

# LIMITATIONS TO OPTICAL/IR INTERFEROMETRY FROM THE GROUND AND SPACE

M. M. COLAVITA

*Jet Propulsion Laboratory  
California Institute of Technology  
4800 Oak Grove Dr., Pasadena, CA 91101, USA*

## **Abstract.**

The primary limitation to ground-based optical/IR interferometry is the turbulent atmosphere, which limits sensitivity by restricting the coherence volume, limits imaging accuracy by corrupting the fringe phase, and limits astrometric accuracy by corrupting the angle of arrival. Various advanced techniques can be used to circumvent these limits to some extent. Sensitivity can be increased with adaptive optics and laser guide stars, which should eventually be able to phase the individual apertures of an interferometer down to some cutoff wavelength, limited by tilt sensing. However, the sky coverage for cophasing the interferometer on an arbitrary object will remain limited at short wavelengths. For imaging, closure-phase techniques, well established in radio interferometry, will be used in next-generation instruments. However, for maximum sensitivity on extended objects, redundant arrays will be needed to cophase the interferometer. For astrometry, the limits to wide-field astrometry set by the atmosphere can be reduced somewhat with two-color techniques, but otherwise do not seem reducible by the techniques now being discussed. However, over narrow fields, the astrometric performance of an interferometer can be quite good. In space, without the corruptions of the atmosphere, the fundamental limitation is photon noise. However, technical issues such as metrology accuracy and practical issues such as maximum affordable baseline length will also limit performance.

**Key words:** long-baseline interferometry –interferometric imaging –astrometry –atmospheric effects

## **1. Introduction**

Atmospheric turbulence limits all visible and infrared observations made from the ground, whether they be imaging or astrometric. However, recent advances in adaptive optics and laser guide stars and new techniques in astrometry can be used to circumvent these limits to some extent. While a ground-based interferometer can never exceed the performance of an identical instrument in space, similar performance should eventually be achievable for certain wavelength bands for certain types of imaging and astrometric observations. In fact, the large cost differential between instruments on the ground and instruments in space is such that considerably larger instruments can be built on the ground, and better performance may indeed be achievable from the ground for certain types of observations.

This brief review discusses the atmospheric limitations to imaging and astrometry on the ground using Michelson interferometers in the visible and near-IR, instrument configurations and observation strategies to deal with these limitations, and a brief comparison with space interferometers. More detail on advanced techniques for interferometry is discussed in (Shao & Colavita, 1992a) and references therein.

## 2. Imaging

The effect of atmospheric turbulence on ground-based imaging is twofold: the reduction of the coherence volume (the product of the atmospheric coherence area  $r_0^2$  and the coherence time  $\tau_0$ ) results in a loss of sensitivity, and the corruption of the Michelson fringe phasor usually makes the raw fringe phase unusable for synthesis imaging. Despite working under these limitations, excellent science has been accomplished with existing instruments. However, future high-performance imaging systems must address both of these issues. The sensitivity issue can be addressed by using adaptive optics to phase the individual apertures of the array, essentially increasing  $r_0$  to the aperture diameter, and by using phase referencing to cophase between apertures, essentially increasing  $\tau_0$  to the available integration time. The corruption of the fringe phase can be addressed by the use of closure phase, as in the radio.

However, there are limits to the application of adaptive optics and phase-reference techniques, and to closure phase techniques. For the former, there are limits to the sky coverage achievable at short wavelengths, discussed below; for the latter, there are limits to source brightness and extension due to the properties of the estimators at visible/IR wavelengths. In particular, closure phase is estimated using the bispectrum in order to yield an unbiased estimate. However, the bispectrum is a sixth-order estimator in electric field, and at visible wavelengths the signal-to-noise ratio (SNR) is proportional to  $(\sqrt{NV})^3$  for low SNR, where  $N$  is the number of photons per coherence volume and  $V$  is the fringe visibility. Thus, SNR falls off rapidly for faint objects when the coherence volume is small ( $N \ll 1$ ) or the object is extended ( $V \ll 1$ ). Thus, for successful imaging of extended objects, the coherence volume must be increased such that the SNR per frame is greater than unity.

