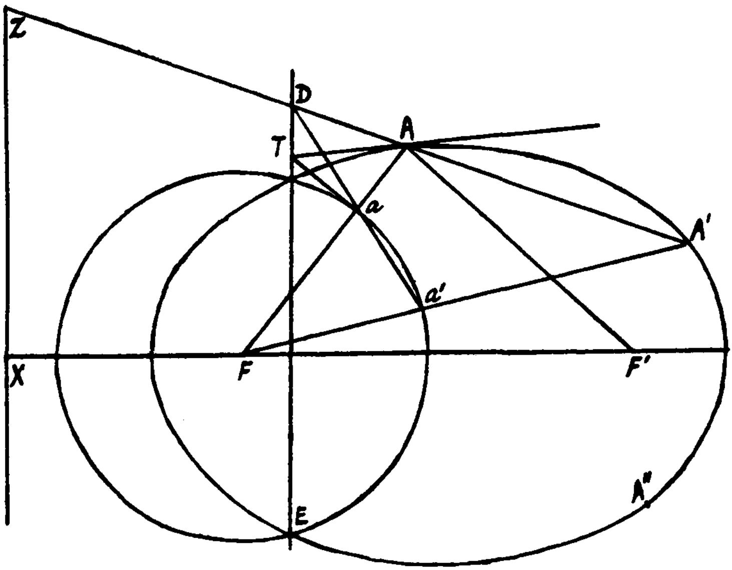
focus F' is got very easily since $A'F$ and $A'F'$ are equally inclined to $A'T$.



[See Housel's *Introduction à la Géométrie Supérieure*; Paris, Ganthiers-Villars, 1865, where a somewhat difficult proof depending on homology by de Jonquières is given.]

R. F. DAVIS.

Common Logarithms calculated by simple multiplication.

The 4-place tables inform us that log $3 = \cdot 4771$. This means that $3 = 10^{\cdot 4771}$ or $3^{10000} = 10^{4771}$; in other words, that there are $4771 + 1$ digits in 3^{10000}. Hence to find log 3 we have merely to raise 3 to the 10000th power and count the digits in the result. This need not be so long a process as one might anticipate, if we use contracted multiplication, and arrange the work suitably. The figures given below form the actual calculation, which took about 10 minutes to complete, and could be done in a shorter time.

A	B	C
3000	0	
3000		
9000	1	
9000		
8100	2	×
8100		
6480		
81		
6561	3	×
6561		
39366		
3280		
394		
7		
4305	4	×
4305		
1722		
129		
2		
1853	5	×
1853		
185		
148		
9		
342		

A	B	C
342	6	
342		
1026		
137		
7		
117	7	×
117		
117		
12		
8		
137	8	
137		
137		
41		
9		
187	9	
187		
187		
149		
13		
349	10	
35		
35		
105		
17		
122	11	×

A	B	C
122		
122		
122		
24		
2		
15	12	
15		
22	13	
48	14	
24	15	×

D
·25
·125
·0625
·03125
·00781
[·00195]
·00049
·00003
·4771

The process consists in squaring 3, squaring the result, and so on to the 15th step which gives $3^{2^{15}}$, *i.e.* 3^{32768} which is more than sufficient.

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Of course we use contracted multiplication, carrying the calculation to 4 digits at first, afterwards to 3 and 2 digits. Column A gives the actual multiplication. Column B gives the number of the step. Column C has a mark × at each step where an extra digit appears on the left. The theory of this arrangement of the work may be thus explained·

$$\log 3 = \frac{1}{2} \log 9 = \frac{1}{2^2} \log 81 = \frac{1}{2^2}(1 + \log 8{\cdot}1)$$

$$= \frac{1}{2^2} + \frac{1}{2^3} \log 65{\cdot}61 = \frac{1}{2^2} + \frac{1}{2^3}(1 + \log 6{\cdot}561)$$

