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A CONSTRUCTION OF THE PAIR COMPLETION OF A QUASI-UNIFORM SPACE

BY

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1. Introduction. Although L. Nachbin introduced the concept of a quasi-uniform space in connection with the study of uniform ordered spaces [7], it was A. Császár who first developed a theory of completion for these spaces. In the review of Császár's work [3], J. Isbell wrote: "A topogenic space is already complete according to the present definition; an unfortunate side effect is that not every convergent filter is Cauchy." This aspect of Császár's theory of completeness seems to have stalled the investigation of its applications, so that even in the study of uniform ordered spaces Császár's important work has often been overlooked. While it may have seemed that Isbell had pointed out the Achilles' heel of Császár's theory, in this paper we outline a construction of Császár's completion for quasi-uniform spaces, which differs from Császár's original construction and the construction given by S. Salbany [8], in which Isbell's objection does not obtain. Our procedure is not so general as Császár's, but it is patterned upon a well-known construction of a uniform completion; consequently by sacrificing a degree of generality that is not required in the study of uniform ordered spaces, we have made Császár's completion for quasi-uniform spaces readily accessible to anyone familiar with the completion of a uniform space. In light of Theorem 16, the pair completion defines a reflection on the category of T_0 quasi-uniform spaces.

Besides allowing the retrieval of many proofs from the classical theory of uniform spaces, a principal attribute of our construction is that it reveals properties of the completion that have hitherto gone unnoticed. In particular, in [5] the construction given here is used implicitly in the study of generalized ordered spaces. Since one of the advantages of our construction is that it makes possible straightforward adaptation of existing arguments from the elementary theory of uniform spaces, we are content in Section 3 of this paper to outline our construction by stating, often without proof, the necessary propositions in their appropriate order.

In Section 4, we consider briefly the relationship between pair completeness and the concept of completeness introduced by J. L. Sieber and W. J. Pervin [9]. The pair completion constructed previously is a main source of counter-examples, which serve to distinguish these two kinds of completeness.

Throughout this paper, if $\mathcal{U}$ is a quasi-uniformity, then $\mathcal{U}^*$ denotes the smallest uniformity that contains $\mathcal{U}$ and $\mathcal{T}(\mathcal{U})$ denotes the topology generated

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by $\mathscr{U}$. Terminology and notation concerning quasi-uniform spaces that is not defined herein may be found in [6].

2. **Definitions and elementary results.** If $\mathscr{F}$, $\mathscr{G}$ and $\mathscr{F} \vee \mathscr{G}$ are filter bases on a set X, then $(\mathscr{F}, \mathscr{G})$ is called a *pair filter base*. The insistence that $\mathscr{F} \vee \mathscr{G}$ be a filter base is the analogue for pair filter bases of the requirement that no filter contain $\varnothing$.

Let $(X, \mathscr{U})$ be a quasi-uniform space and let $(\mathscr{F}, \mathscr{G})$ be a pair filter on X. Then a point p of X is a *cluster point* of $(\mathscr{F}, \mathscr{G})$ provided that $p \in \operatorname{adh}_{\tau(\mathscr{U})}(\mathscr{F}) \cap \operatorname{adh}_{\tau(\mathscr{U}^{-1})}(\mathscr{G})$. If $x \in X$, we let $L(x)$ denote the filter generated by $\{U^{-1}(x) : U \in \mathscr{U}\}$, $R(x)$ denote the filter generated by $\{U(x) : U \in \mathscr{U}\}$ and $\mathscr{N}_x = \operatorname{fil}(L(x) \vee R(x))$. For each $x \in X$, $(L(x), R(x))$ is a pair filter base. A pair filter base $(\mathscr{F}_1, \mathscr{G}_1)$ is *coarser* than $(\mathscr{F}, \mathscr{G})$ provided that fil $\mathscr{F}_1 \subset$ fil $\mathscr{F}$ and fil $\mathscr{G}_1 \subset$ fil $\mathscr{G}$. Two pair filter bases $(\mathscr{F}_1, \mathscr{G}_1)$ and $(\mathscr{F}, \mathscr{G})$ are *equivalent* provided that $(\mathscr{F}_1, \mathscr{G}_1)$ is coarser than $(\mathscr{F}, \mathscr{G})$ and $(\mathscr{F}, \mathscr{G})$ is coarser than $(\mathscr{F}_1, \mathscr{G}_1)$. A filter $(\mathscr{F}, \mathscr{G})$ *converges to* p provided that $(L(p), R(p))$ is coarser than $(\mathscr{F}, \mathscr{G})$. A pair filter base $(\mathscr{F}, \mathscr{G})$ is $\mathscr{U}$-Cauchy provided that for each $U \in \mathscr{U}$ there is an $F \in \mathscr{F}$ and a $G \in \mathscr{G}$ such that $F \times G \subset U$. A Cauchy pair filter base $(\mathscr{F}, \mathscr{G})$ is a *minimal Cauchy pair filter base* provided that if $(\mathscr{F}_1, \mathscr{G}_1)$ is a Cauchy pair filter base coarser than $(\mathscr{F}, \mathscr{G})$, then $(\mathscr{F}_1, \mathscr{G}_1)$ and $(\mathscr{F}, \mathscr{G})$ are equivalent pair filter bases. In the case that $(X, \mathscr{U})$ is a uniform space and $\mathscr{F}$ is a filter on X, then clearly $\mathscr{U} = \mathscr{U}^*$ and $\mathscr{F}$ is a (Cauchy, respectively convergent) filter on X if, and only if, $(\mathscr{F}, \mathscr{F})$ is a (Cauchy, respectively convergent) pair filter base.