### 2.1. PHASING

Traditionally, the reference-star problem in adaptive optics has precluded the application of these techniques to observations at short wavelengths. At visible wavelengths without laser guide stars, it would ordinarily be necessary to find a 10-mag star within the several arc second isoplanatic patch about the target in order to yield sufficient photons per  $r_0$  to run the wavefront sensor. However, with the advent of laser-guide-star techniques (cf. Fugate et al. 1991, Primmerman et al. 1991), the laser spot provides the photons for the high-order corrections, while a natural star is needed only for tip-tilt correction; this star, which can be sensed using the whole aperture, can be as faint as 15–18 mag. As coherence area and isoplanatic angle increase rapidly with wavelength, above some cutoff wavelength,  $\sim 1 \mu\text{m}$  for large apertures, full sky coverage is available, with partial coverage or partial correction available at shorter wavelengths. Adaptive optics with natural guide stars, including the partial correction case, as well as laser-guide-star adaptive optics, are discussed in detail elsewhere in this proceedings.

For application to interferometry, it is worth noting that the SNR needed for tilt correction with laser-guide-star adaptive optics is similar to that needed for

fringe tracking with an interferometer. In other words, if you restrict yourself to objects you could fringe track if the apertures were phased, then the object is bright enough to serve as a tilt reference. Said differently, assuming phased pupils, if you can cophase the interferometer, you can phase it as well. Unfortunately, the converse of this is not true, and cophasing is the more difficult problem for interferometry.

## 2.2. COPHASING

Unfortunately, laser-guide-star technology does not seem applicable to cophasing of interferometers. There are several problems. The primary problem is atmospheric reciprocity, which has the effect of making the laser spot appear stationary to the instrument, and thus prevents it from being used as a tilt or cophasing reference. This is the reason a natural tilt reference is needed for laser-guide-star adaptive optics. In addition to this problem, there would be problems with focus anisoplanatism, as well as the achievable brightness of the laser spot, which would be strongly resolved by the interferometer.

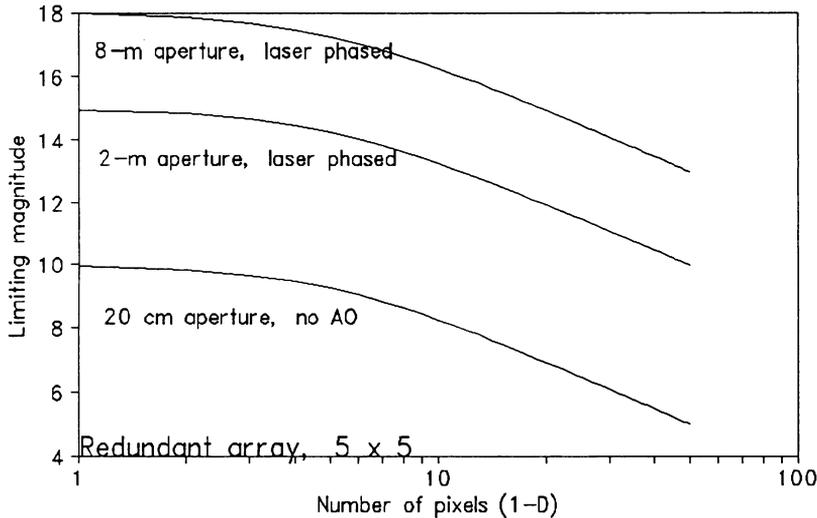
Thus, only a celestial object can be used as a cophasing reference. With a sparse-aperture interferometer, this object must be located within the ordinary isoplanatic patch (not the tilt isoplanatic patch, which is slightly larger), so that the sky coverage for cophasing with phased pupils is less than that achievable with laser-guide-star adaptive optics. However, as with aperture phasing, there is some cutoff wavelength above which full sky coverage is available. The calculations for sky coverage are similar to those for natural-guide-star adaptive optics (cf. Rigaut & Gendron 1992), and an estimate is given below.

wavelength	sky coverage		
	D = 2 m	D = 6 m	D = 10 m
0.6 $\mu\text{m}$	0.1%	1%	3%
1.0 $\mu\text{m}$	0.5%	5%	15%
2.0 $\mu\text{m}$	7%	70%	100%

From the table, it can be seen that significant sky coverage is only available for  $\lambda \lesssim 1.5\text{--}2.0 \mu\text{m}$  with larger apertures. However, in this fully cophased mode, a ground interferometer now has the same basic performance as a space interferometer with optics at the same temperature. Note, that when cophased, closure quantities can be integrated coherently such that the SNR per frame is always greater than unity, and thus there is no sensitivity penalty to measuring the closure phase.

