CORRESPONDENCE.

THE LAW OF HUMAN MORTALITY.

To the Editor of the Assurance Magazine.

SIR,—The importance of the discovery of the law of human mortality is too obvious to require any further explanation, and it is highly gratifying to observe that the labours which have been brought to bear upon the discovery of these laws, and which for a time seemed to have ceased, have lately been renewed. The reader of the *Assurance Magazine* must feel indebted to this periodical for the publication of any essay referring to this question, for even a fruitless endeavour in this direction may be the means of leading another inquirer into the right path; and from this point of view even the contest which has sprung up between Mr. Gompertz and Mr. Edmonds, although not pleasing in itself, may prove of advantage to the question in point.

I have not the least intention of questioning the value of Mr. Edmonds's exertions, and I neither can nor will interfere in the contest as to the priority of the discovery; but when Mr. Edmonds opposes principally Mr. Gompertz on the ground of his formula giving only a numerical approximation, while he asserts to have found out a law of nature—when he places the result of his labours beyond all doubt—when he does not hesitate to assert (*Assurance Magazine*, ix., p. 177), "The truth is, however, that (p), with its three determinate values, is independent of all formulæ, has existed as long as man has existed, and forms part of the foundations of the universe," then I venture to object, begging to add a few remarks. I fear that such assertions as those above mentioned may easily lead the student in an entirely false direction. I think the only point we can arrive at in the present state of science is to discover a formula which gives a numerical approximation to the numbers contained in the table of mortality; any discovery of the natural law of mortality cannot be contained in a formula only—on the contrary, thereby the elements which constitute the formula must be explained in their reference to the effect of death.

I beg permission to add a few explanatory remarks.

When we derive any conclusions from a table of mortality, we thereby pronounce our conviction that the table contains the expression of a natural law, although it may be only a numerical approximation to this law. If we assume (according to Deparcieux), as a general rule, that, out of 814 persons aged 20 years, after the lapse of a year only 806 are alive, this will say neither more nor less than that certain for the present unknown powers, acting in persons aged 20 years in accordance with certain for the present unknown laws, undergo such a change in the course of a year, that the one side of their effect, which we call "life," is thus changed, that while its measure has been expressed by 814 in the beginning of the year, it is only 806 at the end of it, supposing we considered a sufficient number of cases, so that the disturbances from other unknown accidental causes may be left out of consideration. I do not see the least reason to suppose that the phenomenon which we call "life" is the effect of a single cause, but just as little reason to suppose that this phenomenon be the only effect of the force or forces which are its cause. We will call the sum of the

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single forces (resulting force) which are the cause of "life," however, only with reference to this utterance of their activity and with the exclusion of those parts acting in other directions, the "power of life," without troubling ourselves as to this being a single or a composed force, this question being immaterial as to the numerical results. It is obvious that even if we be so fortunate as to discover the law of the changes of this power, we cannot draw any conclusions as to its components, nothing at all being known to us of their nature.

But this becomes still more clear by another peculiarity, which should always be borne in mind. As long as we consider the number given by the table of mortality in its totality as a unit, the "power of life" offers many analogies with other natural forces. It is a changeable force, which can be made known and fixed by its changes, and its intensity is measured by the numbers of the persons alive. But the thing suddenly changes when we divide our field of observation in its single parts—the individuals. Of the 814 persons alive a year ago, 8 are dead and 806 alive; in each individual the "power of life," in the sense in which we have defined this expression, is the same as it has been a year before, and in the 8 persons dead it is entirely extinct. While we have observed a decrease of the power of life in the total number considered as a unit, two states opposed to each other appear in the individuals; they must be considered as each having always the same intensity, as there cannot be a question of a gradual passing from the one state to the other in the sense in which we mean life and death. Therefore, in the individual that expression of the force which we have defined as "power of life" cannot be observed at all; and on closer examination we perceive that always two elements enter in the measure of the "power of life," which, like all forces, can only be measured by its effects. One of these elements consists of the number alive at an earlier age, or, if we prefer it, of the number deceased since a certain time. It follows that the "power of life" can only be measured by a quotient, and is considered proportionate to the number alive. If we try to make this clear, we may assume that the power of life existing in all persons born at a certain period gives a sum which, divided by the number born, has as quotient α; this force is equally distributed over the whole number, leaving to each a quantity equal to α, and this quantity be at the same time necessary and sufficient to permit of that state which we call life. In the time following the whole quantity of the power of life diminishes, but the remaining part contracts itself at the same time, so that the intensity in the individual remains equal to α, and, all fractional values left out of consideration, the remaining quantity of the power of life can no longer be expanded over the whole number of persons, so that some of them contain no power of life at all—*i.e.*, they die—while those alive must be considered to be so always by the same amount of this force from birth to death. If we say that the mortality is greater at 60 than at 20, this must not be understood as affirming that the power of life has a greater intensity in an individual aged 20 than in one aged 60, and for the same reasons we cannot imply an increase of the power of life when we say that the mortality decreases from birth to the age of 10 years. On the contrary, these expressions have reference to quite another quantity, they refer to the proportion of two numbers of persons alive at different equidistant ages, or, what is the same, to the proportion of the power of life at equal intervals of age—*i.e*, the probability to live a certain time.

