arguments in favour of fixed poles arise from working only to first order in e^2, and could be wrong. Hence one can feel fairly secure in treating the photon like a hadron as far as the leading Regge behaviour is concerned.

One interesting consequence of this concerns the electromagnetic mass differences of isotopic multiplets. Cottingham (1963) showed how the first-order electromagnetic contributions to the self energy (= mass) of a particle, given by the photon emission and re-absorption diagram fig. 12.10, can be directly related to the spin-averaged forward Compton scattering amplitude on the given particle by a photon of mass q^2

$$\delta M = \frac{e^2}{2\pi} \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \sum_\mu \frac{A_{\mu\mu}(\nu, q^2)}{q^2 - \mathrm{i}\epsilon} \tag{12.4.16}$$

where $\nu \equiv p.q$. The $A_{\mu\mu}$ can be expressed in terms of a dispersion relation in ν as

$$\hat{A}_{\mu\mu}(\nu, q^2) = \frac{2}{\pi} \int_0^\infty \frac{\nu' \mathrm{d}\nu' D_{sH}(\nu', q^2)}{\nu'^2 - \nu^2} \tag{12.4.17}$$

If the non-flip helicity amplitudes are Regge pole dominated at high energy we expect their even signature $(s + u)$ discontinuities to behave like

$$D_{sH}(\nu, q^2) \to \gamma_i(q^2)\, \nu^{\alpha_i(0)} \tag{12.4.18}$$

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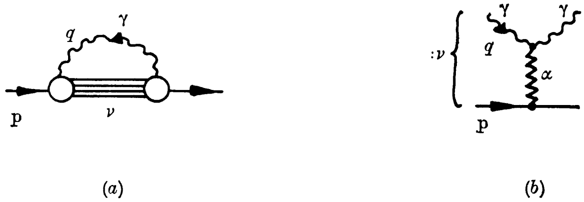

FIG. 12.10 (*a*) The one-photon loop which gives the first order electromagnetic mass re-normalization of the particle propagator. (*b*) Reggeon exchange model for the virtual Compton-scattering amplitude in (*a*).

For example the difference between the neutron and proton electromagnetic masses depends on the dominant even-signature $\Delta I = 1$ exchange, i.e. the A_2 trajectory with $\alpha(0) \approx 0.5$ (Harari 1966), so clearly (12.4.17) will diverge, and hence these equations do not permit one to calculate this electromagnetic mass difference unless one can determine the subtraction constant. However, the $\Delta I = 2$ mass difference $(m_{\pi^\pm} - m_{\pi^0})$ is dominated by $I = 2$ exchange and since no such trajectory is known the dominant (Regge cut?) exchange may well have $\alpha(0) < 0$ so the integral should converge. This may help to account for the fact that $m_{\pi^\pm} > m_{\pi^0}$ as one would expect (i.e. electromagnetic effects add to the mass of the pions) but $m_n > m_p$ which contradicts this expectation. This criterion based on the intercept of the exchanged Reggeon seems to work for the signs of other mass differences as well.

This is just one example of the way in which the known Regge asymptotic behaviour is helpful for understanding the weaker interactions, particularly their dispersion sum rules.

12.5 Deep inelastic scattering

Some of the most interesting results on the structure of hadrons have come from deep inelastic scattering experiments on nucleons, $\mathrm{ep} \to \mathrm{e}X$ and to a lesser extent $\nu\mathrm{p} \to \mu X$. These are treated in the one-photon or one-W exchange approximation as in figs. 12.1. (For reviews of these processes see for example Gilman (1972) and Llewellyn-Smith (1972), respectively, and Landshoff and Polkinghorne (1972).)

