THE TRAINING OF THE MATHEMATICAL TEACHER

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The professional training of the Mathematical Teacher after his
degree course is finished is a principle which has come to stay, and it
therefore behoves all who are concerned in the practical administra-
tion of this principle to do their utmost to make the course of profes-
sional training as valuable as possible for the future teacher. So far
as I know nothing has as yet been done by the Mathematical
Association as a body in helping forward this movement, but surely
the Mathematical Association will fail in its fundamental aims if it
does not take a very active part in helping to decide what are the best
courses of instruction to give to the student-teachers while in train-
ing. The object of the present paper is not to lay down dogmatic
assertions as to what ought to be done, but rather to put forward
suggestions, and thus to act as a nucleus round which the discussion
can revolve. No attempt is made to deal with the training of anyone
except the specialist type of mathematical teacher—the person who in
time to come is destined to teach in the Advanced Departments of
the Secondary Schools in addition to taking his share in the general
mathematical work of the school, and who, if weighed in the balance
and not found wanting, will be called upon to act as Head of a
Mathematical Department, and thus to guide and mould the mathe-
matical destinies of the institution to which he is attached. In
almost every University nowadays there exists a Department of
Education, and within that Department of Education a School of
Mathematical Pedagogy. It is for us to discuss now what ought to
be the type and scope of instruction given in this School of Mathe-
matical Pedagogy.

The normal course of study in the School of Mathematical
Pedagogy is one year, and I would suggest first of all that during this
year the student be not cut off entirely from his general mathematical
interests. Even two hours a week devoted to post-graduate study of
some branch of advanced mathematics in which he is particularly
interested will keep him fresh and not allow him to rust, mathem-
tically speaking. It will also be a welcome change from educa-
tional psychology and class-room practice. The man is to be a
teacher of mathematics, and the first essential of a teacher is to have
something to teach. He must not be allowed to forget the technical
knowledge of his craft.

The next point to consider is, that in order that the future teacher
may have a proper understanding of the methods of teaching
mathematics today, he must have a sound historical knowledge of
the development of mathematical teaching in the past. This requires both a knowledge of the history of the progress of Mathematics regarded as a science—a subject which has been too much neglected by Universities in the past—and it also requires a detailed study of the past methods of giving instruction in Mathematics. Thanks to the labours of Sir T. L. Heath and his scholarly editions of the works of the Greek Mathematical Teachers, it is possible to reconstruct the ancient methods of mathematical instruction in the Greek world and to picture their schools of study. The traditional method of propagating new theorems among the old Greeks is also of interest to young teachers—how the ancient Greek scholar interested both in abstract things and in the selling of his merchandise often cruised about among the islands vending his wares and discussing new mathematical theorems. Then comes the Mathematical World of the Middle Ages and the Renaissance. Take for example the views of Roger Bacon, who flourished about 1300, on the teaching of Mathematics. He strove hard to replace logic in the University curriculum by mathematical and linguistic studies, and it may cheer many a non-mathematician to know that his view that "divine mathematics alone can purge the intellect and fit the student for the acquirement of all knowledge" fell on deaf ears in his own day.