Proposition 1. *Every convergent pair filter base is a Cauchy pair filter base.*

Proposition 2. *Let $(X, \mathscr{U})$ and $(Y, \mathscr{V})$ be quasi-uniform spaces and let $f : X \to Y$ be a quasi-uniformly continuous function. If $(\mathscr{F}, \mathscr{G})$ is a $\mathscr{U}$-Cauchy pair filter base, then $(f(\mathscr{F}), f(\mathscr{G}))$ is a $\mathscr{V}$-Cauchy pair filter base.*

Proposition 3. *Let X be a set, let $\{Y_i, \mathscr{V}_i\}_{i \in I}$ be a family of quasi-uniform spaces and let $\{f_i\}_{i \in I}$ be a family of functions such that for each $i \in I$, $f_i : X \to Y_i$. Let $\mathscr{U}$ be the coarsest quasi-uniformity on X with the property that f_i is quasi-uniformly continuous for each $i \in I$. Then a pair filter base $(\mathscr{F}, \mathscr{G})$ is $\mathscr{U}$-Cauchy if, and only if, $(f_i(\mathscr{F}), f_i(\mathscr{G}))$ is $\mathscr{V}_i$-Cauchy for each $i \in I$.*

Corollary 4. *Let $(\mathscr{F}, \mathscr{G})$ be a $\mathscr{U}$-Cauchy pair filter base on a quasi-uniform space $(X, \mathscr{U})$ and let A be a subset of X. Then $(\mathscr{F}, \mathscr{G})$ induces a $U \mid A \times A$-Cauchy pair filter base on A.*

Corollary 5. *Let $\{X_i, \mathscr{U}_i\}_{i \in I}$ be a family of quasi-uniform spaces. A pair filter base $(\mathscr{F}, \mathscr{G})$ on $\prod_{i \in I} (X_i, U_i)$ is a Cauchy pair filter base if, and only if, for each $i \in I$, $(\pi_i(\mathscr{F}), \pi_i(\mathscr{G}))$ is a $\mathscr{U}_i$-Cauchy pair filter base.*

Proposition 6. *Let $(X, \mathscr{U})$ be a quasi-uniform space and let $(\mathscr{F}, \mathscr{G})$ be a Cauchy pair filter base on X. There is exactly one minimal Cauchy pair filter $(\mathscr{F}_0, \mathscr{G}_0)$*

that is coarser than $(\mathscr{F}, \mathscr{G})$. *A base for* $\mathscr{F}_0$ *is* $\{U^{-1}(F): U \in \mathscr{U}$ *and* $F \in \mathscr{F}\}$ *and a base for* $\mathscr{G}_0$ *is* $\{U(G): U \in \mathscr{U}$ *and* $G \in \mathscr{G}\}$.

