DOUBLE-MODE CEPHEIDS

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INTRODUCTION

As their name implies double-mode (or beat) cepheids form a sub-group of cepheid variables which appear to be pulsating simultaneously in two modes. From the period ratio of the two frequencies, the modes have been identified with the fundamental and first overtone modes of radial pulsation. Petersen (1973) was the first to establish that the two periods can be used to determine the masses and radii of these stars independent of any other knowledge such as luminosities, effective temperatures or gravities. The masses derived by this technique are 2 to 3 times lower than the expected evolutionary masses, given that these stars are normal population I cepheids. It is this result that has stimulated much of the subsequent research on double-mode cepheids.

OBSERVED PROPERTIES OF DOUBLE-MODE CEPHEIDS

Despite intensive searches (Szabados 1977, Pike & Andrews 1979, Henden 1979, 1980, Barrell 1980) to discover further candidates the number of known and confirmed double-mode cepheids remains at eleven. The periods and amplitudes of the light and radial velocity variations of these variables are listed together with relevant references in Table 1. In two cases (VX Pup and BK Cen) the period P has changed significantly from previously published values as a result of the analysis of new observations (Stobie & Balona 1979c). Once the two frequencies, ω_0 and ω_1, of the oscillation have been specified the data are fitted to an nth order Fourier solution

$$X = C - \Sigma_{ij} A_{ij} \cos 2\pi[(i\omega_0 + j\omega_1)(t - t_0) + \phi_{ij}] \qquad (1)$$

where $|i| + |j| \leq n$ and $i \geq 0$. A least squares technique is used to determine the $[2n(n + 1) + 1]$ unknowns C, A_{ij}, ϕ_{ij}. The amplitudes in Table 1 correspond to the terms A_{10} and A_{01} for the visual light curve

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Table I. Periods and amplitudes of double-mode cepheids

Star	P_o	P_1	P_1/P_o	$A(V)_o$	$A(V)_1$	$A(RV)_o$	$A(RV)_1$	References
TU Cas	2.1393	1.5183	0.7097	0.287	0.134	14.1	7.2	GK47 Oo57b WE57 Oo60 WS60 FA77c GR78 Fa79 Ni79 NS79 HS79
U TrA	2.5684	1.8249	0.7105	0.297	0.125	14.7	7.5	Oo57a Ja62 Fa77a FS79 SB79a SB79c
VX Pup	3.0117	2.1390	0.7102	0.166	0.144	6.3	9.0	WS60 Ir61 MI64 St70 SB79a SB79b SB79c
AP Vel	3.1278	2.1998	0.7033	0.275	0.138	13.2	9.8	Oo57a Oo64 Pe76 SB79a SB79c
BK Cen	3.1739	2.2230	0.7004	0.245	0.106	13.2	10.0	Le67 SB79a SB79c
UZ Cen	3.3343	2.3553	0.7064	0.308	0.070	14.7	6.2	Pe76 St76 SB79a SB79c
Y Car	3.6398	2.5595	0.7032	0.266	0.120	11.1	6.8	Oo57a WM58 Ir61 St72 SB79a SB79c
AX Vel	3.6731	2.5928	0.7059	0.116	0.148	3.6	7.5	Oo57a WM58 Ir61 SH72 Pe76 SB79a SB79c Ba80
GZ Car	4.1589	2.9337	0.7054	0.150	0.086	5.3	7.1	Oo57a Pe76 St76 SB79a SB79c
BQ Ser	4.2707	3.012	0.7053	-	-	-	-	We66 Za69 Sz76
V367 Sct	6.2931	4.3847	0.6967	0.143	0.137	-	-	RP63 TR63 KE67 Ta69 MB75 EK75a EK75b MS78 Ba80

Notes: (i) The amplitude quoted is the semi-amplitude from a Fourier analysis of the light and radial velocity variations. In most cases the amplitude has been taken from a second order Fourier fit.

(ii) Y Car is a spectroscopic binary (SB79c). The orbital velocity variation has been removed to obtain the Fourier amplitudes of the pulsation.

and radial velocity observations in most cases fitted to a second order Fourier solution.

The amplitude and energy systematics of double-mode cepheids have been discussed by Faulkner (1977a, b, c) and Stobie & Balona (1979c). Although AX Vel is exceptional in that it is the only known double-mode cepheid whose light amplitude in the first overtone exceeds that in the fundamental mode, this is not true if we consider the radial velocity observations where 3 out of 9 double-mode cepheids dominate in the first overtone. By calculating the energy systematics (estimated from $4\pi^2 \Sigma_{ij} \omega_{ij}^2 A_{ij}^2$ for each mode) it can be shown that there is no preference for domination of either mode. In all cases investigated a considerable fraction of the energy is contained in the interaction terms (i.e. terms of frequency $i\omega_0 + j\omega_1$ where $ij \neq 0$) caused by the non-linear superposition of the two oscillations in the stellar envelope.

