

Numerical study on snow transport and drift formation

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ABSTRACT. A three-dimensional, numerical simulation model for snow transport and drift formation is proposed in which saltation as well as suspension are considered as dynamic behavioral factors of moving snow particles. The procedure for simulation is as follows: (1) Air flow field is simulated solving the Reynolds equations and the continuity equation. (2) Using the result of the air field flow simulation, the blown-snow density field is simulated using the diffusion equations in which the fall velocity of blown snow particles is considered. In the boundary conditions, the particle movement of saltation is taken into consideration. (3) Finally, the snowdrift rate is computed based on the amount of snow particles not transported by saltation. This model was quantitatively tested for the phenomenon of snowdrift development. The computed results showed good agreement with observations.

INTRODUCTION

Uematsu and others (1989) tried to research drifting snow phenomena ranging from air flows to snowdrifts. Their models were two-dimensional except in the case of the suspension phenomenon. In this research the authors were not able to reproduce a snowdrift in a complicated topography, which points out the need for consideration of the suspension phenomenon. Recently there have been a few papers on this topic, but none of much significance. No numerical simulations have considered the air flows which lead to drifting snow as well as both the saltation and the suspended motion of snow particles.

NUMERICAL MODEL

Basic equations

Air flow

Snowdrift development is mainly controlled by the friction velocity u_* , which can be obtained from a velocity profile.

A numerical simulation method developed by Arisawa (1987) is applied to obtain a three-dimensional velocity field. Arisawa's method is a so-called control volume method based on Patanker's algorithm (Patanker, 1981). The Reynolds equations with boundary conditions are numerically solved using this method.

Snow transport

Snow transport is possible by saltation, suspension or creep. As mentioned above, only saltation and suspension are taken into consideration in this simulation.

Up to now, many empirical formulae describing snowdrift transport rate by saltation have been issued

(e.g. Kobayashi, 1972). However, the application of these empirical formulae are limited strictly to the place and time of the observation because they contain empirical constants. Therefore, in view of future applications, a new theoretical equation should be derived from the particle trajectory.

Assuming that the saltation phenomenon occurs continuously, the following four equations for saltation of snow particles are derived. The parameters used in the derivation of the trajectory of saltating particles are shown in Figure 1.

The shear stress τ_0 at the snow surface is expressed by the following formula

$$\tau_0 = \tau_{S0} + \tau_t \quad (1.1)$$

where τ_{S0} is the shear stress caused by collision of saltating particles with the surface and τ_t is the shear stress due to wind. Here, τ_t is assumed to be the critical shear stress, which is consistent with the assumption of Owen (1964).

τ_{S0} can be expressed as

$$\tau_{S0} = M(U_2 - U_1), \quad (1.2)$$

where M is the mass flux per unit area and time, U_2 is the horizontal component of the particle velocity before collision with the surface, and U_1 is the horizontal component of the particle velocity after collision. The

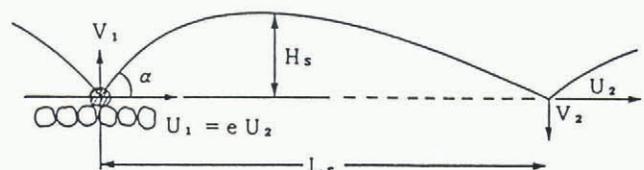


Fig. 1. Sketch for the definition of trajectory parameters.

condition for snow particles undergoing constant saltation can be given by the equation

$$\frac{M}{m} = \frac{N}{L_S} \tag{1.3}$$

Here, m is the mass of a single particle given by $(\beta/6)\pi d^3$, d is the diameter of the particle and β is the density of ice. N is the number of particles passing per unit width per unit time, and L_S is the saltation length. The lefthand side of Equation (1.3) gives the number of particles passing per unit area per unit time, and the righthand side gives the number of particles landing on the surface.

Finally, the snowdrift transport rate q_B is defined as

$$q_B = Nm. \tag{1.4}$$

Using Equations (1.1), (1.2), (1.3) and (1.4), the following theoretical equation for particle transport rate can be derived:

$$q_B = \frac{e \tan \alpha}{1 - e} \sqrt{\frac{2H_S}{L_S}} \sqrt{S + \frac{3}{2} \frac{u_*^2 - u_{*t}^2}{\sqrt{Sgd}}} \sqrt{L_S} d \rho. \tag{1.5}$$

Here, H_S is the saltation height, α is the fly-out angle, e is the restitution coefficient, $S = \rho - 1$, g is the acceleration due to gravity, and u_{*t} is the threshold friction velocity. Although there are many variables in Equation (1.5), each has a physical meaning.