Proof. Let $\mathscr{F}_0$ be the filter generated by $\{U^{-1}(F): U \in \mathscr{U}, F \in \mathscr{F}\}$ and let $\mathscr{G}_0$ be the filter generated by $\{U(G): U \in \mathscr{U}$ and $G \in \mathscr{G}\}$. Then $(\mathscr{F}_0, \mathscr{G}_0)$ is a pair filter base. Let $U \in \mathscr{U}$. There is an entourage V such that $V^3 \subset U$. Since $(\mathscr{F}, \mathscr{G})$ is a Cauchy pair filter base there is an $F \in \mathscr{F}$ and a $G \in \mathscr{G}$ such that $F \times G \subset V$. Let $(x, y) \in V^{-1}(F) \times V(G)$. There is an $f \in F$ and $g \in G$ such that $x \in V^{-1}(f)$ and $y \in V(g)$. Then $y \in V(g) \subset V^2(f) \subset V^3(x)$ and $V^{-1}(F) \times V(G) \subset V^3 \subset U$. Therefore $(\mathscr{F}_0, \mathscr{G}_0)$ is a Cauchy pair filter. Suppose that $(\mathscr{F}_1, \mathscr{G}_1)$ is a Cauchy pair filter that is coarser than $(\mathscr{F}_0, \mathscr{G}_0)$. Let $F_0 \in \mathscr{F}_0$. There exists an $F \in \mathscr{F}$ and an entourage $V \in \mathscr{U}$ such that $V^{-1}(F) \subset F_0$, and there exists an $F_1 \in \mathscr{F}_1$ and $G_1 \in \mathscr{G}_1$ such that $F_1 \times G_1 \subset V$. Since $(\mathscr{F}, \mathscr{G})$ is a pair filter there exists an $x \in F \cap G_1$. Then $F_1 \subset V^{-1}(x) \subset V^{-1}(F) \subset F_0$. It follows that $\mathscr{F}_0 \subset \mathscr{F}_1$. A similar argument establishes that $\mathscr{G}_0 \subset \mathscr{G}_1$.

COROLLARY 7. *Let* $(X, \mathscr{U})$ *be a quasi-uniform space and let* $x \in X$. *Then* $(L(x), R(x))$ *is a minimal Cauchy pair filter base.*

COROLLARY 8. *Let* $(X, \mathscr{U})$ *be a quasi-uniform space and let* $(\mathscr{F}, \mathscr{G})$ *be a Cauchy pair filter base on* X. *If* x *is a cluster point of* $(\mathscr{F}, \mathscr{G})$, *then* $(\mathscr{F}, \mathscr{G})$ *converges to* x.

COROLLARY 9. *If* $(\mathscr{F}, \mathscr{G})$ *is a minimal Cauchy pair filter base on a quasi-uniform space* $(X, \mathscr{U})$, *then* $\mathscr{F}$ *has a basis that is open in* $\mathscr{T}(\mathscr{U}^{-1})$ *and* $\mathscr{G}$ *has a basis that is open in* $\mathscr{T}(\mathscr{U})$.

Let $(X, \mathscr{U})$ be a quasi-uniform space, let $(\mathscr{F}, \mathscr{G})$ be a Cauchy pair filter base and let $(\mathscr{F}_0, \mathscr{G}_0)$ be the minimal Cauchy pair filter that is coarser than $(\mathscr{F}, \mathscr{G})$. Then $(\mathscr{F} \vee \mathscr{G}, \mathscr{F} \vee \mathscr{G})$ is also a Cauchy pair filter and $\mathscr{F} \vee \mathscr{G}$ is a $\mathscr{U}^*$-Cauchy filter. Since $(\mathscr{F}_0, \mathscr{G}_0)$ is coarser than $(\mathscr{F} \vee \mathscr{G}, \mathscr{F} \vee \mathscr{G})$, $(\mathscr{F}_0, \mathscr{G}_0)$ is the minimal Cauchy pair filter that is contained in $(\mathscr{F} \vee \mathscr{G}, \mathscr{F} \vee \mathscr{G})$. By Proposition 6, $\mathscr{F}_0$ is equivalent to $\{U^{-1}(F) : U \in \mathscr{U}$ and $F \in \mathscr{F} \vee \mathscr{G}\}$ and $\mathscr{G}_0$ is equivalent to $\{U(F) : U \in \mathscr{U}$ and $\mathscr{F} \vee \mathscr{G}\}$ so that the following useful result obtains.

