BUCKLING ANALYSIS OF SMART SIZE-DEPENDENT HIGHER ORDER MAGNETO-ELECTRO-THERMO-ELASTIC FUNCTIONALLY GRADED NANOSIZE BEAMS

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ABSTRACT

The present paper examines the thermal buckling of nonlocal magneto-electro-thermo-elastic functionally graded (METE-FG) beams under various types of thermal loading namely uniform, linear and sinusoidal temperature rise and also heat conduction. The material properties of nanobeam are graded in the thickness direction according to the power-law distribution. Based on a higher order beam theory as well as Hamilton's principle, nonlocal governing equations for METE-FG nanobeam are derived and are solved using Navier type method. The small size effect is captured using Eringen's nonlocal elasticity theory. The most beneficial feature of the present beam model is to provide a parabolic variation of the transverse shear strains across the thickness direction and satisfies the zero traction boundary conditions on the top and bottom surfaces of the beam without using shear correction factors. Various numerical examples are presented investigating the influences of thermo-mechanical loadings, magnetic potential, external electric voltage, power-law index, nonlocal parameter and slenderness ratio on thermal buckling behavior of nanobeams made of METE-FG materials.

Keywords: Magneto-electro-thermo-elastic FG nanobeam, Buckling, Nonlocal elasticity theory, Higher order beam theory.

1. INTRODUCTION

The theory of magneto-electro-thermo-elasticity has aroused much interest in many industrial applications, particularly in nuclear device, where there exists a primary magnetic field. In magneto-electro-thermo-elastic (METE) materials, applying heat or a magnetic/electric field results in mechanical deformation due to their unique capability to convert energy among three different forms: Magnetic, electric and mechanical. Several investigations have been performed by considering the interaction between magnetic, thermal and strain fields. Among them, Jiang and Ding [1] presented analytical solutions to study magneto-electro-elastic responses of beams. Chen *et al*. [2] investigated vibrational charactristics of non-homogeneous transversely isotropic magneto-electro-elastic plates. Free vibration of multiphase and layered magneto-electro-elastic beam for $BaTiO_3$-$CoFe_2O_4$ composite is carried out by Annigeri *et al*. [3]. Kumaravel *et al*. [4] researched linear buckling and free vibration behavior of layered and multiphase magneto-electro-elastic (MEE) beam under thermal environment. Applying finite element method, transient dynamic response of multiphase magneto-electro-elastic cantilever beam is presented by Daga *et al*. [5]. Also, Liu and Chang [6] presented a closed form expression for the vibration problem of a transversely isotropic magneto-electro-elastic plate. Razavi and Shooshtari [7] studied nonlinear free vibration of symmetric magneto-electro-elastic laminated rectangular plates. Most recently, Xin and Hu [8] presented semi-analytical solutions for free vibration of layered magneto-electro-elastic beams via three-dimensional elasticity theory.

Recently, various studies in solid mechanics are being performed where the elastic coefficients of materials are no longer constant but they are position-dependent. Therefore, new structural materials such as functionally graded materials (FGMs) as a novel class of advanced composite materials have a non-homogeneous character wherein the composition of each material constituent varies gradually with respect to spatial coordinates. Initially, FGMs were designed as thermal barrier materials for aerospace application

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and fusion reactors, later on, FGMs are developed for military, automotive, biomedical application, semiconductor industry, manufacturing industry and general structural element in thermal environments. These materials are created to provide desirable properties of their individual constituent. For instance, thermal protection structures made of a ceramic/metal functionally graded material show heat and corrosion resistance on the pure ceramic side while maintaining the structural strength and stiffness by the pure metal surface. Magneto-electro-elastic composites are exploited for the construction of magnetic field probes, electric packaging, hydrophones, medical ultrasonic imaging, sensors and actuators [9]. Until now, several investigations are carried out on mechanical responses of structures made from a contribution of magneto-electro-elastic and compositionally graded materials. Pan and Han [9] presented an exact solution for the multilayered rectangular plate made of functionally graded, anisotropic, and linear magneto-electro-elastic materials. Also, Huang *et al*. [10] studied the plane stress problem of generally anisotropic magneto-electro-elastic beams. In another study, three-dimensional static behavior of doubly curved functionally graded (FG) magneto-electro-elastic shells under mechanical load, electric displacement and magnetic flux using an asymptotic approach is investigated by Wu and Tsai [11]. Li *et al.* [12] investigated the problem of a functionally graded, transversely isotropic, magneto-electro-elastic circular plate acted on by a uniform load. Kattimani and Ray [13] investigated active control of geometrically nonlinear vibrations of functionally graded magneto-electro-elastic (FGMEE) plates. Bending of circular magnetoelectroelastic plates with functionally graded material properties using a meshless method is analyzed by Sladek *et al*. [14].

