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Prediction of the COVID-19 transmission: a case study of Pakistan

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Abstract

The world has suffered a lot from COVID-19 and is still on the verge of a new outbreak. The infected regions of coronavirus have been classified into four categories: SIRD model, (1) suspected, (2) infected, (3) recovered, and (4) deaths, where the COVID-19 transmission is evaluated using a stochastic model. A study in Pakistan modeled COVID-19 data using stochastic models like PRM and NBR. The findings were evaluated based on these models, as the country faces its third wave of the virus. Our study predicts COVID-19 casualties in Pakistan using a count data model. We've used a Poisson process, SIRD-type framework, and a stochastic model to find the solution. We took data from NCOC (National Command and Operation Center) website to choose the best prediction model based on all provinces of Pakistan, On the values of log L and AIC criteria. The best model for modelling the total suspected, infected, and recovered COVID-19 occurrences in Pakistan as it has the maximum log L and smallest AIC of the other count regression model. It was also observed that the active and critical cases positively and significantly affect COVID-19-related deaths in Pakistan using the NBR model.

Introduction

Statistical techniques based on the Poisson distribution were used to evaluate all of the cases discussed. The Poisson distribution is a probabilistic distribution that has been used to generate different flexible continuous distributions by compounding methods for modelling survival data relates to counting events. Various overviews based on the Poisson distribution are examined in recent years. For example, Minka et al. [1] introduced the Conway–Maxwell–Poisson (COM-Poisson) distribution for overdispersion and under-dispersion data, in [2] authors also used the COM-Poisson distribution for the fitting discrete data set, the two-parameter distribution called exponential Poisson distribution [3] used for investigating the convergence of the proposed EM scheme, Weibull Poisson distribution studied [4], and Exponentiated Burr XII Poisson distribution for lifetime data set introduced in [5].

Further, the regression model is used to model the relationship between the predictor (X) and response (Y) variables [6, 7]. The regression model is involved in a linear regression model that is normally distributed and a nonlinear regression model that is not normally distributed is employed [8]. The GLM has involved three components: random component, systematic component, and link function [9]. One of the most useable nonlinear regressions modelled with GLM for lifetime data is Poisson regression. The GLM model was developed to model the relationship between the response and predictor variables and implies that the response variable can be assumed to have the Poisson, binomial, exponential, Gamma, and negative binomial distribution [10]. In Poisson regression, the response variable is considered to have a Poisson distribution. Count information, on the other hand, frequently shows over-dispersion in realworld applications that can be calculated with compound Poisson models (CPM). When the variance is considerably higher than the average, then over-dispersion arises. The information is shown to have over-dispersed when this occurs. CPM can lead to an underestimation of standard errors, resulting in an incorrect conclusion. The aim of the simulation study was how well Poisson and negative binomial regression (NBR) performed during data analysis without any overdispersion and even some data with over-dispersion. Poisson regression is most suitable for data without any over-dispersion, according to the findings.

In recent years, a lot of authors used CPM, Poisson regression models (PRMs), and compound Poisson processes (CPP) for various diseases such as a study of the major infection diseases human immunodeficiency virus (HIV) described in sub-Saharan Africa using PRM [11], to analyse the risk levels of mortality and deadly illness among MERS-CoV patients in the Middle East during 2012 to 2015, Poisson regression with strong variation and a bootstrap-based expectation maximisation model were utilised to cope with significant incomplete information [12], an interpretation of infinite divisible distribution using Levy–Khintchine Formula with

different situations described under the modern interpretations of discrete compound Poisson distributions [13] and a study on the German Children's cancer database described using CPP monitoring all children malignancies [14]. For further understanding related to CPP, PR, NBR, and generalised Poisson regression, refer to the following for more information [13, 15, 16]. In 2020, a study elaborated on the period of emergence of the infectious molecular releasing characteristic of SARS-coronavirus-infected people that resembled that of flu patients and contrasts with that of SARS-coronavirus-infected patients [17]. The Model of Poisson autore-gression of the daily new observed cases dynamically adapt its estimates to explain the evolution of contagion in short and long-term dependence on COVID-19 case counts [18].

Moreover, COVID-19 infection is a transferrable disease that is caused by the severe acute breathing disease known as coronavirus and is now a global pandemic that has affected close to millions of people from different countries. According to the World Health Organisation's statistics as of March 30, 2020, [19], the mortality rate of persons who had been diagnosed with cases was an average of 4.6% with ranges from 0.2% to a higher level of 15% depending on the age group, which also depends on the health status of the predisposed person. Some of the studies are available on the COVID-19 under various statistical models, for example, a compound Poisson generalised linear model (CPGLM) illustrated [20] with Monte Carlo and random forest models to examine the effect of meteorological elements on COVID-19 in Bangladesh and an SEIR design introduced with different purposed parameters have been overstated to compensate for global spread, maturity reliability spread and death rates [21]. The COVID-19 pandemic in China had accumulated a large data set on the ecology factors of this emerging outbreak, which could be relevant if the epidemic spreads [22] and other studies related to COVID-19 with Pakistani epidemic situation with different models described in [23-25].