While cophasing for arbitrary fields is possible at long wavelengths, at short wavelengths a cophasing reference for arbitrary fields will not generally be available. In this case there are several options. One is to observe in passive mode, where the bispectrum is integrated in narrow bandwidths set to maintain coherence despite the atmospheric phase fluctuations. However, because of the photon rate and visibility dependence of the bispectrum estimator and the narrow bandwidths required, passive operation exacts a large sensitivity penalty, especially on extended

Fig. 1. Approximate sensitivity of a redundant-aperture interferometer ( $\sim 5 \times 5$ ) for imaging extended objects.



objects. While, similar to partial adaptive optics, there are partial cophasing options which allow the use of wider spectral channels than ordinary passive-mode operation, the highest throughput solution is to cophase on the object itself using a redundant array.

### 2.3. IMAGING WITH REDUNDANT ARRAYS

The basic idea is to design a redundant array such that the longer baselines which resolve the target are spanned by a number of shorter ones which do not (cf. Roddier 1988). Thus, while it is not possible to fringe track directly on the longer baselines because of the low fringe visibility, fringe tracking is still possible on the short baselines over which the object still appears point-like. Thus, by bootstrapping across a number of short baselines, the longer baselines can be approximately cophased. In this case, the situation is as discussed above, where the bispectrum can be coherently integrated until its SNR is greater than unity, so that there is no penalty from the use of the bispectrum rather than the ordinary Michelson phasor. The general behavior of this type of imaging is illustrated in Fig. 1, which for an assumed  $5 \times 5$  redundant array, plots the limiting source magnitude vs. object complexity for several aperture diameters. For compact objects, the limiting magnitude is set by the fringe-tracking limiting magnitude for a point object. However, as the object becomes resolved by the short baselines, the sensitivity degrades rapidly. Thus, redundant arrays are a good solution for moderately bright objects, or those that have a compact core.

## 2.4. IMAGING WITH SPACE INTERFEROMETERS AND CONCLUSIONS

The results of the last few sections can be summarized by comparison with a space interferometer. As limited coherence volume is the main problem with ground-based systems, it is clear that any space interferometer should be phase stable. However, a small phase-stable interferometer will find it hard to compete with a filled-aperture 8–10 m telescope on the ground using laser guide stars at red/near-IR wavelengths. However, the interferometer has advantages at shorter wavelengths where full sky coverage is problematic, and clear advantages in the UV which is unobservable from the ground. For longer baselines and near-to mid-IR wavelengths where full cophasing and phasing is possible on the ground, the space interferometer is not competitive with a ground interferometer unless the apertures are cooled; however, brute-force collecting area on the ground can make up for the some of the cooling advantage if the space apertures are very small. For visible to near-IR wavelengths where general cophasing is not possible on the ground, a long-baseline space interferometer can observe faint, extended objects which are not candidates for self cophasing using redundant arrays.

For the sake of brevity, the above comparison is somewhat simplistic and also doesn't address the reduction to practice of some of the phasing and cophasing concepts for ground interferometers. In addition, there are a few areas of comparison which have not been dealt with. These include high-dynamic range observations using coronagraphic or interferometric techniques to cancel the light (and photon noise) from a bright point source in the field in order to reveal a faint feature or object. Even with adaptive optics and cophasing, the time-varying atmospheric residuals in the wavefront limits the performance of such techniques on the ground. In addition, a space interferometer has no isoplanicity limitations, and in theory large fields can be imaged with a stable PSF (although recent work with HST images has shown the capabilities of non-stationary deconvolution techniques).

## 3. Astrometry

As with imaging, atmospheric turbulence limits the accuracy of all astrometric measurements made from the ground. For the purpose of understanding the astrometric limits from the ground, it's useful to consider three general categories of measurements: (a) wide-angle astrometry (tens of degrees), which is best accomplished with a long-baseline visible or infrared interferometer, (b) narrow-angle astrometry ( $\sim 10$  arcmin), as implemented with long-focus telescopes and CCD or ronchi-ruling back ends, and (c) very-narrow-angle astrometry using a long-baseline IR interferometer. In general, the effects of atmospheric turbulence decrease (nonlinearly) with the size of the field. In addition, over narrow fields, there are significant advantages to long baselines.