Equation (1.5) is checked in section 3 by comparison with observations. The values for L_S , H_S , d and e are given as follows: $e = 0.8$, obtained from the collision experiment on the ice particles against the ice surface (Araoka and Maeno, 1978); the other three are constant values since L_S is a function of a friction velocity, judging from the work of Kikuchi (1981). As a result, Equation (1.5) is reduced to

$$q_B = 7.404 \rho \sqrt{d} (u_*^2 - u_{*t}^2) u_*. \tag{1.6}$$

Equation (1.6) is compared with the data collected by Kobayashi (1972). Figure 2 shows that this semi-theoretical equation compares well with measured values.

Suspension is modeled using the diffusion equation.

The *deposition* (or erosion) rate can be calculated by simulating the drift density Φ_h in the saltation layer. The deposition rate, d , can be expressed as

$$d = |w_f| \Phi_h. \tag{1.7}$$

Here, w_f is the snowfall velocity. In Equation (1.7) the maximum erosion rate is assumed to be equal to the maximum deposition rate.

The maximum erosion rate, e_r , can be expressed as

$$e_r = \frac{q|w_f|}{u_h h}. \tag{1.8}$$

Finally, we can calculate the snowdrift rate, s , as

$$s = -(d + e_r), \tag{1.9}$$

where s is defined as the mass of snow accumulation on a unit horizontal area per unit time.

2. Numerical aspects

Various methods for numerical calculation in fluid

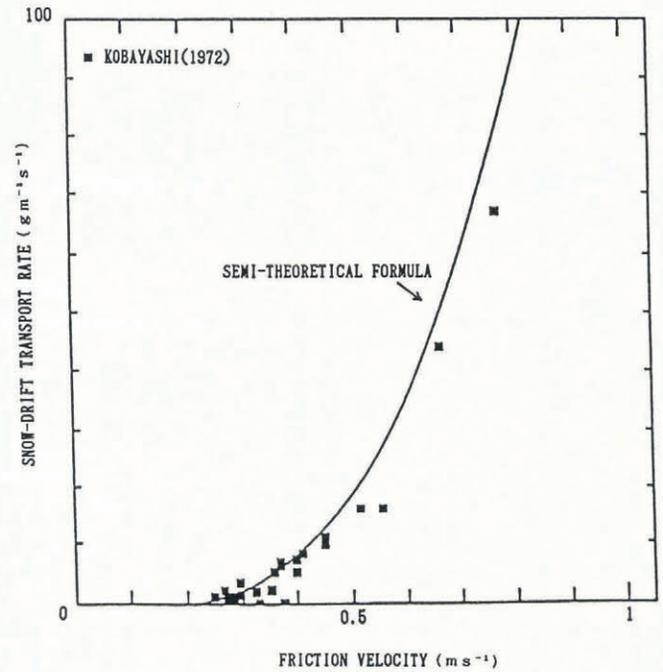


Fig. 2. Snowdrift transport rate against friction velocity by Kobayashi (1972) and the simulated curve of the semi-theoretical formula.

mechanics are found in Roache's textbook (1978). Among them, the "control volume method" is presumed to be the most physically precise. The SIMPLER (Semi-Implicit Method for Pressure-Linked Equations Revised) model, which Patanker (1981) proposed, is used here.

3. Initial and boundary conditions

This simulation is roughly divided into two parts, flow and snowdrift density. Each initial and boundary condition is applied here. To determine the snowdrift density, the diffusion equation must be solved. In doing so, setting the boundary and initial conditions becomes a problem.

Air flow

The boundary conditions are now stated.

At the lower boundary

$$u = v = w = 0. \tag{3.1}$$

At the top of the domain

$$u = u_0, v = v_0, w = 0, \tag{3.2}$$

and

$$\rho = (\rho)_{t=0}, \tag{3.3}$$

where u_0 and v_0 are the initial wind speeds at the top of the domain and $(\rho)_t$ is the initial air density.

