

When, however, we come to deal with annual payments, we find that the rate of interest is involved: that is  $(ni)$  is not a constant, but still very nearly so. For  $\bar{s}_n$  is the equivalent of the amount of an annual payment of  $\frac{d}{\delta}$  in advance for  $n$  years, and we therefore require the value of  $(n\delta)$  where

$$\frac{e^{n\delta} - 1}{n\delta} = \frac{d}{\delta}$$

The value of  $(ni)$  will then be obtained from

$$\log (ni) = \log (n\delta) + \log \left( \frac{i}{\delta} \right)$$

For practical values of  $i$  say .03 and .06 it is found that  $(ni)$  equals 125.03/100 and 124.40/100 respectively.

It is seen, therefore, that  $n(100i)$  is subject to a very small range of variation and the adoption of a general and easily remembered value of 125 as suggested by Mr. Touzel is sufficient for the practical purposes in view.

Yours faithfully,

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Wellington, New Zealand.

6 April 1926.

#### ON THE VALUES OF $P_x$ AT VARIOUS RATES OF INTEREST BY DIFFERENT MORTALITY TABLES.

*To the Editors of the Journal of the Institute of Actuaries.*

DEAR SIRS.—Recently, in checking some monetary tables which I had constructed on the basis of Australian mortality, I was struck with a feature of the function  $P_x$  which appears likely to be of service in certain cases where monetary tables for a given experience are available for a very limited range of rates of interest.

2. I found, when the values of  $P_x$  for any given age at various rates of interest by one mortality table are compared rate by rate with the corresponding values for the same age by another table, that there is roughly a constant difference between these respective values, or more correctly, that these differences, while increasing or decreasing with increases in the rate of interest involved, do so with marked regularity and very slowly.

3. An example will make my meaning clearer. Taking the  $H^M$  (Makeham Graduation) and the  $O^M$  Tables, the corresponding values of  $P_x$  for rates 3 per-cent,  $3\frac{1}{2}$  per-cent and 4 per-cent by each table and the differences between these values are as follows:

Rate of interest	P <sub>25</sub>		Difference (2) - (3)
	H <sup>M</sup>	O <sup>M</sup>	
(1)	(2)	(3)	(4)
3	·01628	·01524	·00104
3½	·01524	·01416	·00108
4	·01431	·01321	·00110

4. Taking similar differences for ages 35, 45, 55 and 65 the following results are obtained :

Rate of interest	DIFFERENCES FOR AGE				
	25	35	45	55	65
3	·00104	·00070	·00069	·00096	·00175
3½	·00108	·00071	·00068	·00096	·00174
4	·00110	·00072	·00068	·00096	·00173

5. The H<sup>M</sup> and O<sup>M</sup> Tables have a great deal in common since each is based on an experience of the British Isles, both relate to assured lives, both are aggregate experiences and both have been graduated by Makeham or modified Makeham methods. It might, therefore, be thought that the feature indicated is related to the general similarity of constitution of the tables compared.

6. To test this I have compared the table which I have recently constructed on the basis of male lives in the Australian general population for the three years 1920, 1921 and 1922, called A<sup>M21</sup>, with the H<sup>M</sup> Table used above, and have been able to use a larger range of rates of interest than was available in the O<sup>M</sup> Table. I have taken the rates 3 per-cent, 3½ per-cent, 4 per-cent, 4½ per-cent, 5 per-cent and 6 per-cent, and have compiled a table for age 25 similar to the first of those given above :

Rate of interest	P <sub>25</sub>		Difference (2) - (3)
	H <sup>M</sup>	A <sup>M21</sup>	
(1)	(2)	(3)	(4)
3	·01628	·01314	·00314
3½	·01524	·01204	·00320
4	·01431	·01107	·00324
4½	·01349	·01021	·00328
5	·01276	·00946	·00330
6	·01155	·00820	·00335

7. Corresponding differences for ages 35, 45, 55 and 65 are compared below with those obtained for age 25.

Rate of interest	DIFFERENCES FOR AGE				
	25	35	45	55	65
3	·00314	·00334	·00386	·00513	·00755
3½	·00320	·00338	·00389	·00517	·00758
4	·00324	·00342	·00392	·00521	·00761
4½	·00328	·00344	·00394	·00523	·00764
5	·00330	·00347	·00396	·00525	·00765
6	·00335	·00350	·00398	·00528	·00769

8. In the comparison of  $H^M$  with  $O^M$ , the differences for the earlier ages increased with increase in the rate of interest, attained identity in the later ages and eventually decreased with increasing rates of interest. In the comparison of  $H^M$  with  $A^{M21}$ , there is an increase throughout with increase in rate of interest.

9. In view of the fact that in all cases these differences have been obtained from values which are themselves adjusted to the nearest fifth place of decimals and that they are consequently subject to a relatively large possible error due to such adjustment, their regularity is all the more striking.

10. These results indicate that if a standard table were constructed and tables of  $P_x$  computed thereon at rates of interest ranging from say, 2 per-cent to 6 per-cent (inclusive) by intervals of ½, all that would be necessary to obtain the same range with approximate accuracy for a new experience would be the construction on the basis of that experience of  $P_x$  at say 4 per-cent, and the calculation of the difference for each age between these values and those of the 4 per-cent standard table. The differences so obtained applied to the other standard tables would give approximately the corresponding values for the new experience.

11. From the regularity of the differences obtained from such dissimilar experiences as the  $H^M$  and the  $A^{M21}$ , it would appear that there is no necessity for the suggested standard mortality table to represent any actual experience. It might, for example, be a composite table, or possibly one so constructed as to give a mathematically ideal representation of the curve of life.

Yours faithfully,

CHAS. H. WICKENS.

Melbourne,

31 March 1926.