### 3.1. WIDE-ANGLE ASTROMETRY

For wide-angle, or absolute astrometry, the atmospheric error is very non-white and is only weakly dependent on integration time ( $t^{-1/6}$ ). With an infinite outer scale,

the error is also independent of baseline length. The limit from the ground is  $\sim 50$ – $100$  milliarcsec (mas) for a 1-min measurement. Repeated measurements decorrelate for separations of order 1 hr, and thus the achievable accuracy for, say, 10 nights of 10 observations per night is  $\sim 5$ – $10$  mas. These numbers are consistent with Mark III observations (Shao et al. 1990). The equivalent visible limiting magnitude for such measurements is  $\sim 10$  mag for a visible-wavelength interferometer, and  $\sim 15$  mag for an IR interferometer; photon noise is not a problem for such bright sources.

At visible wavelengths somewhat higher accuracies are achievable with two-color techniques. The essence of the technique is to observe the fringe position at two widely-separated colors. Because of atmospheric dispersion, the two fringe positions will be different, and the difference will be proportional to the instantaneous atmospheric error, allowing a correction to be made. The process is not significantly different from the use of two-frequency techniques in the radio to correct for ionospheric errors. The achievable gain in astrometric performance is typically a factor of 3–5, limited by water vapor and other effects. Thus,  $\sim 1$  mas yearly accuracies seem to be a fairly hard limit for ground-based wide-angle astrometry.

### 3.2. NARROW-ANGLE ASTROMETRY WITH TELESCOPES

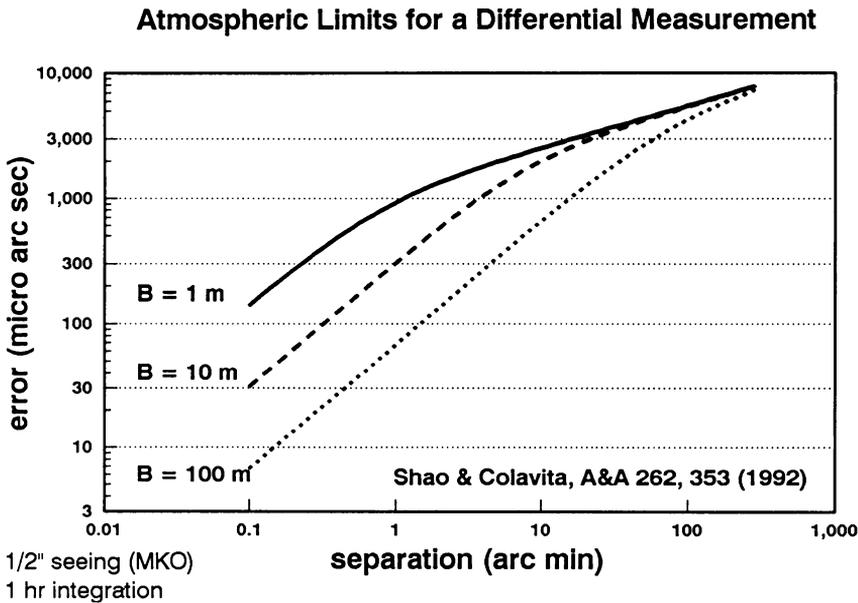
The atmospheric effects are less severe with a differential measurement which measures the difference in the positions of two stars; in particular, the “whitening” effect of a differential measurement leads to an atmospheric error which decreases at the square root of the integration time. The state-of-the-art for differential measurements over fields of  $\sim 10$  arcmin utilizes modest-size telescopes with CCD or ronchi-ruling back ends. For this combination of field and instrument size, the atmospheric error is only weakly dependent on the telescope diameter and the star separation, and the achievable accuracy is  $\sim 1$ – $2$  mas/ $\sqrt{\text{hr}}$ . Photon noise is equal to atmospheric noise at  $\sim 18$  mag for a 1.5 m telescope with a CCD detector.

