# Erratum to: An Exactly Solved Model for Recombination, Mutation and Selection 

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The proof of Lemma 5 in reference [1] implicitly uses a property that is not stated as an assumption. Namely, even if the totally positive operator $W$ satisfies $W=$ $W_{<\alpha} \otimes W_{>\alpha}$, the relations

$$
W_{<\alpha} \circ \pi_{<\alpha}=\pi_{<\alpha} \circ W \quad \text { and } \quad W_{>\alpha} \circ \pi_{>\alpha}=\pi_{>\alpha} \circ W
$$

can only hold if both $W_{<\alpha}$ and $W_{>\alpha}$ are separately norm-preserving, at least for the positive measures. If one needs these relations for all links $\alpha$, each $W_{m}$ in $W=$ $\bigotimes_{m=0}^{n} W_{m}$ must be norm-preserving on $\mathcal{M}_{+}\left(X_{m}\right)$, which was not stated and went unnoticed for a while.

A correct formulation of Lemma 5 thus reads as follows.

Lemma 5 Let $W$ be a strictly positive bounded linear operator on $\mathcal{M}^{\otimes}$ which has a complete tensor product structure, so that $W=W_{0} \otimes \cdots \otimes W_{n}$. Then, given $\alpha \in L$, the elementary recombinator $R_{\alpha}$ commutes with $W$ on $\mathcal{M}_{+}(X)$, if $W_{<\alpha}$ and $W_{>\alpha}$ are normpreserving on $\mathcal{M}_{+}\left(X_{<\alpha}\right)$ and $\mathcal{M}_{+}\left(X_{>\alpha}\right)$. When all $W_{m}$ are separately norm-preserving on $\mathcal{M}_{+}\left(X_{m}\right)$, one has $W R_{\alpha}=R_{\alpha} W$ on $\mathcal{M}_{+}(X)$ for all $\alpha \in L$.

This has no further consequence for Section 4 of the paper, because the mutation operator $Q$ is assumed to be a complete product of site-wise Markov generators, so that $\exp (t Q)$ is automatically norm-preserving on each site space separately.

Unfortunately, this is not so for Section 6 on the combination with a selection operator, as additive selection does not imply this additional property. Imposing sitewise norm-preservation to the semigroup generated by $P$, for instance in Lemma 6, removes the interesting degrees of freedom and brings the lemma back to the case of a Markov semigroup, up to free normalization constants per site. Theorem 6 suffers the same fate - and this part of the paper thus does not cover interesting cases of additive fitness.

[^0]We thank Nick Barton, Matthias Steinrücken, and Yun Song for alerting us to the problem with Theorem 6.

## References

[1] M. Baake and E. Baake, An exactly solved model for mutation, recombination and selection. Canad. J. Math. 55(2003), no. 1, 3-41;

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