COROLLARY 10. *For each minimal Cauchy pair filter* $(\mathscr{F}_0, \mathscr{G}_0)$ *in a quasi-uniform space* $(X, \mathscr{U})$ *there is a* $\mathscr{U}^*$*-Cauchy filter* $\mathscr{F}$ *so that* $\mathscr{F}_0$ *is generated by* $\{U^{-1}(F): U \in \mathscr{U}$ *and* $F \in \mathscr{F}\}$ *and* $\mathscr{G}_0$ *is generated by* $\{U(F): U \in \mathscr{U}$ *and* $F \in \mathscr{F}\}$. *Conversely every* $\mathscr{U}^*$*-Cauchy filter generates a minimal Cauchy pair filter.*

3. **Pair completeness and pair completions.** Throughout the remainder of the paper we restrict our attention to quasi-uniform spaces $(X, \mathscr{U})$ with the property that $\mathscr{T}(\mathscr{U})$ is a T_0-topology (equivalently $\mathscr{T}(\mathscr{U}^*)$ is a Hausdorff topology). This restriction simplifies some of the proofs and makes it possible to strengthen the conclusions of several results.

A quasi-uniform space $(X, \mathcal{U})$ is *pair complete* provided that each Cauchy pair filter base converges.

PROPOSITION 11. *A quasi-uniform space $(X, \mathcal{U})$ is pair complete if, and only if, $(X, \mathcal{U}^*)$ is a complete uniform space.*

Proof. Suppose that $(X, \mathcal{U})$ is a pair complete quasi-uniform space and let $\mathcal{F}$ be a $\mathcal{U}^*$-Cauchy filter. Then $(\mathcal{F}, \mathcal{F})$ is a Cauchy pair filter. Therefore $\mathcal{F} = \mathcal{F} \vee \mathcal{F}$ is a convergent filter in $\mathcal{T}(\mathcal{U}^*)$. Now suppose that $(X, \mathcal{U}^*)$ is a complete uniform space. Let $(\mathcal{F}, \mathcal{G})$ be a Cauchy pair filter base and let $(\mathcal{F}_0, \mathcal{G}_0)$ be the minimal Cauchy pair that is coarser than $(\mathcal{F}, \mathcal{G})$. By Corollary 10, there is a $\mathcal{U}^*$-Cauchy filter $\mathcal{H}$ that generates $(\mathcal{F}_0, \mathcal{G}_0)$. Since $\mathcal{H}$ converges in $\mathcal{T}(\mathcal{U}^*)$, it is evident that $(\mathcal{F}_0, \mathcal{G}_0)$ is a convergent pair filter.

In light of [8, Proposition 3.2], the proposition stated above establishes that pair completeness is the same concept of completeness that is studied in [8], and consequently for quasi-uniform spaces coincides with Csaszar completeness [2]. In Theorem 15, we indicate an alternate construction of Csaszar's pair completion for a quasi-uniform space $(X, \mathcal{U})$; the construction itself, as well as the completion obtained, plays an important role in [5]. In the case that $\mathcal{U}$ is a uniformity our construction coincides with the well-known construction of a uniform completion by means of minimal Cauchy filters.

A quasi-uniform space $(X, \mathcal{U})$ is *pair compact* if, and only if, $\mathcal{T}(\mathcal{U}^*)$ is a compact topology for X [8]. The following results of [2 and 8] are evident from Propostion 11.

PROPOSITION 12. *A quasi-uniform space is pair complete and totally bounded if, and only if, it is pair compact.*

PROPOSITION 13. *A subspace Y of a pair complete quasi-uniform space $(X, \mathcal{U})$ is pair complete, if, and only if, Y is a closed set in $\mathcal{T}(\mathcal{U}^*)$.*

PROPOSITION 14. *The product of any collection of quasi-uniform spaces is pair complete if, and only if, each factor space is pair complete.*

A *pair completion* of a quasi-uniform space $(X,\mathcal{U})$ is a pair complete quasi-uniform space $(Y, \mathcal{V})$ that has a $\mathcal{T}(\mathcal{V})$-dense subspace that is quasi-unimorphic to $(X, \mathcal{U})$.