At the lateral boundaries, the radiation condition of Orlanski (1976) is adopted.

The logarithmic distribution is adopted as the initial condition over the entire domain.

Snow transport

The lower boundary of the suspension layer is in contact with the saltation layer; therefore, the mean snowdrift

density of the saltation layer is considered to be the lower boundary condition.

Strictly speaking, the height of the saltation layer varies with the degree of development of the snowdrift; however, it is assumed to be approximately constant at all points.

The snowfall rate is used as the upper boundary condition for suspension. The upper boundary of suspension is the height where the snowdrift density becomes zero. The upper boundary condition is expressed as

$$\Phi_H = \frac{P_{re}}{|w_f|}, \quad (3.4)$$

where P_{re} is the snowfall rate.

The snowdrift density Φ_H is used as the initial condition over the entire domain.

The other variables

The five elements shown in Table 1 are input values for the other variables in the numerical simulation. The wind speed and snowfall rate are explained as the initial condition (described above). The roughness height is used to give the initial condition for air-flow calculation and at the same time to acquire the friction velocity on the snow surface from the wind-speed distribution obtained. The threshold friction velocity is used to acquire the drifting-snow transport rate of the saltation layer. Furthermore, the falling velocity is used to convert snowfall rate and the drifting-snow transport rate into the snowdrift density.

4. Flowchart of simulation

Figure 3 outlines the flow of the calculation process. First, geographical features are taken as a boundary for the calculation of air flow. The calculation of air flow is repeated until the wind velocity given by the logarithmic distribution converges.

Next, the snowdrift density is calculated using the previously obtained air flow. The calculations are repeated until a converged value is obtained using the snowdrift density obtained from snowfall rate for the initial condition. As a result, the snowdrift density and the snowdrift rate are obtained. A snowdrift is simulated by integrating the snowdrift rate. A snowdrift at an arbitrary time can be obtained by repeating the above calculation for a necessary period of time.

