Part III

$\overline{MS}$ scheme for QCD and QED

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Introduction

As in QED, the evaluation of QCD Feynman diagrams leads (in many cases) to divergent results. Finite physical answers need a regularization and a renormalization of the QCD parameters (vertices, coupling, masses...). However, the renormalization programme of QED [106] cannot be extended to QCD in a naïve way, as contrary to leptons which we can (freely) observe, quarks are off-shell, such that the standard Pauli–Villars regularization [107] and on-shell renormalization successful in QED cannot be used here. There exists different versions of off-shell renormalization schemes, which can be applied to non-Abelian gauge theories. Among them, we shall review the most elegant and powerful one, which is the: *Dimensional Regularization and Renormalization, the so-called* $\overline{MS}$ *scheme* originally proposed by 't Hooft and Veltman, Bollini and Giambiagi and by Ashmore [108,109,123].[1]

The most important feature of the method is the concept of analytic continuation of the dimension of space–time to complex n ($n = 4$ for low-energy space–time). This regularization procedure has the great advantage of preserving the local invariance of the underlying Lagrangian, and allows one to treat, in a gauge-invariant way, divergent Feynman integrals to all orders of perturbation theory.

In the ϵ-regularization procedure, the UV and IR divergencies are transformed into poles in ϵ, where the integrals are performed in $4 - \epsilon$ space–time dimensions). In general the UV poles are of the form:

$$\sum_{p=1} \frac{Z^{(p)}}{\epsilon^p} , \tag{7.1}$$

and will appear as counterterms in the initial Lagrangian. However, these counterterms are not arbitrary as they should obey constraints imposed by the Slavnov–Taylor identities [103,104]. In the case of renormalizable theory like QCD, the $Z^{(p)}$ must be constants or polynomial in the fermion (boson) mass after the introduction of the renormalized parameters.

Finally, the most relevant term entering in the renormalization group programme is the $Z^{(1)}/\epsilon$, while the other $Z^{(p)}$ for $p \geq 2$ are related to each other via the differential equation of the renormalization group equation.

[1] For reviews see e.g. [110–112] .

In the following, we shall discuss successively the dimensional regularization and the renormalization procedure. We shall also compare this $\overline{MS}$ scheme with some other schemes proposed in the literature and discuss the link between these different schemes.