With the four-momenta indicated in the figure we have

$$k^2 = k'^2 = m_e^2 \approx 0 \quad \text{and} \quad q = k - k' \tag{12.5.1}$$

In the laboratory frame (proton at rest) we can write

$$p = (m_p, 0, 0, 0), \quad k = (E, \boldsymbol{k}), \quad k' = (E', \boldsymbol{k}'), \quad q = (E - E', \boldsymbol{k} - \boldsymbol{k}'),$$
$$k^2 = E^2 - \boldsymbol{k}^2 \approx 0, \quad k'^2 = E'^2 - \boldsymbol{k}'^2 \approx 0 \tag{12.5.2}$$

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Hence
$$\nu \equiv p.q = (E-E')\, m_{\mathrm{p}} \tag{12.5.3}$$

gives the energy of the virtual photon, while its mass

$$q^2 = (E-E')^2 - (\boldsymbol{k}-\boldsymbol{k}')^2 = E^2 + E'^2 - 2EE' - \boldsymbol{k}^2 - \boldsymbol{k}'^2 + 2|\boldsymbol{k}|\,|\boldsymbol{k}'|\cos\theta$$
$$\approx -4EE'\sin^2\frac{\theta}{2} \tag{12.5.4}$$

(using (12.5.2)) depends on θ, the scattering angle between the directions of motion of the initial- and final-state leptons. (The reader will note that many authors (e.g. Gilman) define ν without the factor m_{p} in (12.5.3) and take the opposite sign for q^2 in (12.5.4).)

For the scattering process in the bottom of the figure, $\gamma_{\mathrm{v}} + \mathrm{p} \to X$, the effective centre-of-mass energy squared is

$$s \equiv (p+q)^2 = p^2 + q^2 + 2p.q = m_{\mathrm{p}}^2 + q^2 + 2\nu \equiv M_X^2 \tag{12.5.5}$$

using (12.5.3). Averaging over the spins of the electron and proton, the differential cross-section for $\mathrm{ep} \to \mathrm{e}X$ is found to be (Drell and Walecka 1964)

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'} = \frac{4e^4E'^2}{q^4}\left(2W_1(\nu, q^2)\sin^2\frac{\theta}{2} + W_2(\nu, q^2)\cos^2\frac{\theta}{2}\right) \tag{12.5.6}$$

where $\mathrm{d}\Omega$ is the element of solid angle within which the final-state electron of energy E' is detected, and $W_{1,2}$ are the conventionally defined deep inelastic structure functions of the nucleon. They are directly related to the total cross-sections for transversely and longitudinally polarized virtual photons scattering on a proton (σ_{T} and σ_{L} respectively) by

$$\left.\begin{aligned} W_1(\nu, q^2) &= \frac{K_1}{4\pi e^2}\,\sigma_{\mathrm{T}}(\nu, q^2) \\ W_2(\nu, q^2) &= \frac{K_2}{4\pi e^2}\,(\sigma_{\mathrm{T}}(\nu, q^2) + \sigma_{\mathrm{L}}(\nu, q^2)) \end{aligned}\right\} \tag{12.5.7}$$

where
$$K_1 \equiv \frac{1}{m_{\mathrm{p}}}\left(\nu + \frac{q^2}{2}\right), \quad K_2 \equiv m_{\mathrm{p}}\left(\nu + \frac{q^2}{2}\right)\left(\frac{q^2}{m_{\mathrm{p}}^2 q^2 - \nu^2}\right) \tag{12.5.8}$$

As $q^2 \to 0$, $\sigma_{\mathrm{L}} \to 0$ and σ_{T} is the real $\gamma\mathrm{p}$ total cross-section.