$$= \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^3} \log 6{\cdot}561 \text{ and so on,}$$

so that $\log 3 = {\cdot}25 + {\cdot}125 + {\cdot}125 \log 6{\cdot}561$, etc., so that we have simply to add certain powers of ·5, viz., the 2nd, 3rd, 4th, 5th, 7th, 11th and 15th to get the result. This is done in the sum marked D. Note that in calculating the 11th power of ·5 from the 7th we have to divide by 2^4, *i.e.* by 4×4 and it is convenient to do it in two steps, crossing out the intermediate quotient ·00195, which is to be left out in adding. If several logarithms had to be calculated it would be convenient to have the powers of $\frac{1}{2}$ expressed decimally in a column and simply cross out in pencil those not to be added.

Is it too much to ask that every class that uses logarithms should calculate a 4-place table for itself? Of course it would be unnecessary to calculate the 1000 logarithms separately. Half a dozen or fewer would suffice, then a sufficient number of others could be got as products, or fractions whose numerator and denominator are products of the numbers for which the logarithms have been calculated, to enable the rest to be filled in by "proportional parts."

To ensure the correctness of 4 digits, it would be advisable to begin by working to 5 places. Since $2^{10} = 1024$ it is clear that we may reduce the number of places in our calculation by *one* for every three or four steps as has been done in the above example.

A more formal proof of the rule might be stated as follows:

Let N be a number lying between 1 and 10, so that the characteristic of $\log N$ is 0 and its mantissa m. Let m_1 be the mantissa of $2 \log N$, m_2 that of $2^2 \log N$, m_r that of $2^r \log N$.

Then $m=\frac{1}{2}(c_1+m_1)$, $m_1=\frac{1}{2}(c_2+m_2)$, $m_2=\frac{1}{2}(c_3+m_3)$ and so on, where c_r is either 1 or 0 according as the square of antilog m_{r-1} is greater or less than 10.

Thus $m=\frac{c_1}{2}+\frac{c_2}{2^2}+\frac{c_3}{2^3}+\ldots+\frac{c_n}{2^n}+\frac{m_n}{2^n}$.

R. F. MUIRHEAD.

Trigonometrical Ratios of (A ± B).

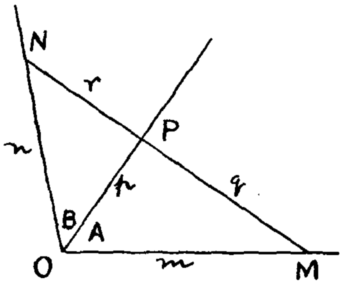

Fig. 1

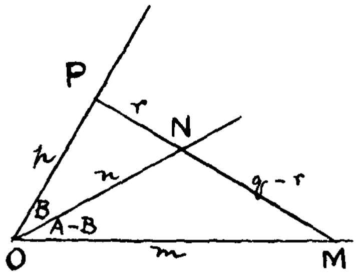

Fig. 2.

The formulae for sin $(A \pm B)$ and cos $(A \pm B)$ may be derived by the following method, the main attraction of which is the simplicity of the figures employed.

Consider the case in which A and B are both acute angles.

Let $\angle MOP=A$ and $\angle PON=B$, then in Fig. 1 $\angle MON=A+B$ and in Fig. 2 $\angle MON=A-B$. Through any point P on OP, the arm common to both angles, draw a perpendicular to OP meeting the other arms in M and N respectively.

Let $OM=m$, $ON=n$, $OP=p$, $MP=q$, $PN=r$.

From the triangle OMN we obtain

$$\sin(A \pm B)=\sin MON=\frac{2\,\triangle OMN}{m\,n}=\frac{p\,(q \pm r)}{m\,n}$$

$$=\frac{q}{m}\cdot\frac{p}{n}\pm\frac{p}{m}\cdot\frac{r}{n}$$

$$=\sin A\cos B \pm \cos A \sin B.$$