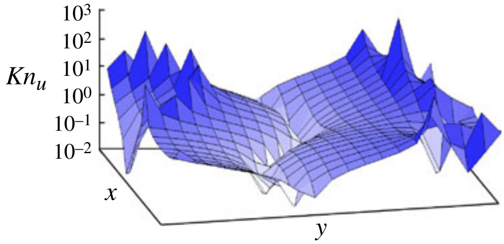
Focus on Fluids journals.cambridge.org/focus

The surprising relevance of a continuum description to granular clusters



M. Y. Louge†

Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY 14853, USA

Nature shuns homogeneity. In turbulent clouds, industrial reactors and geophysical flows, discrete particles arrange in clusters, posing difficult challenges to theory. A persistent question is whether clusters can be modelled with continuum equations. Recent evidence presented by Mitrano *et al.* (*J. Fluid Mech.*, vol. 738, 2014, R2) indicates that suitable equations can predict the formation of clusters in granular flows, despite violating the simplifying assumptions upon which they are based.

Key words: particle-fluid flow, kinetic theory, granular media

1. Introduction

The article by Mitrano *et al.* (2014) addresses the pivotal question of whether particle suspensions can be modelled by continuum equations, despite the inevitable formation of clusters. Whether particles interact with atmospheric turbulence (Chun *et al.* 2005) or flow as a suspension through chemical reactors (Pepiot & Desjardins 2012), they invariably produce clusters that theory cannot ignore. A challenge is to predict the drag on particles, so that their concentration and reaction rates may be calculated. Unfortunately, the average drag force on clustered solids is much less than if particles were homogeneously distributed (Helland *et al.* 2007), a fact that wild geese exploit to fly long distances.

An essential ingredient for cluster formation is inertia, which compels particles to skip fluid streamlines (Maxey 1987). Kinetic energy losses exacerbate this mechanism, either as particles interact with the surrounding fluid (Wylie & Koch 2000), or as they collide inelastically with one another (Hopkins, Jenkins & Louge 1993). Turbulence also congregates particles in regions where their collisions are more frequent than if they were homogeneously distributed (Longmire & Eaton 1992; Duncan *et al.* 2005; Simonin *et al.* 2006; Pan & Padoan 2010; Bec *et al.* 2010).

† Email address for correspondence: MYL3@cornell.edu

doi:10.1017/jfm.2013.650

Numerical simulations reveal details of cluster dynamics by tracking individual particles, either as points subject to a local drag law (van der Hoef *et al.* 2008; Capecelatro & Desjardins 2013), or by resolving the flow around them (Wylie, Koch & Ladd 2003). However, real systems can involve more particles than numerical simulations can directly handle. Treating particles as a continuum allows computation on a larger scale (Agrawal *et al.* 2001; Fox 2012). Yet, a persistent question is whether this can capture cluster formation, particularly when particles cross (Desjardins, Fox & Villedieu 2008). Therefore, because clusters also form in a granular 'gas' of inelastic particles colliding in a vacuum, it is instructive to test, as Mitrano *et al.* (2014) did, whether hydrodynamic equations are relevant to the particle phase without a fluid.

2. Overview

Hydrodynamic equations for granular gases are obtained by extending the Maxwell–Boltzmann kinetic theory to inelastic impacts. A central concept is the granular temperature which, as with a gas of hard spheres, represents the kinetic energy stored in velocity fluctuations. At first, Jenkins & Richman (1988) and others considered weak collisional energy dissipation, mostly with smooth inelastic spheres or disks, but possibly involving frictional interactions as well (Lun 1991; Jenkins & Zhang 2002). The resulting equations reproduced most features of numerical simulations and experiments without gravity (Xu, Louge & Reeves 2003; Xu *et al.* 2009), but they did not apply to highly inelastic systems.

To derive a granular hydrodynamics for higher collisional energy dissipation, Garzó & Dufty (1999) considered a homogeneous cooling process, in which particles steadily lose their initial agitation through collisions, without any mechanism to replenish it. Their calculations uncovered concentration gradient terms in the energy flux that become important at high inelasticity. However, as Goldhirsch & Zanetti (1993) had observed, the cooling process forms clusters, contradicting the homogeneity that Garzó & Dufty (1999) invoked. At first glance, particle clusters also appear to challenge two chief simplifications of the kinetic theory. In the first, particles are required to forget their past rapidly, so the theory can handle reshuffling of their statistical velocity distribution after impacts. Long-lasting coherent clusters cast doubt on this 'molecular chaos' assumption.

In the second simplification, most kinetic theories forbid variations on a scale less than the mean free path between consecutive collisions, so the granular gas is not rarefied. In other words, the Knudsen ratio Kn of mean free path to a gradient length scale is taken to be small. Yet, with a periphery featuring steep variations next to relatively small concentrations, clusters can produce regions of high mean free paths and large Kn based on velocity gradients. In principle, the Navier–Stokes equations can then be refined by expanding the velocity distribution to higher-order in Kn (Agarwal, Yun & Balakrishnan 2001). However, for granular materials, this 'Burnett' expansion must also account for non-trivial effects of particle inelasticity (Sela & Goldhirsch 1998; Kumaran 2006).

