# The electric currents from viscosity in differentially moved plasma 

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#### Abstract

From the two-fluid plasma theory, we derived a kind of current in differentially moved plasma. The current is from the different viscosities between electrons and ions. The higher temperature and lighter mass of the electrons make the viscosity of electrons much stronger than ions. In this way, the electrons will have smaller velocity than ions in the differentially moved layer and contribute a net current in the plasma. The value of the current depends on the temperature and density of electrons in the plasma and the differential velocity $\nabla^{2} v$.


Keywords. Sun, Plasma, Electric Current.

## 1. Two-fluid Models of plasma

The two-fluid plasma equations with viscosity included are given by :

$$
\begin{gather*}
\frac{\partial n_{\alpha}}{\partial t}+\nabla \cdot n_{\alpha} \mathbf{v}_{\alpha}=0  \tag{1.1}\\
\frac{d \mathbf{v}_{\alpha}}{d t}=\frac{q_{\alpha}}{m_{\alpha}}\left(\mathbf{E}+\frac{1}{c} \mathbf{v}_{\alpha} \times \mathbf{B}\right)-\frac{\nabla P_{\alpha}}{n_{\alpha} m_{\alpha}}+\frac{\mathbf{M}_{\alpha \beta}}{n_{\alpha} m_{\alpha}}-\frac{\mu_{\alpha} \nabla^{2} \mathbf{v}_{\alpha}}{n_{\alpha} m_{\alpha}} \tag{1.2}
\end{gather*}
$$

where $\mathbf{v}_{\alpha}, \mathbf{v}_{\beta}$ refers to electrons and/or protons $(e, i), \mu_{\alpha} \nabla^{2} \mathbf{v}_{\alpha}$ is the viscous force acted on protons or electrons, $\mathbf{M}_{\alpha \beta}$ is the collision between electrons and ions, and the electron charge is $q_{e}=-e$ in the proton and electron plasma.

In the steady case, $\frac{\partial B}{\partial t}=0$, that is the same as $E=0$ in plasma, with the Hall current and Bierman's battery ignored, we can have:

$$
\begin{equation*}
\mathbf{j}=\frac{c^{2}}{4 \pi \eta}\left(\frac{1}{c} \mathbf{v}_{i} \times \mathbf{B}-\frac{m_{e}}{n e m_{i}} \mu_{i} \nabla^{2} \mathbf{v}_{i}+\frac{1}{n e} \mu_{e} \nabla^{2} \mathbf{v}_{e}\right) \tag{1.3}
\end{equation*}
$$

The term $\frac{c m_{e}}{n e m_{i}} \mu_{i} \nabla^{2} \mathbf{v}_{i}-\frac{c}{n e} \mu_{e} \nabla^{2} \mathbf{v}_{e}$ is the current from the viscosity effect. In the differential moved plasma, the ions and electrons are supposed to have the same shear velocity gradients. This may cause a net current in the shearing layer and generate magnetic field.

The viscosity for electron and proton in plasma have the form (Braginskii 1957, Spitzer, 1962):

$$
\begin{align*}
& \mu_{i}=2.21 \times 10^{-15} \frac{T_{i}^{5 / 2} A_{i}^{1 / 2}}{Z^{4} \ln \Lambda}\left(\frac{g m}{c m s e c}\right)  \tag{1.4}\\
& \mu_{e}=2.21 \times 10^{-15} \frac{T_{e}^{5 / 2} A_{e}^{1 / 2}}{Z^{4} \ln \Lambda}\left(\frac{g m}{c m s e c}\right) \tag{1.5}
\end{align*}
$$

where $A_{i}, A_{e}$ is the atomic weight of electrons and positive ions, $A_{i}=1$ and $A_{e}=1 / 1836$, $Z$ is the ionic charge, $T e$ and $T_{i}$ are the temperature of electron and ion in K.

The currents which are perpendicular to and parallel to the plasma velocity are,

$$
\begin{equation*}
\mathbf{j}_{\perp}=\frac{\mathbf{c}}{4 \pi \eta} \mathbf{v} \times \mathbf{B} \tag{1.6}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathbf{j}_{\|}=\frac{c^{2} m_{e}}{4 \pi e} P_{m e} \nabla^{2} \mathbf{v} \tag{1.7}
\end{equation*}
$$

The parameter $P_{m e}$ is the magnetic Prandtl number of electrons,

$$
\begin{equation*}
P_{m e}=\frac{\nu_{e}}{\eta}=5.66 \times 10^{-3} T_{e}^{5 / 2} T^{3 / 2} /(n \ln \Lambda) \tag{1.8}
\end{equation*}
$$

where $T_{e}$ is the temperature of electrons in K and n is the electron/ion density in $\mathrm{cm}^{-3}$. $\ln \Lambda$ is usually between $5--20$.

## 2. The Current in Coronal Loops

The current parallel to the velocity of plasma depends on the sheared velocity $\nabla^{2} \mathbf{v}$, and the magnetic Prandtl number of electrons.

$$
\begin{equation*}
\mathbf{j}_{\|}=0.136 P_{m e} \nabla^{2} \mathbf{v}\left(A / m^{2}\right) \tag{2.1}
\end{equation*}
$$

with the velocity in unit of $\mathrm{m} / \mathrm{s}$. And the magnetic Prandtl number of electrons depends strongly on the temperature and density of electrons.

There is a lot of theoretical models and observations for the density and temperature of electrons in the coronal loops (Robb \& Cally 1992; Spadaro et al. 2003). The electron temperature in the loops is about $10^{6} \mathrm{~K}$ and the electron density is about $10^{10} \mathrm{~cm}^{-3}$. Supposing the plasma temperature in the loops is $10^{6} \mathrm{~K}$, we can get the magnetic Prandtl number in the coronal loops is $P_{m e}=10^{11}$.

In a recent observation, $\operatorname{Lin}(\operatorname{Lin}$ et al, 2006) shows that the velocity difference in a field of $13^{\prime \prime}$ is from $230 \mathrm{~km} / \mathrm{s}$ to $320 \mathrm{~km} / \mathrm{s}$. We can estimate $\nabla^{2} \mathbf{v}$ as $\delta v / L^{2} . \delta v$ is the velocity fluctuation in a field with scale $L$. It is about $2.25 \times 10^{-13} \mathrm{~m}^{-1} \mathrm{~s}^{-1}$. The total current in this area will be $2.7 \times 10^{2} P_{m e} A$. If we choose $P_{m e}$ to be $10^{10}$, the current in a coronal loop will be $10^{12} \mathrm{~A}$.

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