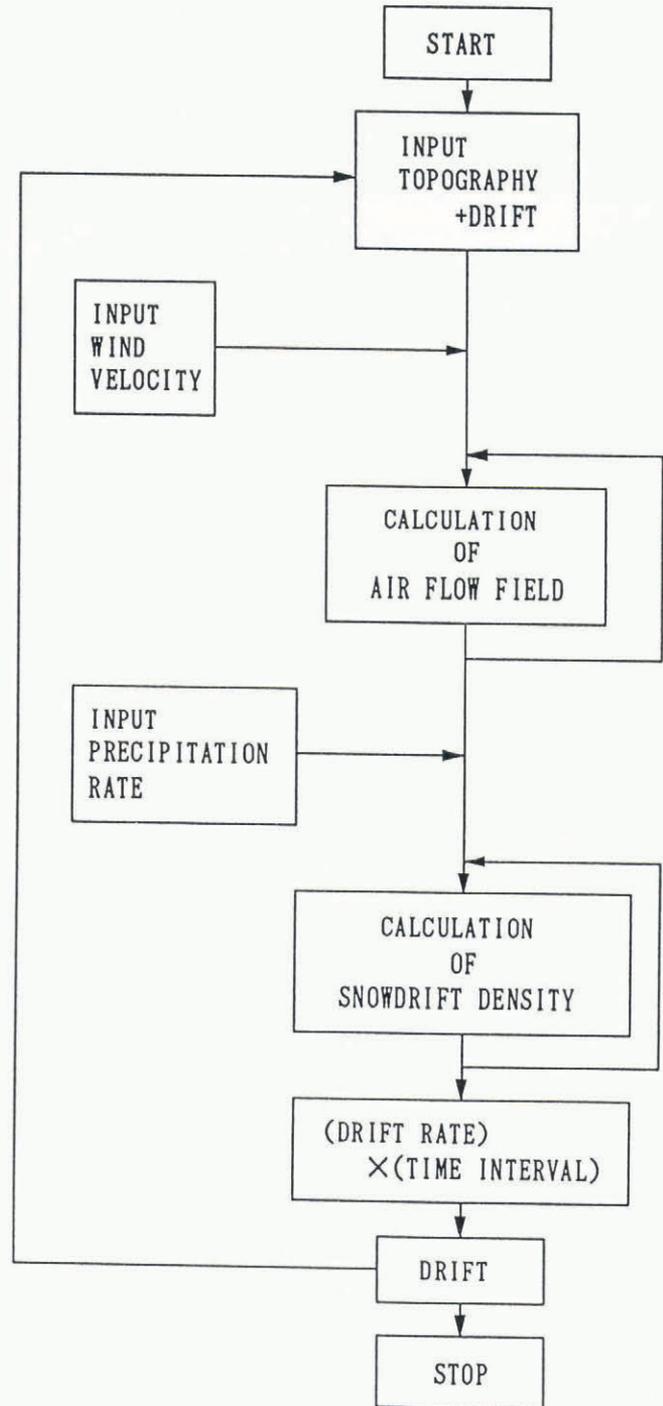


Fig. 3. Flowchart of simulation.

Table 1. Input data

	Input data	Diameter	Diameter	References
		d = 0.3 mm	d = 0.1 mm	
Windspeed	observational data			
Roughness height	0.1 mm	0.08 mm	0.03 mm	Lettau (1969)
Threshold speed	0.20 m s ⁻¹	0.29 m s ⁻¹	0.17 m s ⁻¹	White (1940)
Snowfall velocity	0.50 m s ⁻¹	0.65 m s ⁻¹	0.22 m s ⁻¹	Kawamura (1982)
Precipitation rate	observational data			

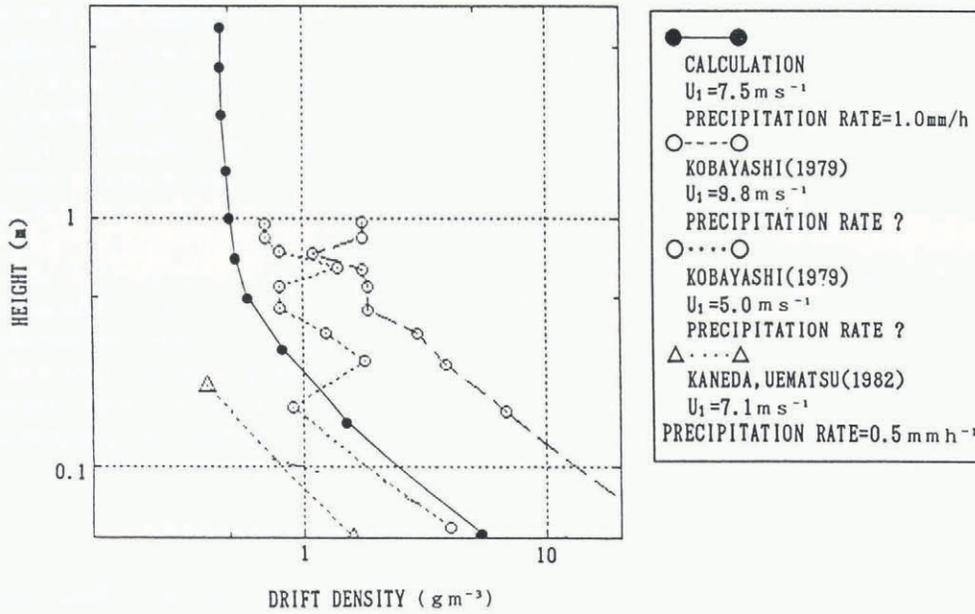


Fig. 5. Comparison between observed and simulated drift density.

layer is 0.7 m for a particle diameter of 0.3 mm, and 4.25 m for that of 0.1 mm. There is almost no change in the height and density of the high-drift density layer. The windward lateral boundary is 40 m from the origin of the snowdrift for the particle diameter of 0.3 mm, and 110 m for that of 0.1 mm. For both particle diameters, the leeward snowdrift rate increases gradually from zero to the snowfall rate towards leeward. The snowdrift rate reaches the snowfall rate at approximately the same point that the drift density becomes constant.

Figure 5 shows the drift-density profile, which was obtained at steady state. The solid circles show the simulation result and the other symbols show observations from the past. The drift density decreases nearly linearly up to a height of 50 cm, and becomes constant above 50 cm with the logarithmic vertical axis. This profile is compared with the observations of Kobayashi (1979), and Kaneda and Uematsu (1982) (Fig. 6). The simulated profile falls between the density profiles for wind speeds of 9.8 m s⁻¹ and 5.0 m s⁻¹ at the height of 1 m above the snow surface (Kobayashi, 1979). Since the simulation was done for a wind speed of 7.5 m s⁻¹, the result is considered adequate.