One interesting systematic effect in Table 1 is that the ratio of amplitudes A(RV)/A(V) is always greater for the first overtone mode than the fundamental mode. This can be understood in terms of the linear pulsation theory developed by Balona & Stobie (1979a) where for radial pulsation one obtains a relation

$$A_{RV}/A_V \quad = \quad 33.01 \quad \omega(R/R_{\odot})(f^2 + 4f \cos \psi + 4)^{-\frac{1}{2}} \tag{2}$$

where ω is the frequency in cycles/day, R is the radius, f is the ratio of flux variation to radius variation and ψ is the phase lag between maximum flux and maximum radius. Thus for a given star A_{RV}/A_V is proportional to ω and this predicts the displacement observed between fundamental and first overtone modes of pulsation. Equation (2) can also be used to estimate $R/R_{\odot}$ since ω, A_{RV}, A_V, f and ψ can be determined from the Fourier analysis of contemporaneous light, colour and radial velocity observations (Stobie & Balona 1979c). Analysis of 8 double-mode cepheids led to a mean radius of $R/R_{\odot} = 33 \pm 2$, quite inconsistent with the radius expected from the period ratios.

Balona & Stobie (1979a) showed that, in the case of single-mode cepheids, there is a correlation between the ratio of the flux variation to the radius variation (f in equation (2)) and the period. Because the period correlates strongly with intrinsic colour we expect that f will also correlate with intrinsic colour as shown in Fig. 1. Exactly this effect is predicted by theoretical pulsation models. The double-mode cepheids also show this correlation in that the value of f decreases as the intrinsic colour increases. The maintenance of this correlation by the double-mode cepheids is indirect evidence that the intrinsic colours have been relatively accurately determined and also that the small variation in intrinsic colour is real.

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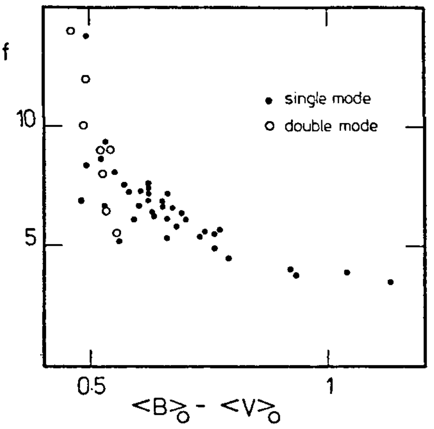

Fig. 1. Correlation of f with intrinsic colour for single-mode and double-mode cepheids

EVOLUTIONARY STATUS OF DOUBLE-MODE CEPHEIDS

The evidence that double-mode cepheids belong to population I is persuasive. From Walraven five-colour photometry Pel (1978) has shown that the line blanketing in double-mode cepheids is identical to that in population I cepheids. From spectroscopic data on 10 out of the 11 known double-mode cepheids, Barrell (1980) has established that the abundance of iron is consistent with the abundance of iron in the solar atmosphere. Furthermore the galactic spatial distribution of double-mode cepheids and their kinematics as deduced from the radial velocities all match those of single-mode cepheids of the young disc (Stobie 1977).

Given that the double-mode cepheids almost certainly are members of population I, the question is what are their masses, radii and luminosities? Unfortunately the one double-mode cepheid (Y Car) known to be a member of a spectroscopic binary does not have the orbit sufficiently well determined nor any information known about the companion in order to shed light on this. However 9 out of the 11 double-mode cepheids have contemporaneous observations of the light, colour and radial velocity curves (Balona & Stobie 1979b, Niva & Schmidt 1979). These data may be used to derive radii by the Baade-Wesselink technique. In both cases. the authors were careful to compare the radii of double-mode cepheids with the radii of single-mode population I cepheids of similar period. Thus even if there are systematic effects in determining Baade-Wesselink radii as a result of the assumptions involved they hopefully will affect the single-mode and double-mode variables in a similar way. The results showed that the radii of the double-mode variables were indistinguishable from the radii of the single-mode variables and hence were inconsistent with the radii expected from the period ratios. This, in turn, implies as a consequence of the period-radius-mass relation that the masses of the double-mode cepheids are consistent with the evolutionary masses expected for population I cepheids of similar period.