Elastic ep scattering (fig. 12.11) clearly requires $M_X^2 = m_{\mathrm{p}}^2$ and so from (12.5.5)
$$q^2 = -2\nu \tag{12.5.9}$$

at which values W_1 and W_2 are related to the proton's electromagnetic

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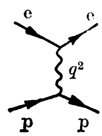

FIG. 12.11 One-photon exchange diagram for ep → ep.

form factors, G_E and G_M, by

$$\left.\begin{aligned} W_1(\nu, q^2) &= -\frac{q^2 G_M^2(q^2)}{4m_p^2}\,\delta\left(\nu+\frac{q^2}{2}\right) \\ W_2(\nu, q^2) &= \left[G_E^2(q^2)-\frac{q^2}{4m_p^2}\,G_M^2(q^2)\right]\left(1-\frac{q^2}{4m_p^2}\right)^{-1}\delta\left(\nu+\frac{q^2}{2}\right) \end{aligned}\right\} \tag{12.5.10}$$

The most remarkable result to come from experiments on deep inelastic scattering is the scaling of W_1 and νW_2 as ν, $|q^2| \to \infty$ (q^2 is negative in the physical region)

$$\left.\begin{aligned} W_1(\nu, q^2) &\to F_1\left(\frac{2\nu}{|q^2|}\right) \\ \nu W_2(\nu, q^2) &\to F_2\left(\frac{2\nu}{|q^2|}\right) \end{aligned}\right\} \tag{12.5.11}$$

where F_1, F_2 are functions which depend only on the dimensionless ratio

$$\omega \equiv \frac{2\nu}{|q^2|} \equiv \frac{1}{x} \tag{12.5.12}$$

and not on the values of ν and $|q^2|$ individually. That is to say, if both ν and $|q^2|$ are varied, keeping their ratio fixed, the values of W_1 and νW_2 are unchanged (see section 10.5 for the concept of scaling).

The most simple explanation of this scaling effect is provided by the parton model (see Feynman 1972) in which the nucleon is imagined to be composed of a number of structureless, point-like, charged particles (partons), i, each carrying a fraction x_i of the total proton momentum

$$p_i = x_i p, \quad 0 \leqslant x_i \leqslant 1 \tag{12.5.13}$$

If the parton is structureless its mass, and its charge, Q_i, will be unchanged by the scattering (fig. 12.12) and so it will give a contribution to the W's like (12.5.10) but without any q^2 dependence of the form factor (so $G_E = Q_i$). So for example

$$W_2^i(\nu, q^2) = Q_i^2\,\delta\left(\nu+\frac{q^2}{2x_i}\right) = \frac{Q_i^2 x_i}{\nu}\,\delta\left(x_i-\frac{|q^2|}{2\nu}\right) \tag{12.5.14}$$

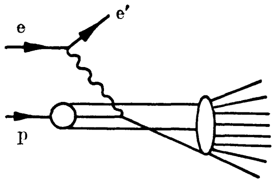

FIG. 12.12 Parton description of ep → eX. The proton is composed of structureless partons (quarks) and the photon is absorbed by one of these partons. In the right-hand blob the quarks recombine to form ordinary hadrons, X.

Then if $f(x)$ is the probability that the parton has a fraction x of the proton's momentum we get

$$\begin{aligned}\nu W_2 &= \sum_i Q_i^2 \int_0^1 \mathrm{d}x_i\, f(x_i)\, x_i\, \delta\left(x_i - \frac{|q^2|}{2\nu}\right) \\ &= \sum_i Q_i^2 x f(x)\big|_{x=|q^2|/2\nu} \equiv F_2\left(x = \frac{|q^2|}{2\nu}\right) \end{aligned} \tag{12.5.15}$$

This result depends crucially on the partons being structureless since otherwise form factors, functions of q^2, would appear in (12.5.14) as they do in (12.5.10), and would destroy the scaling.

The parton model has enjoyed considerable success, not only because it 'explains' scaling but because if the partons are taken to be spin $= \frac{1}{2}$ quarks many features both of the spin dependence and the internal symmetry properties of the cross-sections are accounted for. The main problem is of course that the quarks are not observed, and it is not clear what mechanism can be responsible for the right-hand blob of fig. 12.12 in which all the quarks, including the scattered one, recombine to form conventional hadrons.