COVID-19 has affected whole the world especially in European and African countries, some of the authors described the COVID-19 situations according to their own countries with different statistical techniques such as a study [26] evaluated the metrological factors that influenced the overall coronavirus report as per the generalised Poisson regression in Pakistan, but this study includes the negative impact in Pakistan (202,955 positive cases and 4,118 fatalities) under the CPP and NBR models, a CP model [27] with Poisson subordinator whose random period was an independent Poisson process examined with both exponential and ordinary leaps the first-crossing time case, and a heterogeneous bivariate Weibull regression approach on a CPP of random scale is proposed in [28] for the checking of the several conditions including survivability rate in hereditary hygienic, twins' birth, and client dentures. It is [29] revealed COVID-19 as a zoonotic virus, related to SARS-CoV and MERS-CoV coronaviruses, and investigated the COVID-19 retroviruses initial reproductive rate (R0) employing various stochastic methods.

The selection of a discrete-time count regression model is made with the aim of modelling and predicting the behaviour of COVID-19 across Pakistan and around the world in the future. These models provide key information, exponential growth rate, and the doubling time of the pandemic and provide a flexible modelling approach for non-negative integers [30, 31]. The PRM and a NBR model are from the generalised linear regression family which is widely used for epidemiological studies. This is strongly believed that the models we presented here are beneficial to be able to track the future trend of diseases like COVID-19 and so forth.

Research design and methodology

COVID-19 proposed framework

We should know that how COVID-19 diseases change from one stochastic state to another during the infection period, as discussed in the COVID-19 Conceptual framework. The parameters taken for analysis are Suspected (S(t)), Infected (I(t)), Recovered (R(t)), and Deaths (D(t)) are the four major phases found in a group [34]. A susceptible person who comes into interaction with an infectious individual, as per this theory, is at risk of being infected. An individual who has been infected can either recover or die in case of the virus. So, the total number of (S(t)), (I(t)), ((R(t)), and (D(t)) is defined to be constant in this model. Moreover, it is considered anyone who is infected with the virus got infected instantly, without any latent period among stimulation and infection. There seems to be no compensation for the impacts of isolation or quarantine. The period throughout the research is indicated by the 't'. A proposed framework is used to represent the phases, shown in Figure 1.

The Pakistan Ministry of Health has started to release a special bulletin about COVID-19 infections in Pakistan on 26 February 2020. Data will be taken from the Ministry of Health website. We will use the regression model in Poisson distribution to predict the spread of COVID-19 cases in Pakistan by using data from the beginning until 31 January 2021.

Methodology and background

Poisson regression model

The PRM, which is a nonlinear regression analysis of the Poisson distribution, is often used to estimate a response variable with discrete or count data given several explanatory factors. Poisson regression is said to obtain over-dispersion if the variance value is higher than the mean value. Overdispersion has the same effects as if such data types appeared over-dispersion however Poisson regression was also applied, regression coefficients of model parameters may ensure consistency and although inefficient. This is mostly applied when solving issues where the result of a stochastic function can only take count numbers. The Poisson distribution is one of the distributions that fulfil this constraint. It belongs to the exponential distribution family. Assume that 'B' is the r.v (Coronavirus death ratio) and that the case findings are an event. The 'B' variable approaches a Poisson distribution [31] with parameter omega $\omega > 0$

$$P(B=b) = \frac{e^{-\omega}\omega^n}{B!},$$
(1)

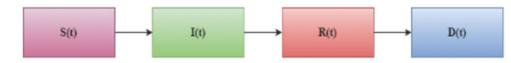


Figure 1. Proposed framework of COVID-19 disease.

where n = 1, 2, 3 is the number of occurrences of an event and ' ω ' is defined as $\omega = \text{Exp}[B]$.

The generalised linear model (GLM) can be stated as

$$g_l = \beta_0 + \beta_1 A_{l1} + \dots + \beta_m. \tag{2}$$

The first function explains how the mean, $\operatorname{Exp}[B_l] = \omega_l$ which depends on the linear predictor $\varphi(\omega_l) = g_l$ while the second function explains how the variance, $V(B_l)$ depends on the mean $V(B_l) = \sigma V(\omega_l)$. Where parameter ' σ ' is a constant, assuming B_l is a Poisson distribution, so

$$B_l \sim \text{pois}(\omega_l)$$
, $\text{Exp}[B_l] = \omega_l$ and $V(B_l) = \omega_l$.

The variance function is defined as follows:

$$V(\omega_l) = \omega$$
 Ranges from $(0, \infty)$.

A log function is given as

$$\ell(\omega_l) = \log_e(\omega_l)$$

Based on the general linear model (GLM), the linear regression must be associated with the dependent variables with a link variable, which is also the link variable between the linear regression in a matrix form and the Stochastic regression line in this case. Assuming a linear model is as follows:

$$B_l = \beta_l A_l + \epsilon_l, \tag{3}$$

where 'A' is n^* (m + 1) vectors of predictors, a column of 1's β is a (m + 1) by 1 vector of uncertain parameter, as well as ϵ is a ($n^* 1$) vectors of error values with

$$\operatorname{Exp}(B/A) = a\beta. \tag{4}$$

Consider the general linear regression, the linked value and its transportation 'B' are as below:

$$G(B) = \log_{e}(B). \tag{5}$$

Hence, it may be expressed as

$$\log_e E(B_A) = a\beta. \tag{6}$$

So that, regarding a Poisson regression with the function ' β ' and '*a*' is its input vector, an expected average of the aligned Poisson distribution is taken as

$$E(B_{A}) = e^{a\beta}.$$
 (7)

If ' B_l ' is a response variable with the predictor variable ' A_l ', the parameter ' β ' may be evaluated by MLE method. Mathematical models may be used to evaluate the system indicated in equation (7), which is obtained the conditional expectation of response and predictor variables by adopting the logarithm transform of the conditional expectation. Moreover, PRMs' MLE of the probabilistic area is usually concave, Gradients-based techniques are used as appropriate estimation procedures.