Crucial to the interpretation of the double-mode cepheids is the longest period one, V367 Scuti, in the open cluster NGC 6649 (Madore & van den Bergh 1975, Madore et al. 1978, Kobayashi et al. 1976). Previously the membership of this cepheid in the cluster has been based on positional coincidence and the agreement between its absolute magnitude determined by cluster membership and that expected from the PLC relationship. Flower (1978), however, cast doubts on its membership and indicated that either the cepheid or the cluster was anomalous in some way. Recently Barrell (1980) has carried out a radial velocity study of the cepheid and some of the cluster members. The mean radial velocity of the cepheid is indistinguishable from that of the cluster. In addition she has resolved the anomaly presented by Flower as the three brightest main sequence stars appear to be Be stars and hence are abnormally bright as a result of their rapid rotation. Thus there is no evidence contradicting the membership of V367 Scuti in NGC 6649. This is crucial since now the same cepheid shows the usual mass anomaly (i) the mass determined by its membership of the cluster, M_{evol} and (ii) the mass, M_{beat}, determined from the two periods, which is a factor of 2 lower than M_{evol}.

The evidence against double-mode cepheids being normal population I cepheids is based on two accounts (i) the masses derived from the period ratios are too low for normal population I cepheids and the determinarion of the masses cannot be this inaccurate (Petersen 1978), (ii) the strength of Hα emission in double-mode cepheids is abnormally high in comparison to single-mode cepheids (Barrell 1980). This latter effect is remarkable, requires independent confirmation and further observation as it does not seem to correlate with phase. It would imply a significant difference in the dynamics of the atmospheres of double-mode and single-mode cepheids

In conclusion the weight of evidence is strongly in favour of double-mode cepheids being normal population I cepheids which, for some reason, happen to be pulsating in two modes simultaneously. If this is accepted then there is no conflict with stellar evolution theory as we do not have to understand how stars of such low mass ($1-2M_\odot$) can penetrate the instability strip during core helium burning. Instead we are saying that the masses predicted from the period ratios are wrong by factors of 2 to 3. Thus two important questions remain to be answered A) what is the cause of the double-mode phenomenon and B) what is the source of the mass discrepancy?

POSITION OF DOUBLE-MODE CEPHEIDS IN THE P-C AND H-R DIAGRAMS

Before attempting to answer these questions it is important to know the position of double-mode cepheids in the period-colour Hertzsprung-Russell diagrams as this observational evidence is crucial to any theories purporting to explain these variables. Table 2 lists the intrinsic colours and absolute magnitudes of 10 of the 11 double-

Table 2. Intrinsic colours and absolute magnitudes of double-mode cepheids

Star	log P_o	$\langle B-V \rangle_o$	$M_{\langle V \rangle}$
TU Cas	0.33	0.473	-2.37
U TrA	0.41	0.551	-2.46
VX Pup	0.48	0.488	-2.90
AP Vel	0.50	0.483	-2.99
BK Cen	0.50	0.518	-2.84
UZ Cen	0.52	0.526	-2.95
Y Car	0.56	0.538	-3.07
AX Vel	0.57	0.463	-3.31
GZ Car	0.62	0.521	-3.34
V367 Sct	0.80	0.54	-3.97

mode cepheids based as far as possible on a uniform dereddening system. For stars which had BVI observations the intrinsic colours were determined using the dereddening technique of Dean et al. (1978). In the cases of TU Cas the V367 Sct no BVI observations were available. For TU Cas the intrinsic colour was obtained from Hα profile measurements (Schmidt 1972) and in the case of V367 Sct from the cluster reddening

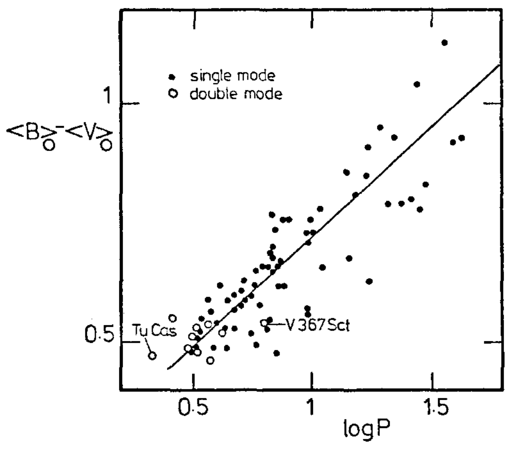

Fig. 2. Intrinsic colours as a function of period for double-mode and single-mode cepheids