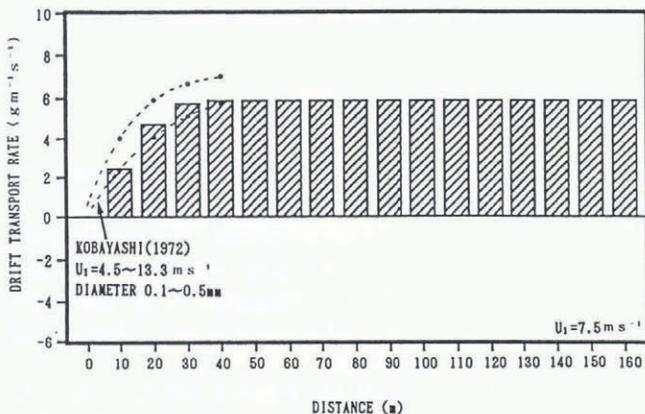


Fig. 6. Comparison between observed drift transport rate (Kobayashi (1972)) and simulated rate.

Figure 6 shows the variation of snowdrift transport rate relating to the distance from the origin of the snowdrift. The snowdrift transport rate increases in the leeward direction and becomes constant for 30 m and beyond. This tendency is consistent with the observations of Kobayashi (1972), which were obtained at wind speeds of 4.5–13.3 m s⁻¹ at 1 m above snow surface with particle diameters of 0.1–0.5 mm. Thus, this newly developed model agrees well with past observations. Takeuchi (1980a) observed that snow transport rate becomes constant 350–400 m from the origin of the snowdrift while Kobayashi (1972) observed that point to be 30–60 m from the origin. This contradiction may be explained by the difference in particle diameter, as shown above.

DISCUSSION

The theoretical formula of the snowdrift transport rate by saltation is examined first. Equation (1.5) for the snowdrift transport rate by saltation contains the following coefficients and constants: saltation distance; saltation height; angle of departure; restitution coefficient; diameter of snow particle; air density; density of snow particle; friction velocity; and threshold friction velocity at which the snow particles begin to move. It is important to know which coefficients or constants make a main contribution to the snowdrift transport rate.

First, the ratio of the saltation height to the saltation length ranges from 0.1 to 0.4 as shown in Figure 7; therefore, the snowdrift transport rate ranges from 1.6 to 3.1. Next, the angle of departure is roughly in the range from 20° to 80°, and the transport rate varies from 0.4 to 5.6. Moreover, the diameter of the snow particle is in the range of 0.1–0.5 mm, reported by Kobayashi (1972), and the snowdrift transport rate varies from 0.01 to 0.02. The coefficient of restitution for the snow particle and the ice surface is roughly in the range of 0.2–0.9 by Araoka and Maeno (1978), therefore, the snowdrift transport rate