Let b_l be a random variable and has a positive integer like l=1,2,3,...,n where n denotes the total number of observations. Hence $b_l \sim$ Pois dist. [31]. So, the Probability Mass Function (PMF) will be

$$P(B_{l} = b_{l}) = \frac{\omega_{l}^{b_{l}} e^{-\omega_{l}}}{b_{l}!},$$
(8)

$$P(B=b) = f(b) = \frac{e^{-\omega}\omega^{\nu}}{b!},$$
(9)

where $b_l = 0, 1, 2, ...$ with mean and variance are given as

$$\operatorname{Exp}[b_l] = V(b_l) = \omega.$$

In equation (4), the estimated average of a Poisson Process is defined as

$$\operatorname{Exp}(B_{A}) = e^{a\beta} = \omega = \operatorname{Exp}(b_{l}),$$

whereas the values of the independent factor $\beta = (\beta_1, \beta_2, \dots, \beta_m)$ be k – dimension vectors of logistic coefficients are unknown, while the average of the estimated stochastic process is presented as $\exp(B_A)$ and $\operatorname{Var}(B_l)$ is given as $V(B_A)$.

The following three factors are indicated by using Poisson regression, which is designed to fit such a rate of fatalities in Pakistan:

 A_1 = Suspected cases A_2 = Infected cases A_3 = Recovered cases.

$$\log(\theta) = \beta_0 + \beta_1 A_1 + \beta_2 A_2 + \beta_3 A_3.$$
(10)

The parameter ' β ' shows the average variation in the independent factor ' A_l ' in the logarithmic equation (8) that may be evaluated using the maximum likelihood estimator (MLE) approach.

Negative binomial regression

To cope with over-dispersion in count data, NBR is being used. As populations are usually varied, specifically distinct to the assumption involved in commonly use basic stochastic methods, overdispersion is a usual feature of the real statistical approach. In this research, the NBR model [32] is stated as follows:

$$\operatorname{Prob}[B_{l} = b] = \sum \operatorname{Prob}(B_{l} = b_{l}/\theta_{l})f(\theta_{l})d_{\theta l}$$
$$= \frac{\Gamma\left(b_{l} + \frac{1}{\alpha}\right)}{\Gamma(b_{l} + 1)\Gamma\left(\frac{1}{\alpha}\right)} \left[\frac{1}{1 + \alpha\omega_{l}}\right]^{\frac{1}{\alpha}} \left[\frac{\alpha\omega_{l}}{1 + \alpha\omega_{l}}\right]^{b_{l}},$$
for l,0,1,2, (11)

where mean is given as

$$\omega_l = \operatorname{Exp}(b_l) = V_l \left[e^{\sum \alpha a_{lm} \beta_m} \right], \text{ for } l = 1, 2, 3, \dots, n.$$

While the variance of b_l is given as

$$V(b_l) = \omega_l + \alpha \omega_l^2.$$

If $\alpha > 0$, the method might be called a dispersion parameter. The PRM may be treated with a limited form of the NBR model with $\alpha \propto 0$. The MLE statistic is written as

$$\log(\beta, \alpha) = \sum \left(\sum_{r=1}^{b_l} \log(1 + \alpha r) \right) - b_l \log(\alpha) - \log(b_l!)$$

+ $b_l \log(\alpha \omega_l) - (b_l + \alpha^{-1}) \log(1 + \alpha \omega_1).$ (12)

A partially differential of the MLE w.r.t (β , α) can be used to evaluate the parameter (β , α). The NBR may not involve the mean = variance but it controls over-dispersion, which happens whenever the value of Variance is larger than the mean value. The objective is to determine a model with the lowest number of parameters that minimises the Negative likelihood, as shown in the equation below:

$$AIC = 2R - 2lnL, \tag{13}$$

in which '*L*' denotes the likelihood ratio and '*R*' is the number of model parameters.

Results and discussion

Finding and discussion

The methods used in this analysis are PRM and NBR which comply mostly with the study's particular target. The count variables are the total number of fatalities that are the response data, also with events of total suspected, infected, and recovered COVID-19 occurrences as the predictor variables. Continuous count data is used for all the variables.

Application

From 26 February 2020 to 1 January 2021 secondary information is attained from the National Command and Operation Center (NCOC) website for total suspected, infected, recovered, and deaths COVID-19 occurrences.

A set of count methods, like the PRM and the NBR model have been used to estimate the total number of COVID-19 suspected, infected, recovered, and deaths cases in Pakistan. The data set is taken from 26 February 2020 to 1 January 2021.

Table 1 shows that mean and standard deviation at Mean = 5,166.33 and SD = 4,888.679 for each suspected because of coronavirus. Same as the mean and standard deviation of infected occurrences of coronavirus infection is Mean = 57,580.18 and SD = 60,088.140. The standard deviation mean and of the recovered coronavirus occurrences in the study reveals that Mean = 51,266.59 and SD = 55,753.951. Thus, the mean and standard deviation of the death occurrences of coronavirus is Mean = 1,233.06 and SD = 1,223.407. According to the findings thus far away, the large mean estimate of Covid-19 infected and recovered cases in Pakistan throughout the situation under analysis has resulted in comparatively large cases of coronavirus-associated fatalities.