### 3.3. NARROW-ANGLE INTERFEROMETRIC ASTROMETRY

What is the source of the error in a differential measurement? Qualitatively, light from each star follows a different path through the atmosphere, and it is this deviation from a common path which introduces the error. Consider the case of two stars  $0.5^\circ$  apart: at the top of the turbulent atmosphere,  $h \sim 10$  km, the rays from the two stars are separated by 100 m, which is ordinarily much greater than the baseline length  $B$ . In this regime, the proportionality of the error is as  $\theta^{1/3}$ , where  $\theta$  is the star separation. However, if the field is narrowed or the baseline is increased so that  $\theta h < B$ , then the error behavior changes radically, and is now strongly dependent on both the star separation (as  $\theta$ ) and the baseline length (as  $B^{-2/3}$ ). This behavior is illustrated in Fig. 2, which gives the atmospheric error in 1 hr of integration time for a Mauna Kea turbulence profile.

The error behavior for very small fields suggests a new type of astrometric measurement for detecting exoplanets or other “ac” motions using a long-baseline infrared interferometer (Shao and Colavita 1992b). The basic idea is to employ a dual feed at each aperture of the interferometer and measure the difference in

Fig. 2. Error behavior of a differential astrometric measurement as a function of field angle and baseline length.



delay between the fringes on two stars simultaneously. If the observations are made in the infrared at  $2.2 \mu\text{m}$ , then phase referencing can be used within the  $2.2 \mu\text{m}$  isoplanatic patch (15–20 arcsec) of a relatively bright target to increase sensitivity in order to always find usable nearby reference stars. A detailed example is worked in the above reference; in summary, for a 200-m instrument, the atmospheric limit for stars separated by  $15''$  is  $\sim 10 \mu\text{as}/\sqrt{\text{hr}}$ . The photon noise limit is also  $\sim 10 \mu\text{as}/\sqrt{\text{hr}}$  for stars of typical visual magnitude 20.5; stars of this magnitude should be available within the assumed  $15''$  of the target, which itself is assumed brighter than  $\sim 16 \text{ mag}$ .

#### 3.4. ASTROMETRY WITH SPACE INTERFEROMETERS AND CONCLUSIONS

The approximate capabilities and sensitivities of ground-based visible/near-IR astrometry, discussed above, are summarized below.

Measurement	Instrument	Field	Atmospheric Accuracy	Sensitivity
wide angle	IR interf.	$20^\circ$	$15 \text{ mas}/\sqrt{\text{night}}$	16 mag
wide angle	2-color vis. interf.	$20^\circ$	$5 \text{ mas}/\sqrt{\text{night}}$	10 mag
narrow angle	1.5 m telescope	$10'$	$1\text{--}2 \text{ mas}/\sqrt{\text{hr}}$	18 mag
very narrow angle	dual-object IR interf.	$15''$	$10 \mu\text{as}/\sqrt{\text{hr}}$	16 mag

The comparison with space interferometers is much simpler for astrometry than for imaging. Interferometers outside of the atmosphere are now metrology and

photon-noise limited; proposed space interferometers such as OSI and POINTS could provide accuracies of  $<10 \mu\text{as}$  over wide fields, which is at least of factor of 100 better than ground instruments. Wide-field operation is important for astrophysics in order to be able to view extragalactic references. While ground interferometers can approach the accuracy of space interferometers over very narrow fields, the limited references available within the narrow fields will probably restrict the technique to detecting "ac" motions, such as the wobble of a star attributable to a binary companion or exoplanet. Finally, while laser guide stars and other techniques show significant promise for ground-based imaging, at the present time they do not appear to offer a means of increasing the accuracy wide-angle ground-based astrometry.

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### Discussion:

#### *Westerhout:*

You calculate atmospheric limits of 3 mas/hr, and quote Gatewood. With the Flagstaff 61-inch telescope and a CCD array, we routinely get differential accuracy of 3 mas in one 4 minute exposure over a 3 - 5 arcminute field. Does that agree with your estimates?

#### *Colavita:*

I think there is still reasonable agreement. Gatewood's numbers, as I recall, were for somewhat larger fields than you use: going from 3' to 24' corresponds to a factor of 2 in accuracy. I suspect the difference in seeing between Pittsburg and Flagstaff accounts for the other factor of 2 difference between your result, extrapolated to one hour, and the number I gave. The theoretical values depend critically on the detailed height-dependent turbulence profile, not just the total seeing, so you would expect large site-to-site differences.