THEOREM 15 [2 and 8]. *Every quasi-uniform space has a pair completion.*

Proof. Let $(X, \mathcal{U})$ be a quasi-uniform space and let $\tilde{X} = \{\mathcal{F} : \mathcal{F}$ is a minimal $\mathcal{U}^*$-Cauchy filter on $X\}$. For each $U \in \mathcal{U}$, let $\tilde{U} = \{(\mathcal{F}, \mathcal{G})$: there exists $F \in \mathcal{F}$ and $G \in \mathcal{G}$ such that $F \times G \subset U\}$. It is easily seen that $\{\tilde{U} : U \in \mathcal{U}\}$ is a base for a quasi-uniformity $\tilde{\mathcal{U}}$ on $\tilde{X}$. Let $U \in \mathcal{U}$ and let $V \in \mathcal{U}$ such that $V^3 \subset U$. Then $(x, y) \in U$ whenever $(\eta_x, \eta_y) \in \tilde{U}$ and $(\eta_x, \eta_y) \in \tilde{U}$ whenever $(x, y) \in V$. Thus the map $i : (x, u) \to (\tilde{x}, \tilde{u})$ defined by $i(x) = \eta_x$ is a quasi-uniform embedding.

In order to prove that $(\tilde{\mathscr{U}})^*$ is complete, we note that $(\tilde{\mathscr{U}})^*$ is in fact the Hausdorff completion of the uniformity $\mathscr{U}^*$.

Let $U \in \mathscr{U}$. Then $\tilde{U} \cap (\tilde{U})^{-1} = (U \cap U^{-1})^{\sim}$, thus $(\tilde{\mathscr{U}})^* = (\mathscr{U}^*)^{\sim}$. From this equality and the well-known construction of the Hausdorff completion by means of minimal Cauchy filters [1, Pages 134–140], it follows readily that $(\tilde{\mathscr{U}})^*$ is the Hausdorff completion of $\mathscr{U}^*$.

Since it has been established in the proof outlined above that $(\tilde{\mathscr{U}})^*$ and $(\mathscr{U}^*)^{\sim}$ denote the same uniformity, we will use the unambiguous symbol $\tilde{\mathscr{U}}^*$ hereafter.

Note that a function $h:(X, \mathscr{U}) \to (Y, \mathscr{V})$ is $\mathscr{U}-\mathscr{V}$ quasi-uniformly continuous if, and only if, h is $\mathscr{U}^{-1}-\mathscr{V}^{-1}$ quasi-uniformly continuous. Thus h's being $\mathscr{U}-\mathscr{V}$ quasi-uniformly continuous also implies that h is $\mathscr{U}^*-\mathscr{V}^*$ uniformly continuous.

Theorem 16. *Let $(X, \mathscr{U})$ be a quasi-uniform space, $(Y, \mathscr{V})$ be a pair complete quasi-uniform space and let $f: X \to Y$ be a quasi-uniformly continuous function. Embed X as a subset of $\tilde{X}$ by identifying x with η_x for each $x \in X$. Then there exists a unique $\mathscr{T}(\tilde{\mathscr{U}}^*)-\mathscr{T}(\mathscr{V}^*)$ continuous extension $g: X \to Y$ and this extension is $\tilde{\mathscr{U}}-\mathscr{V}$ quasi-uniformly continuous.*

Corollary 17. *Any two pair completions of a quasi-uniform space are quasi-unimorphic.*

Proposition 18 [8, Theorem 4.5] *Let $(X, \mathscr{U})$ be a pair compact quasi-uniform space. If $\mathscr{V}$ is a quasi-uniformity on X such that $\mathscr{T}(\mathscr{U}) = \mathscr{T}(\mathscr{V})$ and $\mathscr{T}(\mathscr{U}^{-1}) = \mathscr{T}(\mathscr{V}^{-1})$, then $\mathscr{U} = \mathscr{V}$.*

4. **Completeness and pair completeness.** In reaction to J. Isbell's review of [2], J. L. Sieber and W. J. Pervin introduced a concept of completeness that differs from the concept of pair completeness with which we have been concerned. Although, as we have seen, Isbell's objection can be resolved by a slight modification in terminology, the definition of completeness given by Sieber and Pervin has gained wide acceptance. We pause briefly, therefore, to consider some relationships that exist between these two concepts of completeness.

Let $(X, \mathscr{U})$ be a quasi-uniform space. A filter $\mathscr{F}$ on X is a *$\mathscr{U}$-Cauchy filter* [9] provided that for each $U \in \mathscr{U}$ there is a $x \in X$ such that $U(x) \in \mathscr{F}$, and $\mathscr{U}$ is *complete* provided that every $\mathscr{U}$-Cauchy filter converges.