Figure 2 shows the graphical representation of suspected cases of COVID-19 infection in different provinces of Pakistan, such as, Punjab, Baluchistan, Sindh and KPK.

Figure 3 shows the graphical representation of infected cases of COVID-19 infection in different provinces of Pakistan, such as, Punjab, Baluchistan, Sindh and KPK.

Figure 4 shows the graphical representation of recovered cases of COVID-19 infection in different provinces of Pakistan, such as, Punjab, Baluchistan, Sindh and KPK.

Descriptive statistic					
Mean SD Minimum Maximum					
Suspected	5,166.33	4,888.679	0	21,515	
Infected	57,580.18	60,088.140	0	247,249	
Recoveries	51,266.59	55,753.951	0	224,664	
Deaths	1,233.06	1,223.407	0	4,747	

Figure 5 shows the graphical representation of death cases of COVID-19 infection in different provinces of Pakistan, such as, Punjab, Baluchistan, Sindh and KPK.

The omnibus test shows the actual model to the null model that used a test of likelihood-ratio chi-square. The present framework overcomes the null hypothesis since its *p*-value is less than 0.05. It is a likelihood ratio test to perceive all the explanatory variables (suspected, infected, and recovered COVID-19 cases) are significantly outperforming the intercept-only analysis.

Table 2 shows the explanatory variables (i.e., suspected, infected, and recovered cases) have 0.000 *p*-value (p < 0.05), in Poisson regression (PR) and NBR models, which indicate an overall statistically significant model.

The condition of equi-dispersion in Poisson regression analysis, the mean should be equal to the variance. Therefore, the claim is hardly observed, over-dispersion is frequent in many situations. Though over-dispersion is perhaps the most common issue, it is really essential to examine this number firstly. The deviance for all of the models is near to '1.2', which is admissible. Whenever the amount of deviance is reduced by the degree of freedom (df), it should be larger than '1.2' to identify over-dispersion; meanwhile, under-dispersion occurs whenever the amount of deviance is lowered with df is smaller than '1.2'.

Table 3 shows the deviance/df and Pearson chi-square/df are both more than '1.2', implying that the numbers of COVID-19 occurrences in Pakistan are over-dispersed, based on the PRM. The analysis is performed by NBR to get the better solution as this approach may handle the dispersion parameter, and the resulting log-likelihood and Akaike informatics criteria (AIC) results are (-209,814.870 and 419,637.741) respectively.

Table 4 indicates both amounts of Pearson chi-square/df and deviance/df are below '1.2' (i.e., 1.069 and 0.687, respectively), implying that the amount of COVID-19 occurrences in Pakistan are under-dispersion, and have log-likelihood and -9,664.914 and 19,337.828, respectively.

The reported COVID-19 death cases are a count, and then this form of data seems to have a Poisson distribution. To analyse the variables that impact the number of COVID-19 cases of deaths, the Poisson regression modelling is used. The maximum likelihood estimator (MLE) technique is commonly used to predict the parameters of Poisson regression.

Table 5 shows *p*-values for all parameters that are smaller than 0.05, indicating that the parameters β_1, β_2 and β_3 have a significant impact on results. Consequently, the PRM is given as

 $b_l = \exp(5.941 + 0.00009605A_1 + 0.00003278A_2 - 0.000003883A_3).$

Explanatory factors that determine the coronavirus fatalities in Pakistan are the coronavirus suspected (A_1) , infected (A_2) and recovered (A_3) occurrences.

The NBR is often used to predict count observation as well as obtain dispersion measurement to tackle the limitation of overdispersion related to the PRM. Consequently, the NBR model is given as

 $b_l = \exp(5.370 + 0.00008899A_1 + 0.00001533A_2 - 0.000002329A_3).$

Table 6 reveals that basically, factors are statistically significant at 5% level, also p < 0.05. With exception of the recovered cases, the NBR model shows that two of the COVID-19 factors are positive. This indicates that when using the NBR model to estimate the parameter, a 1% rise in suspected and infected COVID-19 instances might result in a 0.009% and 0.002% higher in COVID-19-related

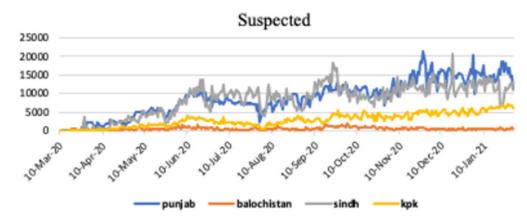


Figure 2. Suspected cases in Punjab, Baluchistan, Sindh and KPK.

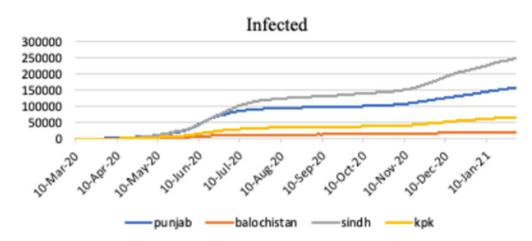


Figure 3. Infected cases in Punjab, Baluchistan, Sindh and KPK.