However, our main interest is in the Regge properties of these results. Since the W's are, apart from the kinematical factors in (12.5.7), γp total cross-sections, we can hope to describe these cross-sections by making Regge models of the γ_{V}p elastic scattering amplitude, as in fig. 12.13 (cf. fig. 10.23). The Regge limit is $\nu \to \infty$, q^2 fixed (so $x \to 0$), and we expect

$$\sigma_{\mathrm{T}}, \sigma_{\mathrm{L}} \sim \sum_k \nu^{\alpha_k(0)-1} \tag{12.5.16}$$

so that

$$\left.\begin{aligned} W_1(\nu, q^2) &\sim \nu\sigma_{\mathrm{T}} \to \sum_k \beta_1^k(q^2)\left(\frac{\nu}{s_0}\right)^{\alpha_k(0)} \\ \nu W_2(\nu, q^2) &\sim |q^2|\,(\sigma_{\mathrm{T}} + \sigma_{\mathrm{S}}) \to \sum_k |q^2|\,\beta_2^k(q^2)\left(\frac{\nu}{s_0}\right)^{\alpha_k(0)-1} \end{aligned}\right\} \tag{12.5.17}$$

So, if the leading singularity is the Pomeron with $\alpha_{\mathrm{P}}(0) = 1$, both

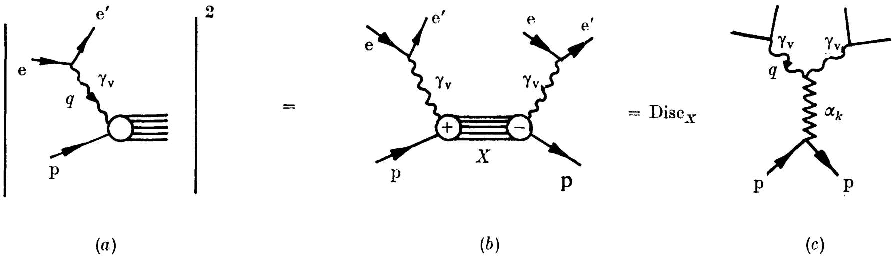

FIG. 12.13 Reggeon exchange description of deep-inelastic ep scattering in terms of $\gamma_V p \to \gamma_V p$. The leading trajectories which can be exchanged in the elastic amplitude are k = P, f and A_2.

νW_2 and $xW_1 \to$ constant as $x \equiv q^2/2\nu \to 0$. The region where scaling occurs ($|q^2|, \nu \to \infty, x$ fixed) may overlap the Regge asymptotic region ($\nu \to \infty, q^2$ fixed). This clearly depends on the behaviour of the couplings $\beta^P_{1,2}(q^2)$ as $|q^2| \to \infty$, but if they are to overlap we need

$$\beta^P_1(q^2) \xrightarrow[|q^2|\to\infty]{} \frac{g_1}{(|q^2|)^{\alpha_P(0)}}, \quad \beta^P_2(q^2) \to \frac{g_2}{(|q^2|)^{\alpha_P(0)}} \tag{12.5.18}$$

where g_1, g_2 are constants, so that W_1, $\nu W_2 \to$ scaling function of x only, i.e.

$$\left.\begin{aligned} W_1(\nu, q^2) &\xrightarrow[x\to 0]{} \frac{g_1}{(2s_0 x)^{\alpha_P(0)}} \\ \nu W_2(\nu, q^2) &\xrightarrow[x\to 0]{} \frac{g_2}{(2s_0 x)^{\alpha_P(0)-1}} \end{aligned}\right\} \tag{12.5.19}$$

This accords with the behaviour found in field-theory models with a point-like electromagnetic coupling (Abarbanel, Goldberger and Trieman 1969).