Mitrano *et al.* (2014) suggest that such a laborious approach might be avoided, while preserving clusters as a natural instability of the hydrodynamic equations. To show this, they juxtapose results from simulations and a numerical integration of the hydrodynamic equations that Garzó & Dufty (1999) derived for small Kn (figure 1). The linear stability analysis of Garzó (2005) yields a scale that Mitrano *et al.* (2012) had interpreted as a cluster length. As Brilliantov *et al.* (2004) suggested, clusters materialize in systems large enough to contain them. In short, continuum equations at

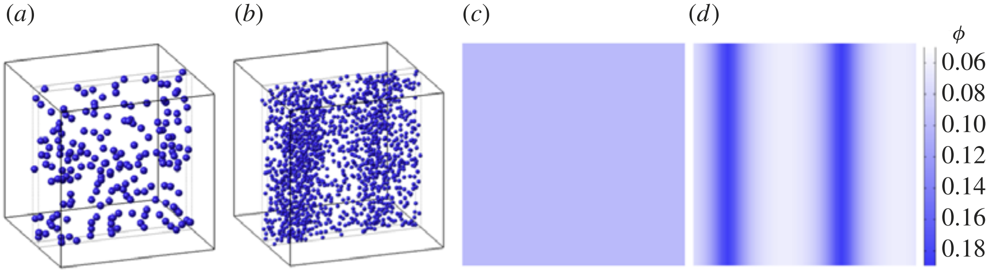


FIGURE 1. Mitrano *et al.* (2014) juxtapose (*a,b*) simulations and (*c,d*) solutions of the equations of Garzó & Dufty (1999) having similar features. For a large enough domain, clusters form in simulations (*b*), and they also arise as instabilities of the continuum theory (*d*).

the Navier–Stokes order initiate realistic granular clusters. This surprises Mitrano *et al.* (2014) for two reasons. First, vortices produce velocity correlations contradicting the molecular chaos assumption. Then, clusters comprise regions of large *Kn* that stretch equations beyond limits of their derivation.

3. Future

The article of Mitrano *et al.* (2014) fits within a fertile line of observations and models on granular inhomogeneities. Their remarks suggest that complicated extensions of hydrodynamic equations to finite Knudsen number may not be necessary to capture cluster onset. This is reminiscent of a debate in gas dynamics, whereby adjustments to the Navier–Stokes equations (Brenner 2005) could describe shock waves (Greenshields & Reese 2007), thus avoiding Burnett-order considerations despite failures of the Navier–Stokes equations (Alsmeyer 1976). This simplification will be welcomed by those attempting to unify theories of clusters arising from fluid–particle interactions and granular collisions away from walls.

References

AGRAWAL, K., LOEZOS, P. N., SYAMLAL, M. & SUNDARESAN, S. 2001 The role of meso-scale structures in rapid gas–solid flows. *J. Fluid Mech.* **445**, 151–185.

AGARWAL, R. K., YUN, K.-Y. & BALAKRISHNAN, R. 2001 Beyond Navier–Stokes: Burnett equations for flows in the continuum-transition regime. *Phys. Fluids* **13**, 3061–3085.

ALSMEYER, H. 1976 Density profiles in argon and nitrogen shock waves measured by the absorption of an electron beam. *J. Fluid Mech.* **74**, 497–613.

BEC, J., BIFERALE, L., CENCINI, M., LANOTTE, A. S. & TOSCHI, F. 2010 Intermittency in the velocity distribution of heavy particles in turbulence. *J. Fluid Mech.* **646**, 527–536.

BRENNER, H. 2005 Navier-Stokes revisited. *Physica* A **349**, 60–132.

BRILLIANTOV, N., SALUEÑA, C., SCHWAGER, T & PÖSCHEL, T. 2004 Transient structures in a granular gas. *Phys. Rev. Lett.* **93**, 134301.

CAPECELATRO, J. & DESJARDINS, O. 2013 An Euler–Lagrange strategy for simulating particle-laden flows. *J. Comput. Phys.* **238**, 1–31.

CHUN, J., KOCH, D. L., RANI, S. L., AHLUWALIA, A. & COLLINS, L. R. 2005 Clustering of aerosol particles in isotropic turbulence. *J. Fluid Mech.* **536**, 219–251.

DESJARDINS, O., FOX, R. O. & VILLEDIEU, P. 2008 A quadrature-based moment method for dilute fluid-particle flows. *J. Comput. Phys.* **227**, 2514–2539.

DUNCAN, K., MEHLIG, B., ÖSTLUND, S. & WILKINSON, M. 2005 Clustering by mixing flows. *Phys. Rev. Lett.* **95**, 240602.