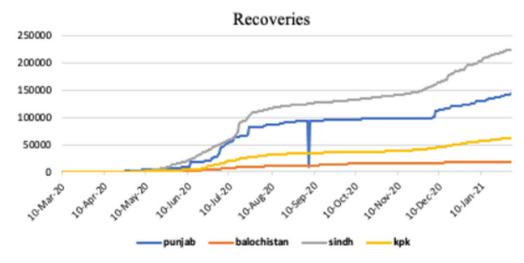


Figure 4. Recoveries in Punjab, Baluchistan, Sindh and KPK.

deaths in Pakistan. COVID-19 patients that are suspected and infected have a positive effect on the deaths related to coronavirus in Pakistan. Consequently, at the 5% level, the recovered occurrences have a negative value (-0.000002329), which implies a 1% increase in recovered coronavirus occurrences in Pakistan

throughout the time period under consideration may occur in a -0.0002% reduction in the number of deaths caused by infectious disease in Pakistan and outcome often reveals that recovered occurrences may have an adverse impact of the coronavirus fatalities.

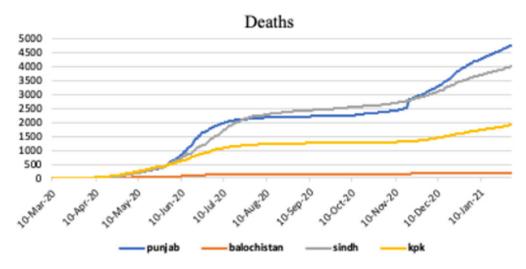


Figure 5. Deaths in Punjab, Baluchistan, Sindh and KPK.

Table 2. Omnibus test of COVID-19 cases in Pakistan

Omnibus test					
Models	Likelihood ratio chi-square	df	Significance		
Poisson regression	1,198,004.616	3	0.010		
Negative binomial regression	1,300.107	3	0.030		

Table 3. Association of variables with model selection (PRM)

Goodness-of -fit					
	Value	df	Value/ <i>df</i>		
Deviance	409,680.895	1,261	324.886		
Pearson chi-square	362,033.002	1,261	287.100		
Log–likelihood	-209,814.870	1,261	-166.388		
Akaike's information criterion (AIC)	419,637.741	1,261	332.782		

Table 4. Association of variables with model selection (NBR)

Goodness-of-fit					
	Value	df	Value/ <i>df</i>		
Deviance	1,473.910	1,261	1.069		
Pearson chi-square	866.075	1,261	0.687		
Log-likelihood	-9,664.914	1,261	-7.664		
Akaike's information criterion	19,337.828	1,261	15.335		

Table 7 shows the association of variables with a model selection of the two counts data, specifically PRM and NBR. The AIC is often used to evaluate approaches. AIC is a way to estimate the likelihood of a model that can forecast results depending on in-sample fits. As compared to another method, a model performs best if its AIC is the lowest.

The NBR has the smallest AIC and likelihood values, as stated by the consequence: (5,244.866, -2,618.433) in Punjab, (3,491.162, -1,741.581) in Baluchistan, (5,252.483, -2,622.242) in Sindh, (4,971.439, -2,481.719) in KPK and (19,337.828, -9,664.914) in Pakistan, and is a selected count model for modelling suspected, infected and recovered coronavirus epidemic occurrences in all Provinces of Pakistan.

Comparison of COVID-19 cases between male and female

The study analysed the effect of the COVID-19 death rates on gender. However, females have a better immune system as compared to males. So, male occurrences are more severe than female occurrences. Males with COVID-19 are at a higher risk for adverse outcomes and deaths. The statistical analysis of the COVID-19 data set shows that men suffer a significantly higher fatality rate than women.

Table 8 shows the mean and standard deviation of females at (Mean = 1,446.59 and SD = 1,368.817) for each suspected because of COVID-19. Same as the mean and standard-deviation of infected occurrences of coronavirus (Mean = 16,122.44 and SD = 16,824.680). The mean and standard deviation of the recovered occurrences of coronavirus in the study reveal that Mean = 13,840.42 and SD = 15,559.324. Thus, the mean and standard deviation of the death occurrences of COVID-19 are Mean = 345.26 and SD = 342.552. According to the findings thus far away, the large mean estimate of COVID-19-infected and recovered (female) cases in Pakistan during the period under analysis has resulted in comparatively large cases of COVID-19-associated deaths.

Table 9 shows that the explanatory factors (i.e., suspected, infected, and recovered occurrences of coronavirus) have a 0.000 *p*-value (i.e., p < 0.05), showing a statistically significant appropriate assessment in PRM and NB).

Table 10 demonstrates that the amount of deviance/df and Pearson chi-square/df are both more than '1.2', implying that the number of COVID-19 (female) occurrences in Pakistan is overdispersed, based on PRM. The analysis is performed by NBR to get the better solution as this approach may handle the dispersion parameter, and the resulting log-likelihood and AIC results are -66,865.403 and 133,738.807, respectively.

Table 11 indicates that the amounts of deviance/df and Pearson chi-square/df are both below '1.2' (i.e., 1.036 and 0.775, respectively), implying that the amount of COVID-19 (female) occurrences in Pakistan are under-dispersion, having log-likelihood and AIC numbers -8,253.522 and 16,515.044, respectively.

