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ELEMENTS OF ENRICHED AND QUANTUM CATEGORY THEORY

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The thesis consists of five research papers. All of them are based on a monoidal category $\mathcal{V}$. The first [1] generalizes the famous embedding theorem for ordinary categories to categories with their homs enriched in $\mathcal{V}$. All the others are dedicated to the study of quantum categories in $\mathcal{V}$. The first paper introduces regular enriched categories and proves an enriched version of the famous embedding theorem by M. Barr, which also generalizes the Freyd–Mitchell embedding theorem for abelian categories.

In the second paper [2] it is shown explicitly how the two known definitions of quantum categories are equivalent. It defines the notion of functor between quantum categories. The outcome is a category qCat. Furthermore, it develops various technical tools for working with quantum categories.

The third paper [5], with Steve Lack and Ross Street, develops a fragment of Hopf algebra theory in the context internal to an autonomous monoidal bicategory in the sense of the formal theory of (co)monads. Specifically, it defines Hopf comonads as monoidal comonads satisfying a certain property corresponding to the property of bialgebra that the fusion map is invertible. This leads to a new (logically equivalent) definition of quantum groupoid.

The fourth paper [3] studies the Takeuchi product $\times_A$, first considering an abstract setting of a general monoidal category $\mathcal{V}$, and then covering some special cases. The concept of module for 'many object' generalizations of Hopf structures is discussed. Such modules can also be thought of as quantum analogues of profunctors between small categories.

The fifth paper [4] shows how the Tannaka theory of bialgebroids is related to weak bialgebras, which are the same as bialgebroids with a separable Frobenius base

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monoid. A Tannaka representation theorem is obtained for a separable Frobenius fiber functor, which is the appropriate kind of fiber functor for the Tannaka theory of weak bialgebras.

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