Proposition 19. *Let $(X, \mathscr{U})$ be a T_1 quasi-uniform space such that both $\mathscr{U}$ and $\mathscr{U}^{-1}$ are complete. Then $(X, \mathscr{U})$ is a pair complete quasi-uniform space.*

Proof. Let $\mathscr{F}$ be a $\mathscr{U}^*$-Cauchy filter. Then $\mathscr{F}$ is both a $\mathscr{U}$-Cauchy filter and a $\mathscr{U}^{-1}$-Cauchy filter so that there is a $p \in X$ and a $q \in X$ such that $\mathscr{F}$ converges to

p in $\mathcal{T}(\mathcal{U})$ and $\mathcal{F}$ converges to q in $\mathcal{T}(\mathcal{U}^{-1})$. Since $\mathcal{T}(\mathcal{U})$ is a T_1 topology, $p=q$ and $\mathcal{F}$ converges to p in $\mathcal{T}(\mathcal{U}^*)$.

The following example shows that in the previous proposition it is necessary that both $\mathcal{U}$ and $\mathcal{U}^{-1}$ be complete.

Example. Let X be the closed unit interval with its usual Euclidean topology and let $\mathcal{P}$ be the Pervin quasi-uniformity on X. Then $\mathcal{P}$ is complete, but, as $\mathcal{P}^*$ is totally bounded and $\mathcal{T}(\mathcal{P}^*)$ is discrete, it follows from Proposition 12 that $(X, \mathcal{P})$ is not pair complete.

Example. A pair complete quasi-uniform space $(X, \mathcal{U})$ such that neither $\mathcal{U}$ nor $\mathcal{U}^{-1}$ is complete. Let X be the set of non-zero real numbers. For each $t \in (0, 1)$ let $U_t = \Delta_X \cup \{x, y) \mid t > y \geq x \geq -t\}$ and let $\mathcal{U}$ be the quasi-uniformity for which $\{U_t \mid 0 < t < 1\}$ is a base. It is easily seen that neither $\mathcal{U}$ nor $\mathcal{U}^{-1}$ is complete, whereas $\mathcal{U}^*$ is the discrete uniformity and is complete.

It is possible to use the pair completion of a totally bounded quasi-uniform space $(X, \mathcal{U})$ to establish the existence of a completion of $(X, \mathcal{U})$ in the sense of Sieber and Pervin. For if $(X, \mathcal{U})$ is totally bounded, then $(X, \tilde{\mathcal{U}}^*)$ is compact and since $\mathcal{T}(\tilde{\mathcal{U}}^*)$ is finer than $\mathcal{T}(\tilde{\mathcal{U}})$, $\mathcal{T}(\tilde{\mathcal{U}})$ is compact. It follows that $\tilde{\mathcal{U}}$ is complete in the sense of Sieber and Pervin [9], and it is clear that $(X, \mathcal{U})$ is quasi-unimorphic to a dense subset of $(\tilde{X}, \tilde{\mathcal{U}})$. Unfortunately, our last result shows that the completion $(\tilde{X}, \tilde{\mathcal{U}})$, obtained in this manner, is not a T_1 completion unless $\tilde{\mathcal{U}}$ is already a uniformity.

Proposition 20. *Let $(X, \mathcal{U})$ be a T_1 quasi-uniform space such that $\mathcal{T}(\mathcal{U}^*)$ is compact. Then $\mathcal{U}$ is a uniformity.*

Proof. Since $\mathcal{T}(\mathcal{U})$ is T_1 and both $\mathcal{T}(\mathcal{U})$ and $\mathcal{T}(\mathcal{U}^{-1})$ are compact, $\mathcal{T}(\mathcal{U}) = \mathcal{T}(\mathcal{U}^{-1}) = \mathcal{T}(\mathcal{U}^*)$ [4, Corollary to Proposition 3.4]. By Proposition 18, $\mathcal{U} = \mathcal{U}^*$.

Corollary 21. *Let $(X, \mathcal{U})$ be a totally bounded quasi-uniform space. Then $(\tilde{X}, \mathcal{T}(\tilde{\mathcal{U}}))$ is T_1 if, and only if, $\mathcal{U}$ is a uniformity.*

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