Epidemiology and Infection

Table 5. Multiple terms test between diagnose disease and respondence in Pakistan (PRM)

Parameter estimates (PRM)							
			95% wald con	fidence interval	Нур	oothesis test	
Parameter	В	SE	Lower	Upper	Wald chi-square	df	Significance
(Intercept)	5.941	0.0017	5.938	5.944	11,706,409.09	1	0.000
Suspected	9.605E-5	2.3205E-7	8.559E-5	8.650E-5	137,502.939	1	0.040
Infected	3.278E-5	8.7030E-8	3.108E-6	3.449E-6	1,419.038	1	0.010
Recoveries	-3.883E-6	8.9952E-8	3.707E-6	4.059E-6	1,863.456	1	0.30
(Scale)	1						

Table 6. Multiple terms test between diagnose disease and respondence in Pakistan (NBR)

Parameter estimates (NBR)							
			95% wald confi	dence interval	Нур	othesis test	:
Parameter	В	SE	Lower	Upper	Wald chi-square	df	Significance
(Intercept)	5.370	0.0489	5.274	5.466	12,034.059	1	0.030
Suspected	8.899E-5	1.3963E-5	6.162E-5	0.000	40.617	1	0.010
Infected	1.533E-5	3.5393E-6	8.391E-6	2.226E-5	18.756	1	0.000
Recoveries	-2.329E-6	3.2563E-6	-8.712E-6	4.053E-6	0.512	1	0.050
(Scale)	1						
(Negative Binomial)	1						

Table 7. Selected count data model for COVID-19 cases

		Мо	dels	
	Poissor	regression	Negative bin	omial regression
Provinces	AIC	Log-likelihood	AIC	Log-likelihood
Punjab	39,984.215	-19,988.107	5,244.866	-2,618.433
Balochistan	4,018.361	-2,005.180	3,491.162	-1,741.581
Sindh	79,922.877	-39,957.439	5,252.483	-2,622.242
КРК	38,215.096	-19,103.548	4,971.439	-2,481.719
Overall	419,637.741	-209,814.870	19,337.828	-9,664.914

Table 8. Descriptive statistics of COVID-19 (female) cases in Pakistan

Descriptive statistics					
Mean SD Minimum Maximum					
Suspected	1,446.59	1,368.817	0	6,024	
Infected	16,122.44	16,824.680	0	69,230	
Recoveries	13,840.42	15,559.324	0	62,906	
Deaths	345.26	342.552	0	1,329	

Table 12 shows the *p*-value of all parameters are lower than 0.05, indicating the parameters β_1, β_2 and β_3 have a significant impact on results. Therefore, the PRM is given as

 $b_l = \exp(4.615 + 0.0007A_1 + 0.00004415A_2 - 0.00001190A_3).$

Table 9. Omnibus test of COVID-19 (female) cases in Pakistan

	Omnibus test		
Models	Likelihood ratio chi-square	df	Significance
Poisson regression	357,878.762	3	0.050
Negative binomial regression	1,456.218	3	0.030

Explanatory factors which determine the coronavirus fatalities in Pakistan are the coronavirus suspected (A_1) , infected (A_2) and recovered (A_3) occurences.

Consequently, the NBR is given as

 $b_l = \exp(3.997 + 0.0004569A_1 + 0.00001589A_2 - 0.00001538A_3).$

Table 13 reveals that almost all factors are significant statistically at a 0.05 level of significance, and *p*-values of all paramaters are lower than 0.05. Except for recovered cases, the NBR model shows that two of the COVID-19 (female) factors are positive. This indicates that when using the NBR model to estimate the parameter, a 1% rise in suspected and infected COVID-19 (female)

Table 10. Association of variables with model selection (PRM) of COVID-19 (female) cases in Pakistan

Goodness-of-fit					
Value df Value/df					
Deviance	125,351.956	1,308	95.835		
Pearson chi-square	110,179.713	1,308	84.235		
Log-likelihood	-66,865.403	1,308	-51.120		
Akaike's information criterion (AIC)	133,738.807	1,308	102.247		

 Table 11. Association of variables with model selection (NBR) of COVID-19 (female) cases in Pakistan

Goodness-of-fit					
Value df Value/df					
Deviance	1,747.278	1,308	1.036		
Pearson chi-square	1,013.607	1,308	0.775		
Log-likelihood	-8,253.522	1,308	-6.310		
Akaike's information criterion (AIC)	16,515.044	1,308	12.626		

instances might result in a 0.05% and 0.002% higher in COVID-19-related deaths in Pakistan. COVID-19 patients that are suspected and infected have a direct positive impact on deaths related to COVID-19 in Pakistan. Consequently, at the 5% level, the recovered occurrences have a negative value (-0.00001538), which implies that a 1% rise in recovered coronavirus occurrences in Pakistan under the period of analysis may occur in a (-0.002%) reduction in deaths caused by COVID-19 in Pakistan, and the outcome often reveals that recovered occurrences may have a negative impact of the COVID-19 (female) deaths.

Table 14 shows that there is a mean and standard deviation of males at Mean = 3,719.74 and SD = 3,519.862 for each suspected because of coronavirus infection. Same as the mean and standard deviation of infected occurrences of coronavirus are Mean = 41,457.74 and SD = 43,263.461. The mean and standard deviation of the recovered occurrences of coronavirus disease upon the study reveals that Mean = 35,589.65 and SD = 40,009.649. Thus, the mean and standard deviation of the death occurrences of coronavirus infection are Mean = 887.79 and SD = 880.855. According to the findings thus far away, the large mean estimate of coronavirus infected and recover (male) occurrences in Pakistan during the period under analysis resulted in a comparatively large amount of COVID-19-associated deaths.

Table 15 shows that the explanatory factors (i.e., suspected, infected, and recovered occurrences of coronavirus) have a 0.000 p-value (i.e., p < 0.05), showing a statistically significant appropriate assessment in PRM and NBR.