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corrected to the reddening of a cepheid (Madore & van den Bergh 1975). In Fig. 2 the periods and intrinsic colours of double-mode cepheids are compared with those of single-mode cepheids of population I, also derived by the same dereddening technique of Dean et al. (1978). It is apparent that the double-mode cepheids all lie in a narrow temperature range and are not confined or concentrated to either the red or blue edges of the instability strip. In order to compare the same set of stars in the H-R diagram we have assumed that both the double-mode (fundamental period) and single-mode cepheids obey the same PLC relationship derived by Martin et al. (1979).

$$M_{\langle V \rangle} = -3.80 \log P + 2.70 [\langle B_o \rangle - \langle V_o \rangle] - 2.39$$

As expected Fig. 3 shows that the double-mode cepheids are concentrated in a narrow vertical strip in the H-R diagram which extends from the blue side of the strip (at the high luminosity end) to the red side of the strip (at the low luminosity end). These results are almost identical to those presented by Barrell (1980) where a totally different technique (Hα profile)) of determining the intrinsic colour was used.

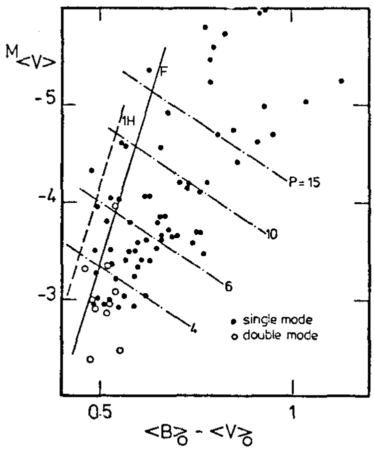

Fig. 3. H-R Diagram of double-mode and single-mode cepheids

CAUSE OF THE DOUBLE-MODE PHENOMENON

A number of theories have been put forward to explain the double-mode pulsation. The stars may be in a transition region between fundamental and first overtone pulsation. The difficulty with this explanation is that Stellingwerf (1975b) has shown, from the rate of evolution scross the strip and the non-linear mode switching rates, that an e-folding time of about 80 years is expected. This is much too short in comparison to the number of double-mode cepheids. However, particularly relevant in this connection is to search for changes in modal content of double-mode cepheids as a function of epoch. Two double-mode cepheids so far have shown changes in modal content. In U TrA it was observed that the first overtone mode grew at the expense of the fundamental mode with the total energy remaining constant (Faulkner 1977a, Faulkner & Shobbrook 1979). In TU Cas the first overtone decreased in amplitude and the total energy also decreased (Hodson et al. 1979, Niva 1979). In V367 Sct there was no observed change in amplitude over a period of 50 years (Madore et al. 1978).

Stellingwerf (1975a) has calculated models which indicate the existence of a region of stable mixed-mode behaviour near the red edge of the instability strip. This, however, would appear to conflict with the observational evidence (Figs. 2, 3) where there is no preference for either the blue or the red side of the strip. A more promising theory is that presented by Simon (1979, 1980) and also considered by Petersen (1979, 1980), where the existence of the double-mode phenomenon in cepheids is caused by a resonance interaction, $\omega_0 + \omega_1 \cong \omega_3$. This resonance theory has the attraction of naturally explaining the extremely narrow observed range of period ratios. Whether it also predicts a nearly vertical region in the H-R diagram where resonance occurs, remains to be seen. A difficulty that is encountered by both Simon's theory and Stellingwerf's (1975a) theory is that, as yet, no non-linear model has been found in a stable mixed-mode pulsation (Simon et al. 1980).

SOURCE OF THE MASS DISCREPANCY

It seems clear that the source of the mass discrepancy occurs either in a modification to the structure of the stellar envelope or in some inadequacy in our understanding of pulsation theory. Cogan (1977) found that the period ratio of first overtone to fundamental modes of pulsation can be substantially reduced by deep envelope convection and greater values of the mixing length parameter. It is difficult, however, to see how this can explain the M_{beat} discrepancy as double-mode cepheids are not concentrated to the red side of the instability strip. Cox et al. (1977) have shown that the inclusion of rotation cannot change the period ratio sufficiently to resolve the mass discrepancy. Faulkner (1977c) has suggested that, because of the observed high degree of mode interaction in double-mode cepheids, period changes may occur due to the non-linear mode interaction. However, non-linear effects on the theoretical periods of a double-mode cepheid have been investigated by Cox et al. (1978a). They conclude that the first overtone to fundamental period ratio agrees with the linear theory value independent of the stage of mode-switching and hence independent of the non-linear coupling.