All through these centuries there is the record—clear here and dim there—not only of how Mathematics was taught, but also of the position of the subject in a liberal education. A knowledge of this important branch of history is surely essential to the future teacher in connection with his life's work, if he is to be an intelligent craftsman and not merely a dull purveyor of well-known mathematical facts. Lastly, there is the great movement of reform in the teaching of Mathematics in the present day. Sir Isaac Newton is not usually regarded as a man who would have been an ideal schoolmaster, but his view of an efficient Mathematical Department in a properly equipped Secondary School—drawn up for Christ's Hospital—deserves a prominent position on the walls of every school. It is a vision of a school mathematical department—at once idealistic and practical—that is but rarely realised even at the present day. The history of mathematical teaching in Great Britain from earliest times is but little known. A certain amount is known of the syllabuses of knowledge required of pupils in bygone days in some of the great and ancient English schools, but scholarly research on the whole subject has not yet been undertaken. Surely this is a good historical subject for some graduate in the Department of Mathematical Pedagogy of a University who wishes to qualify for his Ph.D. degree. Enough has been said, however, to show that the training of the future mathematical teacher is incomplete if he does not possess a sound historical knowledge of the history and development of the Methods of Mathematical Teaching and the place of Mathem-
We come now to the more technical aspect of the young teacher's training. In his school of Mathematical Pedagogy he will naturally ask for guidance and advice in his future teaching of such subjects as Ratio and Proportion in Geometry, Theory of Parallels, the fundamental theorems of the Calculus and so forth. A rigorous study of these subjects is impossible for young boys, but they can and do profit by a course of geometrical study in which intuition is allowed to play an important part. The question arises therefore, "Where does intuition stop and rigour begin?" The teacher is being continually forced to face this question and to face it honestly. Take for example the question of Ratio and Proportion—the crowning glory of Greek Geometry. Boys cannot face rigorous proofs based on logical definition and deduction, but they ought not to be put off with a welter of rigorous logic here, sheer intuition there and proofs that are vicious elsewhere. The teacher must himself understand his ground before he is in a position to give satisfactory presentation of the subject to boys. To do so, he must have made a profound study of Ancient Greek Geometry, of the attempts through all the centuries to get at a simpler explanation of the Greek treatment of ratio and proportion, of the work of Dedekind, etc., on the modern conceptions of measurement, the correspondence between numbers and points on a line, and so forth. Then and only then is the teacher in a position to know where he stands himself, and therefrom to devise a simple treatment of the subject for boys in which intuition and rigorous logic play each a legitimate but definitely assigned role. Take again the theory of Parallels. The teacher must have a real knowledge of the inherent difficulties of this problem and not merely a vague notion that there are difficulties somewhere, which are best overcome by using theorems got from some printed book. The teacher must know where he stands, and to do so he must have studied at his school of Mathematical Pedagogy the Greek Geometry of Parallels and the later historical researches of Lobacheffsky and Bolyai on Non-Euclidean Geometry. He will then be in a scientific position to disentangle Logic and Intuition and so to devise a treatment suitable for his boys, where they will have nothing to unlearn later on. Finally, consider the Calculus. Much print and more discussion have been wasted at pedagogic conferences in trying to give a suitable presentation of the "limit" in finding a differential coefficient than in almost any other subject except Congruence Theorems in Geometry. I can offer no solution, but I do feel that we ought to save the young teachers of the future from many of our avoidable errors of the past by insisting that during their course of training they should be equipped with the necessary scientific knowledge and outlook without which no satis-
factory solution is possible. For this is necessary a profound study of the elements of Modern Analysis.

It may be argued that all this ought to be known as part of the University Course of Mathematics and that all the School of Mathematical Pedagogy has to do is to superpose the clinical experience and the "bedside manner." But this is not so. The object of a University Course in Mathematics is not to turn out a technical expert in some of the applications of mathematics, such as mathematical teaching, but to give the student a general training in the subject of mathematics, and the branches chosen for the student will depend partly on the taste and special studies of the University Lecturers and partly on those of the student himself. The course at the Training College has, however, a specific end in view, namely to equip the teacher-in-training with those precise branches of mathematics which are necessary for a complete understanding of what he has to teach in a Secondary School and to correlate the science of mathematics with the science of psychology, and both with the human material at the various ages of life at a Secondary School. It is a specialised subject, and requires lecturers who have made a special study of these aspects of mathematical, psychological and educational study.

I would suggest in conclusion, therefore, that the course of training at the School of Mathematical Pedagogy to which the student is attached should comprise:

I. Clinical Mathematics, that is, teaching practice in the Classroom.
III. Those more philosophical branches of Mathematics which are essential to an intelligent understanding of the inherent difficulties underlying those parts of Mathematics taught in a Secondary School.
IV. Post-graduate lectures on Mathematics, or research, for two or three hours a week.

If the course is less complete than this, our students have asked us for bread and we have given them a stone.

[Vol. X, No. 147, 1920.]

DISCUSSION ON "HOW TO KEEP TEACHERS OF MATHEMATICS IN TOUCH WITH MODERN METHODS AND DEVELOPMENTS"

The discussion was opened by the Rev. E. M. Radford, who said that every sincere teacher of Mathematics would desire to possess that freshness and inspiration which alone can make his work of