Table 16 demonstrates that amount of Pearson chi-square/df and deviance/df are both more than '1.2', implying that the number of COVID-19 (male) occurrences in Pakistan is over-dispersed, based on PRM. The analysis is performed by NBR to get a better solution as this approach may handle the dispersion parameter, and

Table 12. Multiple terms test between diagnose disease and respondence (female) in Pakistan (PRM)

Parameter estimates								
			95% wald con	fidence interval	Нур	othesis test		
Parameter	В	SE	Lower	Upper	Wald chi-square	df	Significance	
(Intercept)	4.615	0.0033	4.608	4.621	2,002,553.706	1	0.000	
Suspected	0.0007	1.5632E-6	0.000	0.000	40,519.410	1	0.057	
Infected	4.415E-5	5.8732E-7	1.300E-5	1.530E-5	580.499	1	0.046	
Recoveries	-1.190E-5	6.0770E-7	1.071E-5	1.309E-5	383.677	1	0.027	
(Scale)	1							

Table 13. Multiple terms test between diagnose disease and respondence (female) in Pakistan (NBR)

Parameter estimates (NBR)								
95% wald confidence interva					l Hypothesis test			
Parameter	В	SE	Lower	Upper	Wald chi-square	df	Significance	
(Intercept)	3.997	0.0478	3.903	4.090	6,992.882	1	0.000	
Suspected	4.569E-4	5.0417E-5	0.000	0.000	36.575	1	0.050	
Infected	1.589E-5	1.2910E-5	4.058E-5	9.119E-5	26.045	1	0.030	
Recoveries	-1.538E-5	1.1826E-5	-3.855E-5	7.804E-6	1.690	1	0.020	
(Scale)	1							
(Negative binomial)	1							

Table 14. Descriptive statistics of COVID-19 (male) cases in Pakistan

Descriptive statistics								
Mean SD Minimum Maximum								
Suspected	3,719.74	3,519.862	0	15,491				
Infected	41,457.74	43,263.461	0	178,019				
Recoveries	35,589.65	40,009.649	0	161,758				
Deaths	887.79	880.855	0	3,418				

Table 15. Omnibus test of COVID-19 (male) cases in Pakistan

Omnibus test						
Models	Likelihood ratio chi-square	df	Significance			
Poisson regression	920,295.977	3	0.040			
Negative binomial regression	1,462.118	3	0.000			

Table 16. Association of variables with model selection (PRM)

Goodness-of-fit							
	Value	df	Value/df				
Deviance	322,215.894	1,308	246.342				
Pearson chi-square	283,300.674	1,308	216.591				
Log-likelihood	-165,882.729	1,308	-126.822				
Akaike's information criterion	331,773.459	1,308	253.649				

Table 17. Association of variables with model selection (NBR)

Goodness-of-fit						
	Value	df	Value/ <i>df</i>			
Deviance	1,915.229	1,308	1.064			
Pearson chi-square	1,023.804	1,308	0.783			
Log-likelihood	-9,488.507	1,308	-7.254			
Akaike's information criterion (AIC)	18,985.015	1,308	14.515			

Table 17 indicates that amounts of deviance/df and Pearson chisquare/df are both below '1.2' (i.e., 1.064 and 0.783), respectively, then it is implying that the number of COVID-19 (male) occurrences in Pakistan are under-dispersion, having log-likelihood and AIC numbers are (-9,488.507, and 18,985.015) respectively.

Table 18 shows the parameters of the *p*-values are lower than 0.05, indicating that the parameters $\beta_1, \beta_2, \text{and}\beta_3$ have a significant impact on results. Consequently, the PRM is given as

$$b_l = \exp(5.559 + 0.00087A_1 + 0.000005503A_2 - 0.000004631A_3).$$

Explanatory factors which determine the coronavirus fatalities in Pakistan are the coronavirus suspected occurrences (A_1) , infected occurrences (A_2) , and recovered occurrences (A_3) .

Therefore, the NBR is given as

 $b_l = \exp(4.937 + 0.00067A_1 + 0.000002579A_2 - 0.000006080A_3).$

Table 19 reveals that almost all the parameters are significant statistically at the 5% level of significance, and the parameters of the *p*-values are all less than 0.05. Apart from the recovered cases, the NBR model shows that two of the COVID-19 factors are positive. This indicates that when using the NBR model to estimate the parameter, a 1% rise in suspected and infected COVID-19 (male) instances might result in a 0.067% and 0.003% higher in COVID-19 related deaths (male) in Pakistan. COVID-19 patients that are suspected and infected have a direct positive impact on deaths related to COVID-19 in Pakistan. Consequently, at the 5% level, the recovered occurrences have a negative value (-0.000006080)which implies that a 1% rise in recovered COVID-19 occurrences in Pakistan under the period of analysis may occur in a (-0.0006%)reduction in deaths caused by COVID-19 in Pakistan, and the outcome often reveals that recovered occurrences may have a negative impact of the Covid19 (male) deaths.

Table 20 shows that the NBR has the smallest AIC and log-likelihood values, as stated by the consequence: (18,985.015, -9,488.507) in males, respectively, while (11,515.044, -6,053.522) in females and is the selected count data model for modelling the suspected, infected and recovered Coronavirus wide-spread occurrences between males and females in Pakistan. Hence the resulted AIC and log-likelihood shows that males are more prone than females.