The only theory which has been able to account successfully (see, however, Cogan 1978) for the M_{beat} mass discrepancy as well as the M_{bump} mass discrepancy is that due to Cox and co-workers (Cox et al.

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1977, 1978b). This theory requires a chemically inhomogeneous envelope with the inclusion of a helium rich layer (Y = 0.75) throughout the hydrogen and helium convection zones (T < 70,000°K). Even if such a layer could be produced initially by a hydrogen deficient stellar wind, the difficulty is maintaining it as it is completely unstable because of the inverted μ-gradient.

Presently no theory is generally accepted as the source of the mass discrepancy. Given that double-mode cepheids are normal population I cepheids then it is important to consider the relationship of the M_{beat} mass discrepancy to that of any other mass discrepancies in cepheids, in particular the M_{bump} mass discrepancy which appears to be the only other really significant mass discrepancy. Thus whatever theory is proposed it will be necessary to consider its impact on both these mass discrepancies.

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DISCUSSION

J. COX: I understand that no double-mode Cepheids have been found in other galaxies. Is this because no one has looked, or for some other reason?

STOBIE: *Certainly it is harder to pick them up there because obser-*vationally you are looking at much fainter stars and so it is harder to pick up the extra scatter in the light curve. Some of these stars in our galaxy show an extremely large scatter, sometimes of half a magnitude at maximum light and I would have thought that even photographically if something like that existed in an external galaxy we might have picked it up, but I don't think we can say that they definitely don't exist. It is clear that we have to look a lot harder, possibly with photoelectric observations. Maybe the difficulty is that it has mostly been photographic work. Of course, you are talking about looking quite faint. They are going to be down about 16 or 17th magnitude.

CONNOLLY: We recently looked at how the surveys were done in the search for variable stars. Double mode Cepheids being at the faint end of the normal Cepheid region could easily be missed. In looking for RR Lyrae stars they ignored anything too bright and again would miss the double mode Cepheids and so, in both kinds of surveys, they could have been totally overlooked.

COGAN: Ben Gascoigne pointed out to me that one of the short period Cepheids that he observed in the LMC had quite a large scatter in the data. It was larger than others of similar magnitude. He suggested that perhaps this is, in fact, a beat Cepheid. I think no one has looked at it since he did 15 years ago, so there is simply not enough data.

CONNOLLY: That is a very peculiar Cepheid. It seems to be kind of climbing up the instability strip. I have observed it about three times and he observed it over two seasons. It seems to be slowly getting brighter and redder.

A. COX: There is a paper by Takeuti saying that if there is a lot of convection there will be a lot of H-alpha emission. Nobody has mentioned it. Maybe Takeuti will. I don't know what the status is, but he proposed that if you have a lot of helium enhancement you would have stronger convection, stronger surface activity and, therefore, stronger H-alpha emission. Am I right?

TAKEUTI: Yes.

STOBIE: As far as I understand, the emission that has been detected previously in single mode Cepheids has generally been correlated with phase. You find the greatest emission, if any at all, on the rising branch. In the case of double mode Cepheids, there seems to be no correlation with phase.

A. COX: Strong all of the time?

STOBIE: Strong sometimes, other times absent. There is no correlation.

COGAN: Because of the beat phenomenon in double mode Cepheids and the incommensurability of the periods you occasionally get a very strong sharp rise from minimum to maximum. It is conceivable that at these times one might get H-alpha emission but not later. It would, of course, take a lot of observations to sort that particular pattern out.

STOBIE: Barrell's observations do not correlate with these phases.

SIMON: I would like to make the same plea that I always make at these meetings with regard to double mode Cepheids. We need more extensive observations for both light and velocity variations; good enough so that one can fit both curves with a nonlinear Fourier scheme and be confident that the coefficients are physically meaningful. I think it is the size of the coefficients of the various terms and in their phase relations that one can read the key to modal selection.

A. COX: Somebody, maybe both speakers asked Norm Simon, where is the resonance line? I told Walt Fitch I didn't think it was anywhere near. Will you say in public, Norman, where are the resonance lines?

SIMON: It depends on what mass you want.

A. COX: Evolutionary masses.

SIMON: Evolutionary masses and normal composition? Then the resonance lines are far away. Let me say something about the resonance hypothesis. In the work that Art, Steve, and I did, the result that we get said that the best place to be for double mode pulsations is as far away from the resonance as possible, which is not very promising for the resonance hypothesis. I think that as long as we can not make a double mode pulsator on the computer, we are in trouble.