It should be noted that though fig. 12.13(c) looks like the triple-Regge diagram fig. 10.23(c) it is really quite different. Fig. 10.23 involves trajectories $\alpha_i(t), \alpha_j(t)$ so that the angular momentum changes as a function of t, while fig. 12.13(c) involves the photons $\gamma(q^2)$ which remain at $J = 1$ even though q^2 is varied. So rather different information is obtained from electro-production. However, if we adopt a multi-peripheral type of model (fig. 12.14) it is only at the top end that the couplings will be affected by q^2 (because there are only short-range correlations) and many features should be hadron-like, such as multiplicities etc. It is the photon's fragmentation region which should exhibit most of the differences (see Cahn (1974) for a more detailed discussion).

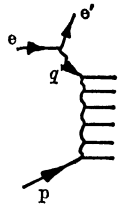

FIG. 12.14 Multi-peripheral model for ep $\rightarrow$ e'X.

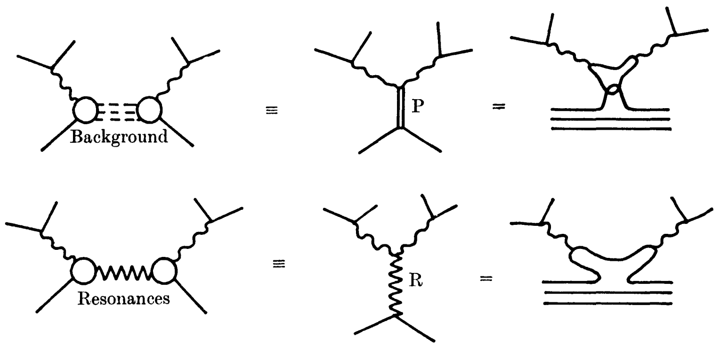

FIG. 12.15 Duality diagrams for ep $\rightarrow$ eX.

Regge theory also predicts that the principal non-constant corrections to (12.5.19) will stem from $k = \mathrm{R}$ (f and A_2) with $\alpha_{\mathrm{R}}(0) \approx 0.5$. Two-component duality suggests that the duality properties will be as in fig. 12.15, with the P dual to background, b, and R dual to the resonances, r (Harari 1970). Since the Regge region is found to overlap the scaling region this implies strong constraints on the resonance-production cross-sections as a function of q^2, and hence on the transition form factors for $\gamma_{\mathrm{V}}(q^2) + \mathrm{p} \rightarrow \mathrm{r}$. The way in which the resonance contributions are smoothed to the scaling behaviour as $|q^2|$ (and hence ν) is increased for fixed ω is shown in fig. 12.16. This is just like the smoothing in the Veneziano model with $\mathrm{Im}\{\alpha\} \neq 0$ (see fig. 7.6). In fact the Veneziano model has enjoyed some success in fitting the ν, q^2 dependence of the R term (Landshoff and Polkinghorne 1970, 1971). There is, however, a rather fundamental problem that the Veneziano model is constructed with factorized Reggeon couplings, whereas here we need a fixed-spin $J = 1$ coupling, so even if we project out $J = 1$ on one leg of the Veneziano model problems concerned with the difference between elementary and composite particle couplings arise (see Drummond 1972).

