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**"The Ambiguous Case" in the Solution of Triangles.** —Given $a$, $b$ and A, there are sometimes two distinct solutions, sometimes one, and sometimes none. The geometrical explanation is a commonplace, from which it appears that two solutions occur if $b > a > b\sin A$. [But see correction below.]

The usual method of solution is to calculate B from the equation $\sin B = b\sin A/a$, whence $B = B_1$ or $180° - B_1 \equiv B_2$, where $B_1$ is the acute angle whose sine is $= b\sin A/a$.

It is not usually noticed, however, that in the actual calculations there is a point where the number of solutions existing can be inferred without any subsidiary calculation or consideration. This is at the stage when, B having been determined provisionally as $B_1$ or $180 - B_1$, we calculate C from the formula $C = 180° - B - A$. We get two values, $C_1 = 180° - B_1 - A$ and $C_2 = 180 - (180° - B_1) - A$, or, say, $C_1 = B_2 - A$, $C_2 = B_1 - A$. If these values are both *positive* there are two solutions, if one is negative there is but one solution, viz., the positive value. The geometrical interpretation is obvious.

But further, both $C_1$ and $C_2$ may be negative (when A is obtuse), and this is a point commonly overlooked in the text-books; in fact, the statement that $b > a > b\sin A$ is a sufficient condition for two solutions (or for any solution) is inexact. If A is obtuse there cannot be two solutions, and there is one only if the condition $a > b$ is fulfilled.

The criterion afforded by the sign of C is sufficient for every case; and as it is simple and complete, and arises naturally in the process of solution, it ought, I think, to take the first place in teaching. The only other point to note is that if $b\sin A/a > 1$ there is of course no angle B for which $\sin B = b\sin A/a$, and therefore no solution. This point also arises in the course of the actual calculations required.

Of course when the subject is being taught for the first time the usual geom rical discussion is quite in place. But I think it should be made clear that the ordinary process of calculation furnishes naturally all the information required.

R. F. MUIRHEAD.

**A method of calculating logarithms, using merely the ordinary laws of indices.**—*To determine logarithms to the base* 1·1.—If $N = a^x$, $x$ is the logarithm of N to the base $a$. Hence, if $N = 1{\cdot}1^x$, $x$ is the logarithm of N to the base 1·1, and, by plotting different values of $x$ with the corresponding values of N, a logarithmic curve may be drawn. This curve will furnish an approximate logarithm of any number to the base 1·1.

*To plot the curve*: $y = 1{\cdot}1^x$.—The method of determining values of $y$ to satisfy this equation is exceedingly simple, resolving itself into a series of multiplications by 11. This may be readily done by putting down the last figure and taking the sums of the consecutive figures in pairs. This process may be carried to any required degree of accuracy.

| $y$. | $x$. |
|---|---|
| 11 | 1 |
| 121 | 2 |
| 1331 | 3 |
| 14641 | 4 |
| 161051 | 5 |
| 177156 | 6 |