If we denote the probability that a person aged x will live the interval of time Δ, by $\frac{L_{(x+\Delta)}}{L_{(x)}}$, this is evidently equal to

$$\frac{L_{\left(x+\frac{\Delta}{n}\right)}}{L_{(x)}} \cdot \frac{L_{\left(x+\frac{2\Delta}{n}\right)}}{L_{\left(x+\frac{\Delta}{n}\right)}} \cdot \frac{L_{\left(x+\frac{3\Delta}{n}\right)}}{L_{\left(x+\frac{2\Delta}{n}\right)}} \cdots \frac{L_{(x+\Delta)}}{L_{\left(x+\frac{(n-1)\Delta}{n}\right)}};$$

therefore,

$$\log \frac{L_{(x+\Delta)}}{L_{(x)}} = \log\left[\frac{L_{\left(x+\frac{\Delta}{n}\right)}}{L_{(x)}}\right] + \log\left[\frac{L_{\left(x+\frac{2\Delta}{n}\right)}}{L_{\left(x+\frac{\Delta}{n}\right)}}\right] + \log\left[\frac{L_{\left(x+\frac{3\Delta}{n}\right)}}{L_{\left(x+\frac{2\Delta}{n}\right)}}\right] + \log \ldots$$

and, if we take n infinitely large,

$$\log \frac{L_{(x+\Delta)}}{L_{(x)}} = \int_x^{x+\Delta} \frac{d.L_{(x)}}{L_{(x)}}.$$

Now $\frac{d\,L_{(x)}}{L_{(x)}}$ is nothing else but the probability to die in the next moment taken inversely, $1 - \frac{L_{(x+dx)}}{L_{(x)}} = \frac{L_{(x)} - L_{(x+dx)}}{L_{(x)}} = \frac{-d\,L_{(x)}}{L_{(x)}}$, while the logarithm of the probability to live still in the next moment is $d.L_{(x)}$. These probabilities form, either consciously or unconsciously, the first step in all our considerations as to mortality—we refer to them when we speak of an increase or a decrease in the power of life—and in reality they contain all changes of this power, although they do not directly express them.

If we denote the probability to die in the next moment by the derived function $\phi'_{(x)}dx$, the number of persons dying out of $L_{(x)}$ in the next moment must be $L_{(x)}\phi'_{(x)}dx$; and as the change is a decrease, $\frac{d\,L_{(x)}}{L_{(x)}} = -\phi'_{(x)}dx$, $\log k.L_{(x)} = -\phi_{(x)}$, $k.L_{(x)} = e^{-\phi_{(x)}}$, k being the constant of integration. It is obvious that if it is possible to express the law of mortality by a complete function of the age x (with the only exception of the logarithmic function), the analytical form of $L_{(x)}$ will be $e^{-\phi(x)}$, while all other forms of $L_{(x)}$ would express that the mortality either must be a logarithmic function, or cannot be given in a complete form at all; and thus we are induced, in our examinations of the numbers alive, to use the form $e^{-\phi(x)}$, which considerations lead us to choose the logarithms instead of the numbers in our investigations as to the nature of $\phi_{(x)}$.

The difference of two of these logarithms, $\log L_{(x+\Delta)} - \log L_{(x)}$, is, as we have seen, the integral of $\phi'_{(x)}dx$ in the limits x and $x+\Delta$ taken inversely; and in this value, better in $\phi'_{(x)}$, we must look for the qualities so well characterized by Mr. Edmonds; but it ought to be borne in mind that we cannot expect to find anything but numerical results and numerical laws.

I must beg your indulgence for having trespassed so largely on your space,

And remain,
Sir,
Yours most obediently,

Hamburg, 15*th August*, 1861.* WILHELM LAZARUS.

* We are enabled at length to find room for this letter, referred to in our "Notice to Correspondents," October, 1861.—Ed. *A. M.*