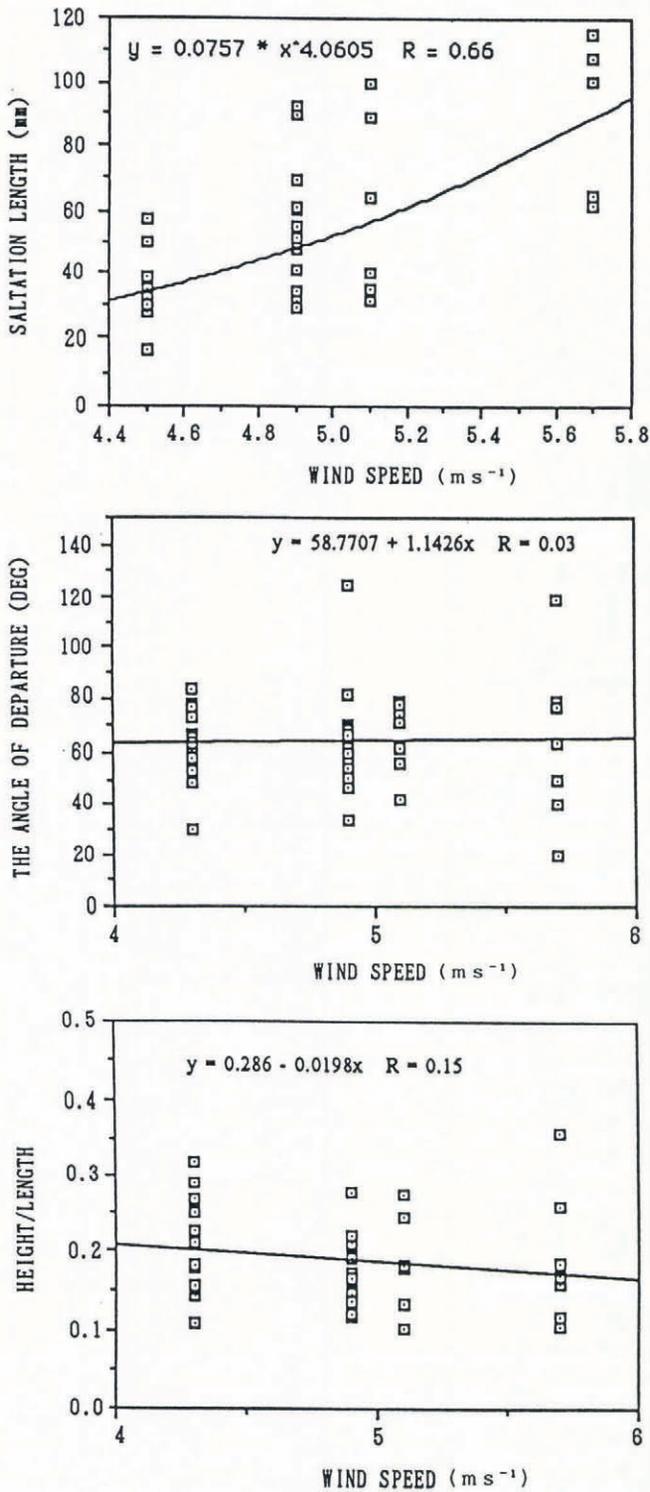


Fig. 7. Saltation length, angle of departure and the ratio of saltation height to saltation length against wind speed, obtained by the wind tunnel experiment (Kikuchi, 1981).

varies from 0.25 to 9.0. The threshold friction velocity calculated with the diameter of the particle by White's formula (1940) is in the range of 0.2–0.4 m s⁻¹, and the snowdrift transport rate ranges from 0.04 to 0.16. The friction velocity is in the range of 0.2–1.0 m s⁻¹, and the snowdrift transport rate varies from 0.04 to 1.0. When the maximum value of the contribution is divided by the minimum value, the resultant value indicates the degree of change in the snowdrift transport rate. The resulting ratios are: 1.9 for the ratio of saltation height to saltation

length; 14 for the angle of departure; 2 for the diameter of the particle; 4.5 for the restitution coefficient; 4 for the threshold friction velocity; and 25 for the friction velocity. These values for the friction velocity, the angle of departure, restitution coefficient, and the threshold friction velocity are larger than 4; however, the angle of departure as well as the restitution coefficient are distributed near the average values as shown in Figure 7 and do not influence the snowdrift transport rate. On the other hand, the range of the friction velocity is very common, as reported by many researchers. Its accuracy must influence the presumed accuracy of the snowdrift transport rate. Therefore, it is important to compute the friction velocity precisely in this simulation model.

CONCLUDING REMARKS

In this study, a numerical simulation model was applied to the drifting snow and snowdrift phenomena. The following results were obtained:

A new theoretical expression for the saltation movement of snow particles is derived from the ballistic study, and thereby, the amount of drifting snow transport in the saltation layer is calculated. The computational results show good agreement with an observed result.

A three-dimensional numerical simulation model is developed for drifting snow and snowdrifts. The simulated flow is determined as follows: (1) Air flow is obtained by solving Reynolds equation and the continuity equations. (2) Using the airflow obtained, the diffusion equation is solved by introducing a semi-theoretical expression of saltation for a lower boundary condition. (3) From the computed solution of the diffusion equation, the accumulation of snow is defined as those snow particles that are not transported by saltation.

The developed numerical model was applied to the drifting snow phenomena on level ground. Various observed snowdrifts were reproduced using this model. In the case where the snowdrift itself changes physical conditions of the air flow, the model is run repeatedly, giving the newly formed drifting snow surface as the lower boundary.

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