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METASTABLE SETS IN OPEN DYNAMICAL SYSTEMS AND SUBSTOCHASTIC MARKOV CHAINS

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The focus of this thesis is dynamical systems in which typical trajectories (1) have a nonzero probability of exiting the state space and (2) before exiting, tend to remain in one proper subset of the state space for a long time. The first property defines an *open* dynamical system and the second property is called *metastability*. Sets in which trajectories remain for a long time are called metastable or almost-invariant sets. The major contribution of this thesis is the development of techniques to locate and characterise metastable sets in open dynamical systems.

In closed dynamical systems, there are well-established connections between the spectrum of the Perron–Frobenius operator and the metastability properties of the system. After introducing the research aims in Chapter 1, we review the existing literature and establish notation in Chapter 2. One can use the eigenfunctions of the transfer operator to *locate* metastable sets, and one can *derive bounds* on the maximal invariance ratio of a set in terms of the second largest eigenvalue of a discretised version of the Perron–Frobenius operator. In Chapters 3 and 4, we extend these techniques to open dynamical systems. Chapter 3 introduces a new closing operation for open systems that has a minimal effect on the metastability properties, and allows us to apply existing techniques for closed systems to locate metastable sets and to derive bounds on the maximal invariance ratio in terms of the second largest eigenvalue of the new operator. In Chapter 4, we derive bounds on the metastability and the conductance of substochastic Markov chains, which can be related to discretised transfer operators for open dynamical systems. Both conductance and metastability quantify how well subsets of states interact and mix.

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Finally, in Chapter 5, we apply some of the techniques developed in previous chapters to a global ocean model. The dynamics of the ocean surface circulation is known to contain attracting regions such as the great oceanic gyres and the associated garbage patches. Understanding the shape and extent of the basins of attraction of these regions sheds light on the question of the strength of connectivity of different regions of the ocean, which helps in understanding the flow of buoyant material like plastic litter. Using short flow time trajectory data from a global ocean model, we create a Markov chain model of the surface ocean dynamics. Using our Markov chain model, we compute net surface upwelling and downwelling, and verify that it matches observed patterns of upwelling and downwelling in the real ocean. We then analyse the Markov chain to characterise the connectivity of the surface of the ocean using both absorption probabilities and eigenvector methods.

Some of these results have been published in [1, 2].

References

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