

REVIEWS

Studies in Risk Theory with Numerical Illustrations concerning Distribution Functions and Stop Loss Premiums by H. BOHMAN & F. ESSCHER.

Remarks by Harald Bohman

In the collective theory of risk there are two functions which are of special interest. One is the distribution of the total amount of claims during a fixed period. The other is the Stop Loss Premium. These two functions are usually based on two assumptions. We start with the distribution function of the number of claims and with the distribution function of the amount of one claim, conditioned by the event that one claim has occurred. This latter distribution is usually called the claim distribution.

The two distributions are usually assumed to be statistically independent of each other. The numerical calculation of the two distributions, namely the distribution of the total amount of claims and the Stop Loss Premium, are as a rule very complicated. It was in 1932 that Frederik Esscher wrote the paper on the approximation method which has since then been connected with his name. This approximation method has been widely used in the collective theory of risk. It has been known from the results of special cases that the method could be expected to give very good approximate values of the functions in question, at least when the skewness of the claim distribution is not too large. People interested in collective risk theory have however always felt a certain uneasiness that nothing definite was known as to how precise these approximate values really are. From a Scandinavian point of view it was especially observed that this feeling of uneasiness was felt by Harald Cramér in 1955 when he wrote his monograph on collective risk theory in connection with the centenary of Skandia Insurance Company. Cramér compared the results of an Edgeworth expansion and an Esscher approximation in a special case and concluded that it would be of interest to know the strength of these two methods.

By the end of the 1950's a Swedish group interested in collective risk theory joined together to form a committee with the aim of studying this problem. The committee has had as its members Gunnar Benktander, Harald Bohman, Harald Cramér, Fredrik Esscher, Ulf Genander, Carl Philipson and Carl-Otto Segerdahl. The reason that the committee was so optimistic as to hope for a solution where a solution had not hitherto been found was the high capacity of available electronic computers.

The committee has now finished their work and they have charged Fredrik Esscher and me with the task of producing the report on their activities, their failures and their results. This report has now been printed. In fact it was printed just a week ago thanks to the efforts of the editor of the *Skandinavisk Aktuarietidskrift*, Mr. Segerdahl. We have brought with us here to London a number of copies of the report to make it available as soon as possible to those who are interested in collective risk theory.

Since it has not been possible to send the report on beforehand to the

members of this Congress it is, of course, impossible to discuss it here in any detail. Let me only give a very brief summary.

We start with five different claim distributions. One is a life insurance claim distribution. One is a non industrial fire claim distribution. These two claim distributions are based on actual experience among Swedish insurance companies during the last decade. The two distributions are both fairly dangerous. There are small probabilities that very large claims shall occur but these small probabilities cannot be neglected. The other three distributions are either modifications of the first one or purely fictitious distributions which we have chosen in order to make possible a comparison between different choices of claim distributions. The first problem was to calculate the exact values of the two functions that interested us, namely the distribution of the total amount of claims and the Stop Loss Premium. From a numerical point of view this is not so easy a problem even for a modern electronic calculator. We did find a method to perform the calculation. The results arrived at are, of course, not exact in the precise meaning of the word but they are fairly accurate. As a rule the tails of the distributions in question are estimated with an error not exceeding two or three per cent.

Our second problem was to compare the exact results with approximations of various types. We have three types of approximations. First, the well known Edgeworth expansion with two terms. Second, approximation by an incomplete gamma function with a suitable choice of the parameter. Third, the Esscher approximation, where we have both the original form and three modifications which are refinements of the method. The number of claims is supposed to have either a Poisson distribution or a negative binomial distribution. The results are presented in a number of tables. We hope that many conclusions might be drawn from the tables. From the observations we have made I will mention the following:

The approximation of the total amount of claims by an incomplete gamma function is surprisingly good. We have not been able to explain why this is so. Still, for one claim distribution, both the artificial ones and those based on experience, this approximation is surprisingly good.

The Esscher approximation is very good, especially for large values of the argument. When the expected number of claims is small, say less than 500, it might be difficult, even with the Esscher approximation to get good approximation.

As a general rule it can be said that the difficulties in getting accurate approximations increase when the skewness of the claim distribution increases. We have examples where the coefficient of skewness is about 35 and where the expected number of claims is equal to 100. Those cases are really difficult to handle. None of the approximation methods dealt with in our paper can be said to give satisfactory results in those cases.

CASUALTY ACTUARIAL SOCIETY

The 1962 Proceedings of the Casualty Actuarial Society (Volume 39) contains seven papers presented at the Society's meetings, reviews of these papers and of papers presented in 1961, a Presidential Address, an Invitational Address and reports on Seminars and a Panel Discussion.