A second fundamental family, the 'Dirichlet ideal norms', is described in terms of the action of $T$ on elements of the form
\[ \int_M \overline{\alpha_k(s)} f(s) \, d\mu(s), \]
where $f \in L_2(M, \mu, X)$.

The book gives a very systematic and thorough account of the theory generated by these ideas, first for general orthonormal systems and then for particular ones. Many of the natural orthonormal systems are sets of characters on compact abelian groups, which allows some unification of treatment. Each of the following systems is given an extended treatment: the Rademacher system; the Gauss system; the Fourier system $e^{ikt}$ and the discrete Fourier system; the Walsh system; the Haar system (leading to ‘martingale-type ideal norms’).

A general theme is the search for equivalence, or asymptotic equivalence, between the norms resulting from these systems. A further variation is to introduce unconditionality by inserting factors $\varepsilon_k \in \{-1, 1\}$ on the right-hand side in the definition quoted above. In this context we meet yet another subspecies, the ‘UMD ideal norms’.

This agenda throws up a multitude of hard problems and deep theorems, as well as a certain amount of routine verification. Truly 'geometric' notions such as $C^2$-convexity and $J$-convexity make their appearance at the right time, so that in the end the coverage of Banach space geometry is indeed quite wide.

It must be said that the book does not lend itself to casual dipping. The system of fixed notations is highly logical but not very user-friendly. Each ideal norm, each orthonormal system and each class of operators is accorded its fixed notation; the index lists 65 such classes of operators, each denoted by a group of Gothic characters which most readers will not be able to reproduce in handwriting. Anyone interested in a later topic, say the Rademacher system, will need to start by investing some time in mastering the notation and the earlier, more general sections.

However, the book contains an enormous amount of information, and by taking ideal norms as the unifying thread, the authors have achieved a distinctive and systematic way of organizing this material. For serious students of the subject area it will serve as an invaluable reference book for years to come.

G. J. O. JAMESON

LAM, T.-Y. Lectures on modules and rings (Graduate Texts in Mathematics, vol. 189, Springer, 1999), xxiii + 557 pp., 0 387 98428 3 (hardback), $46.

The title of this book, and the first sentence 'An effective way to understand the behavior of a ring $R$ is to study the various ways in which $R$ acts on its left and right modules', indicate the approach to Ring Theory to be followed throughout this book. The primary object of interest is the study of rings via their categories of modules. This is in marked contrast with the author's earlier book in the same series A first course in noncommutative rings (GTM 131, Springer, 1991), where the rings themselves are firmly in the foreground at all times.

The book under review can be taken either as an adjunct to the earlier book, or as a basis for an independent course of study. There are seven chapters. The first three chapters are devoted to basic theory of modules with a view to homology, the chapter headings being: 1. Free modules, projective modules and injective modules; 2. Flat modules and homological dimension; 3. More theory of modules. This takes one about halfway through the book. There are then two chapters on the localization theory of rings: 4. Rings of quotients, which concentrates on Goldie Theory; 5. More rings of quotients, which considers the maximal ring of quotients and the Martindale ring of quotients. Chapter 6 deals with Frobenius and Quasi-Frobenius rings and Chapter 7 considers Morita Theory.
Throughout the book, theory is motivated by many worked examples in the text, and there are many exercises provided for the reader to do. For example, before Morita equivalence is developed, there is a thorough discussion of the relation between the category of modules over a ring $R$ and the category of modules over the ring of matrices $M_n(R)$. This approach makes the book well-suited for individual study. Also, the author’s writing style makes the book easy to read. The reviewer strongly recommends this book for anyone with an interest in studying this area of algebra.

T. H. LENAGAN


Semi-classical analysis is concerned with relationships between quantum mechanics and classical mechanics when Planck’s constant $h$ goes to zero. In the Schrödinger operator $-h\Delta + V$, describing the motion of an electron under the influence of an electric field with potential $V$, the main quantum mechanical objects of study are the wave functions (eigenfunctions) and energy levels (eigenvalues), while the classical ones are given by the trajectories of the associated classical Hamiltonian $p(x, \xi) = \xi^2 + V(x)$, that is, the integral curves of the corresponding Hamiltonian field $H_p = 2\xi \cdot (\partial / \partial x) - V'(x) \cdot (\partial / \partial \xi)$. In recent years considerable progress has been made in the understanding of many of the spectral problems within semi-classical analysis through the application of the results and techniques of microlocal analysis; a good example of this is the work of Hörmander, Duistermaat-Guillemin, Ivrii and others on the asymptotic distribution of eigenvalues of elliptic operators on compact manifolds and bounded domains, in which the error terms in the asymptotic formulae depend on the dynamical properties of the Hamiltonian flow. However, while microlocal analysis has had a profound influence on semi-classical analysis, the benefits have not flowed in one direction only, and the relationship between the two subjects is a symbiotic one. Microlocal analysis was originally motivated by problems in partial differential equations, but there are features which are reminiscent of quantum mechanics. For instance, the uncertainty principle has a vital role in both theories. In microlocal analysis the analogues of the objects of quantum mechanics are the pseudo-differential and Fourier integral operators, while the classical ones are those of symplectic geometry such as Poisson brackets and canonical transformations.

These lecture notes give an up-to-date account of recent developments in semi-classical analysis, many of them by the authors and their collaborators, and include necessary background material from microlocal analysis. The following selection of chapter headings gives the flavour: local symplectic geometry, the WKB method, the method of stationary phase, the tunnel effect and interaction matrix, h-pseudodifferential operators, trace class operators and applications of the functional calculus, spectral theory for perturbed periodic systems.

Researchers and graduate students in mathematical analysis are the intended readership, but anyone with an interest in the current state of the mathematics of quantum mechanics will get a lot out of this book. The pace of delivery is brisk, but this has made it possible for a lot of material to be compressed into a relatively short space.

W. D. EVANS