PERIODIC SOLUTIONS FOR THE ECCENTRICITY AND INCLINATION FIRST ORDER RESONANCE

MARISA A. NITTO AND WAGNER SESSIN

Depto. de Mecânica do Vôo e Orbital
Instituto Tecnológico de Aeronáutica
12225, São José dos Campos, SP, Brasil

1. Introduction

For the first order resonance, the problem of the motion of two small masses around a primary body can be of three different types: eccentricity, inclination or eccentricity-inclination. The eccentricity type resonance problem has been the subject of several works since Poincaré(1902). The inclination type resonance problem was studied by Greenberg(1973) who used a particular reference system to obtain an integrable auxiliary system. Sessin and Ferraz-Mello(1984) studied the eccentricity type resonance problem considering the eccentricities of the orbits of the two small masses. Sessin(1991) study the inclination type resonance problem for an arbitrary reference system. In this paper we will study a dynamical system that includes both types of resonance. This study is based in the models developed by Sessin and Ferraz-Mello(1984) and Sessin(1991). The resulting system of differential equation is non-integrable; thus, the families of trivial periodic solutions are studied.

2. The Auxiliary System

Consider the dynamical system defined by two planets P_1, P_2 and the Sun with masses m_1, m_2, M, respectively, where $m_i << M (i = 1, 2)$ and commensurable mean motions in the ratio $p + 1 : p$. Only gravitational forces act on this system. The eccentricities and inclinations are considered small. The disturbing function is developed in the classical way up to the first order in the ratio of the masses and second order in the eccentricities and inclinations. P_2 is supposed to be external with respect to P_1 and short-periodic terms are neglected. The system of differential equations that defines the auxiliary system is

$$\frac{d\,(x, y_i, z_i)}{dt} = \frac{\partial F_1}{\partial\,(\theta, \varpi_i, \Omega_i)} \;, \frac{d\,(\theta, \varpi_i, \Omega_i)}{dt} = -\frac{\partial F_1}{\partial\,(x, y_i, z_i)} \;, \quad (i = 1, 2) \qquad (1)$$

where the averaged Hamiltonian developed in the neighbourhood of the exact resonance is (Sessin and Ferraz-Mello,1984)

$$F_1 = A_0 x^2 + A_1 e_1 \cos(\theta + \varpi_1) + A_2 e_2 \cos(\theta + \varpi_2) + A_3 s_1^2 + A_4 s_2^2$$
$$+ A_5 s_1^2 \cos 2(\theta + \Omega_1) + A_6 s_2^2 \cos 2(\theta + \Omega_2)$$
$$+ A_7 s_1 s_2 \cos(2\theta + \Omega_1 + \Omega_2) + A_8 s_1 s_2 \cos(\Omega_1 - \Omega_2) \;, \qquad (2)$$

S. Ferraz-Mello (ed.), Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, 231–232.

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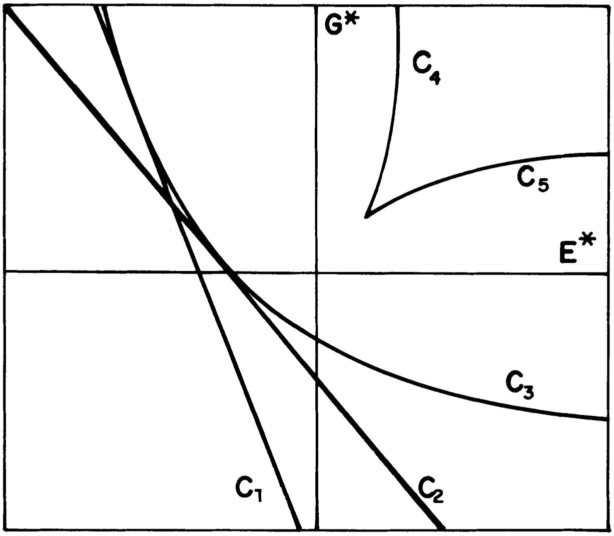

Fig. 1. The Plane (E^*, G^*)

where $e_i = \sqrt{-2y_i}$, $s_i = \sqrt{-2z_i}$, x is related with the semi-major axes, $\theta = p\lambda_1 - (p+1)\lambda_2$ is the critical argument, ϖ_i, Ω_i are the longitude of the pericenters and the longitude of the nodes,respectively, and A_j (i=1,2; j=0,1,...,8) are physical constants. The auxiliary system is reduced to three degree of freedom with two first integrals E and G, and, consequently is non-integrable. Therefore, it is only possible to calculate the families of trivial periodic solutions as done by Sessin and Ferraz-Mello(1984) or Sessin(1991). These families of trivial periodic solutions are determined by the equations defined by the first integrals using normalized constants of integration E^* and G^*, presented in Figure 1. The curves C_1 and C_2 correspond to unstable periodic solutions. The others curves may correspond to unstable or stable periodic solutions depending on the roots of third degree polynomials obtained from the first integrals E^* and G^*.

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