FOX, R. O. 2012 Large-eddy-simulation tools for multiphase flows. *Annu. Rev. Fluid Mech.* **44**, 47–76.

GARZÓ, V. 2005 Instabilities in a free granular fluid described by the Enskog equation. *Phys. Rev.* E **72**, 021106.

GARZÓ, V. & DUFTY, J. W. 1999 Dense fluid transport for inelastic hard spheres. *Phys. Rev.* E **59**, 5895–5911.

GOLDHIRSCH, I. & ZANETTI, G. 1993 Clustering instability in dissipative gases. *Phys. Rev. Lett.* **70**, 1619–1622.

GREENSHIELDS, C. J. & REESE, J. M. 2007 The structure of shock waves as a test of Brenner's modifications to the Navier–Stokes equations. *J. Fluid Mech.* **580**, 407–429.

HELLAND, E., BOURNOT, H., OCCELLI, R. & TADRIST, L. 2007 Drag reduction and cluster formation in a circulating fluidised bed. *Chem. Engng Sci.* **62**, 148–158.

VAN DER HOEF, M. A., VAN SINT ANNALAND, M., DEEN, N. G. & KUIPERS, J. A. M. 2008 Numerical simulation of dense gas–solid fluidized beds: a multiscale modelling strategy. *Annu. Rev. Fluid Mech.* **40**, 47–70.

HOPKINS, M. A., JENKINS, J. T. & LOUGE, M. Y. 1993 On the structure of three-dimensional shear flows. *Mech. Mater.* **16**, 179–187.

JENKINS, J. T. & RICHMAN, M. W. 1988 Plane simple shear of smooth inelastic circular disks: the anisotropy of the second moment in the dilute and dense limits. *J. Fluid Mech.* **192**, 313–328.

JENKINS, J. T. & ZHANG, C. 2002 Kinetic theory for identical, frictional, nearly elastic spheres. *Phys. Fluids* **14**, 1228–1235.

KUMARAN, V. 2006 The constitutive relation for the granular flow of rough particles, and its application to the flow down an inclined plane. *J. Fluid Mech.* **561**, 1–42.

LONGMIRE, E. K. & EATON, J. K. 1992 Structure of a particle-laden round jet. *J. Fluid Mech.* **236**, 217–257.

LUN, C. K. K. 1991 Kinetic theory for granular flow of dense, slightly inelastic, slightly rough spheres. *J. Fluid Mech.* **233**, 539–559.

MAXEY, M. R. 1987 The gravitional settling of aerosol particles in homogeneous turbulence and random flow fields. *J. Fluid Mech.* **174**, 441–465.

MITRANO, P. P., GARZÓ, V., HILGER, A. M., EWASKO, C. J. & HRENYA, C. M. 2012 Assessing a hydrodynamic description for instabilities in highly dissipative, freely cooling granular gases. *Phys. Rev.* E **85**, 041303.

MITRANO, P. P., ZENK, J., BENYAHIA, S., GALVIN, J., DAHL, S. & HRENYA, C. 2014 Kinetic-theory predictions of clustering instabilities in granular flows: beyond the small Knudsen regime. *J. Fluid Mech.* **738**, R2, 12 pages.

PAN, L. & PADOAN, P. 2010 Relative velocity of inertial particles in turbulent flows. *J. Fluid Mech.* **661**, 73–107.

PEPIOT, P. & DESJARDINS, O. 2012 Numerical analysis of the dynamics of two- and three-dimensional fluidized bed reactors using an Euler–Lagrange approach. *Powder Technol.* **220**, 104–121.

SELA, N. & GOLDHIRSCH, I. 1998 Hydrodynamic equations for rapid flows of smooth inelastic spheres, to Burnett order. *J. Fluid Mech.* **361**, 41–74.

SIMONIN, O., ZAICHIK, L. I., ALIPCHENKOV, V. M. & FÉVRIER, P. 2006 Connection between two statistical approaches for the modelling of particle velocity and concentration distributions in turbulent flow: the mesoscopic Eulerian formalism and the two-point probability density function method. *Phys. Fluids* **18**, 125107.

WYLIE, J. J. & KOCH, D. L. 2000 Particle clustering due to hydrodynamic interactions. *Phys. Fluids* **12**, 964–970.

WYLIE, J. J., KOCH, D. L. & LADD, A. J. C. 2003 Rheology of suspensions with high particle inertia and moderate fluid inertia. *J. Fluid Mech.* **480**, 95–118.

XU, H., LOUGE, M. & REEVES, A. 2003 Solutions of the kinetic theory for bounded collisional granular flows. *Contin. Mech. Thermodyn.* **15**, 321–349.

XU, H., VERBERG, R., KOCH, D. L. & LOUGE, M. Y. 2009 Dense, bounded shear flows of agitated solid spheres in a gas at intermediate Stokes and finite Reynolds numbers. *J. Fluid Mech.* **618**, 181–208.