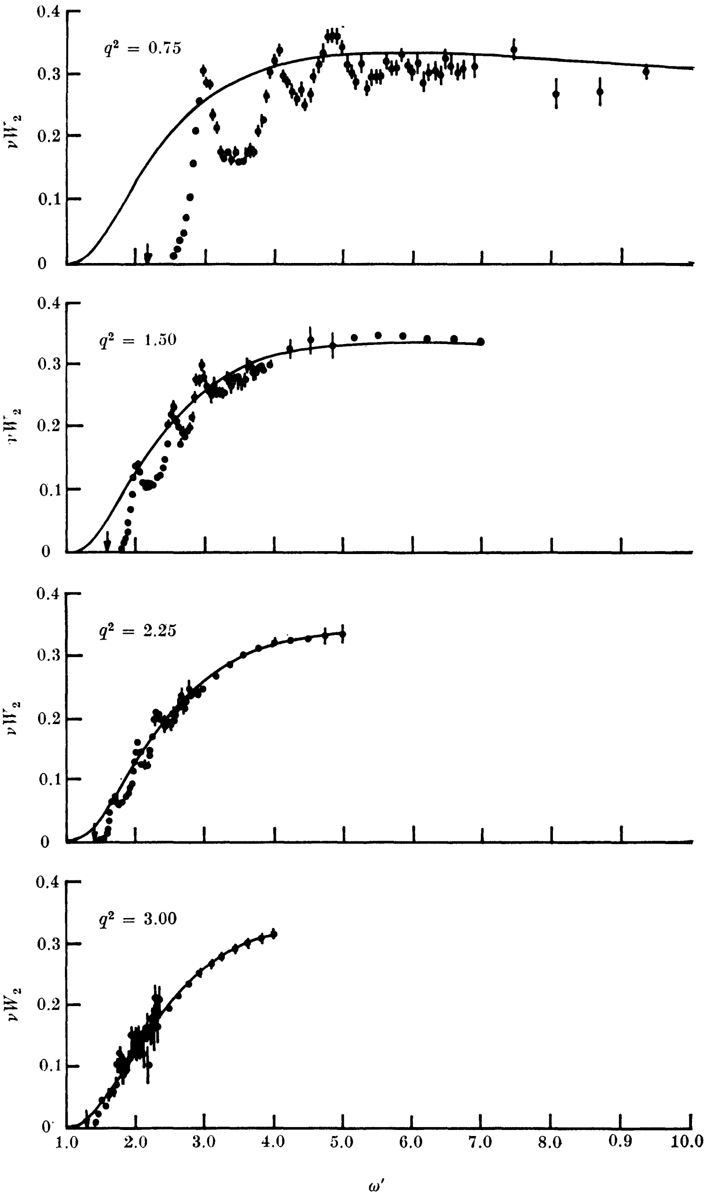

Fig. 12.16 Plot of $\nu W_2(\nu, q^2)$, at various values of $|q^2|$ (GeV units), versus $\omega' \equiv \omega + m_N/q$. The solid line is the scaling curve found for higher values of $|q^2|$ and the resonance oscillations converge to this line as $|q^2|$ increases. (From Gilman 1972.)

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In deep inelastic neutrino scattering, $\nu \mathrm{p} \to \mu X$, there is an extra structure function because parity is not a conserved quantity, and the differential cross-section reads, instead of (12.5.6),

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'} = \frac{G^2E'^2}{2\pi^2 m_{\mathrm{P}}^2}\left(2W_1(\nu,q^2)\sin^2\frac{\theta}{2} + W_2(\nu,q^2)\cos^2\frac{\theta}{2} \mp \frac{E+E'}{m_{\mathrm{p}}}W_3(\nu,q^2)\sin^2\frac{\theta}{2}\right) \quad (12.5.20)$$

with $\mp$ for $\nu, \bar{\nu}$ scattering, respectively. The extra function W_3 is odd under C_{n} and so vanishes as $\nu \to \infty$ since P exchange is not possible. Again the quark–parton model is rather successful, and Regge theory has been used to predict the high ν behaviour (see Llewellyn-Smith 1972). The most interesting results for Regge theory may come at higher energies (if these can be achieved) because the phenomenological Fermi theory, which can be used only for the first order in G, will violate unitarity for $E_{\nu\,\mathrm{lab}} > 10^5\,\mathrm{GeV}$, and so there must be unitarity corrections, and these will presumably heed the restrictions on fixed poles discussed in section 12.4.

In conclusion, Regge theory has so far had only a modest though honourable role to play in weak interactions, mainly because it is still possible to work only to first order in the weak coupling (e^2 or G). But when the time comes to construct a proper unitary theory of the weak interactions of hadrons, or even perhaps a unified theory of all the interactions, Reggeization will be a crucial ingredient.