Table 18. Multiple terms test between diagnose disease and respondence (male) in Pakistan (PRM)

Parameter estimates (NBR)							
			95% wald con	fidence interval	Нур	othesis test	
Parameter	В	SE	Lower	Upper	Wald chi-square	df	Significance
(Intercept)	5.559	0.0020	5.555	5.563	7,472,469.618	1	0.000
Suspected	0.00087	3.7909E-7	0.000	0.000	104,183.125	1	0.040
Infected	5.503E-6	1.4244E-7	5.223E-6	5.782E-6	1,492.343	1	0.031
Recoveries	-4.631E-6	1.4738E-7	4.342E-6	4.920E-6	987.251	1	0.037
(Scale)	1						

Table 19. Multiple terms test between diagnose disease and respondence (male) in Pakistan (NBR)

Parameter estimates (NBR)							
	Hypothesis test						
Parameter	В	SE	Lower	Upper	Wald chi-square	df	Significance
(Intercept)	4.937	0.0477	4.843	5.031	10,698.219	1	0.000
Suspected	0.00067	1.9607E-5	8.005E-5	0.000	36.514	1	0.030
Infected	2.579E-5	5.0227E-6	1.595E-5	3.564E-5	26.373	1	0.000
Recoveries	-6.080E-6	4.5988E-6	-1.509E-5	2.934E-6	1.748	1	0.010
(Scale)	1						
(Negative binomial)	1						

 $\label{eq:constraint} \textbf{Table 20.} Selected count data model for COVID-19 cases between male and female$

		Мос	dels		
	Poisson	regression	Negative binomial regressi		
Genders	AIC	Log-likelihood	AIC	Log-likelihood	
Male	331,773.459	-165,882.729	18,985.015	-9,488.507	
Female	133,738.807	-66,865.403	11,515.044	-6,053.522	

Conclusion

We have used two logistic count regression models to predict the third wave of Covid-19. These models proved very beneficial in the prediction of Covid-19. We have fitted two selected count Regression models on suspected, infected, and recovered cases as factors that impact the novel coronavirus disease 2019, (COVID-19) deaths in Pakistan. The competitive results indicate that the proposed reliability-based regression model has higher performance in predicting the deterioration of COVID-19 patients compared to the classic accuracy-based regression model [33]. During the period under analysis, we provide an overview of the (suspected, infected, recovered and deaths) COVID-19 occurrences observed in different provinces of Pakistan from the period of 26 February 2020 to 31 January 2021. The study shows that the PRM analysis is unable to handle over-dispersion, so another method of the PRM (logistic regression) including the NBR is applied in the prediction. It can be determined from the outcomes of the model's selection process which include log L and AIC. So, the NBR has the lowest value of AIC and greatest value of log L from the two-selection process, it is considered as the better model. The positively significant impact of suspected and infected occurrences over the coronavirus fatality rate in different provinces of Pakistan, such as Punjab, Baluchistan, Sindh and KPK, is a unique result of this study.

For male cases by PRM model: ($\beta_1 = 0.00087, p < 0.05$) a 1% rise in suspected occurrences resulted in a 0.09% of rising in fatality number in Pakistan, whereas ($\beta_2 = 0.00006, p < 0.05$) indicates that 1% rise in infected occurrences resulted in a 0.006% of rise in the fatalities in Pakistan. For male cases by NBR model: ($\beta_1 = 0.00067, p < 0.05$) a 1% rise in suspected occurrences resulted in a 0.07% of rise in the fatalities in Pakistan whereas ($\beta_2 = 0.000026, p < 0.05$) reveals that 1% rise in critical occurrences resulted in a 0.003% of rise fatality's number in Pakistan.

Female cases by PRM model: ($\beta_1 = 0.0007, p < 0.05$) indicates that 1% rise in suspected occurrences resulted in 0.07% of rise in

several fatalities, whereas ($\beta_2 = 0.000044$, p < 0.05) indicates that a 1% rise in infected occurrences resulted in 0.004% of rise in the number of fatalities in Pakistan. Female by NBR model: ($\beta_1 =$ 0.00045, p < 0.05) indicates that 1% rise in suspected occurrences resulted in 0.05% of rise in the number of fatalities whereas ($\beta_2 =$ 0.000016, p < 0.05) reveals that 1% rise in infected occurrences resulted in a 0.002% of the rise in the number of fatalities in Pakistan.

Whenever there is evidence of over-dispersion in the counts, the NBR is said to be an appropriate model for evaluating the factors that impact the coronavirus fatalities in various provinces. The suggestions based on the current study and results are NBR analysis must be utilised to model the daily suspected, infected, and recovered occurrences as variables affecting coronavirus disease-related deaths and NCOC should be focused more on suspected and infected instances as they are the main factors of COVID-19 related fatalities in Pakistan. This study was done on the data of non-vaccinated people while a similar kind of study can be done on vaccinated people, and we can make a comparison of the efficacy of the vaccine in the future.

The same proposed prediction model may be built separately for different countries with unique infrastructure, atmospheric and demographic conditions attributes, and symptomatic patients. The model can be used by the different organisations that are researching different contagious diseases to be focused more on suspected and infected cases as they are the main factors of COVID-19-related deaths in Pakistan. The study recommended that the Pakistani government, Ministry of Health, and administration should give more awareness and protections for societies, and they should also open more COVID-19 laboratory testing centres. Generally, the obtained results of this study may help Pakistani decision-makers put a short-term future to face this pandemic.

Data availability statement. The data are available to the authors upon a reasonable request.

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Competing interest. The authors declare none.

Ethical standard. All procedures were performed per the relevant guidelines in the manuscript.

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