To understand the mechanical behavior of nanostructural elements applied as components NEMS, two kinds of nonlocal models are proposed, *i.e.* nonlocal strengthening model and nonlocal weakening model. Nonlocal strengthening model [15] states that nanostructural stiffness is enhancement with stronger nonlocal effects, while nonlocal weakening model [16-18] asserts an opposite conclusion. These theories are introduced to overcome the defects of classical continuum theory which is unable to describe the size-dependency of structures at nano scales. Therefore, some researchers have studied static and dynamic charactristics of nanobeams by using nonlocal elasticity theories. Li *et al.* [19] showed that both two nonlocal models are correct and are applicable in analysis of nanoscale structures. Based on nonlocal strengthening model, Li *et al*. [20] studied static behavior of ultra-thin beams with nanoscale thickness. Also, Li *et al*. [21] analyzed longitudinal dynamic behaviors of nanorods/nanotubes using the hardening nonlocal approach. Based on the nonlocal weakening model, Şimşek and Yurtcu [22] provided analytical solutions for bending and buckling of FG nanobeams based on the nonlocal Timoshenko beam theory. Rahmani and Jandaghian [23] studied buckling of functionally graded nanobeams based on a nonlocal third-order shear deformation theory. Zemri *et al*. [24] analyzed mechanical responses of FG nanobeams using a refined shear deformation theory. Ebrahimi and Barati [25] studied vibration aalaysis of third-order FG nanobeams. Also, Ebrahimi and Salari [26] conducted thermo-mechanical analysis of FG nanobeams subjected to various thermal loads. It is clear that the effects of magnetic and electric fields are neglected in these works. Free vibration behavior of magneto-electro-elastic (MEE) nanobeams using nonlocal theory and Timoshenko beam theory is studied by Ke and Wang [27]. In this article, it is supposed that the MEE nanobeam is subjected to the external electric potential, magnetic potential and uniform temperature rise. In another study, Ke *et al*. [28] investigated the free vibration behavior of magnetoelectro-elastic (MEE) nanoplates based on the nonlocal theory and Kirchho ff plate theory. Li *et al*. [29]analyzed buckling and free vibration of magnetoelectroelastic nanoplate resting on Pasternak foundation based on nonlocal Mindlin theory. Ansari *et al*. [30] studied forced vibration behavior of higher order shear deformable magneto-electro-thermo elastic (METE) nanobeams based on the nonlocal elasticity theory in conjunction with the von Kármán geometric nonlinearity. Wu *et al*. [31] researched surface effects on anti-plane shear waves propogating in nanoplates made from magneto-electro-elastic materials. According to the literature, there is no work investigating the effects of thermal loading on buckling responses of size-dependent METE-FG nanobeams. Since thermal buckling is an undesirable phenomena [32] which causes instability of structures, there is a strong need to analyze the buckling of METE-FG nanobeams in thermal environments.

This paper investigates thermal buckling of nonlocal magneto-electro-thermo-elastic FG beams by using Eringen's nonlocal elasticity theory. The governing differential equations are derived by implementing Hamilton's principle and also the Navier solution method is adopted solve these stability equations. Magneto-electro-thermo-elastic properties of the FG nanobeams are supposed to be variable through thickness based on power-law model. In the present article, the mentioned task is accomplished, including the following novelties:

1. A higher order parabolic beam theory is used to capture the effect of shear deformation which provides more accurate results than classical beam theory.
2. Four types of thermal loading including uniform, linear and sinusoidal temperature rise as well as heat conduction through the beam thickness are considered.
3. The influences of both magnetic and electric fields are investigated on the stability of smart nanobeams in thermal environments for the first time.

2. THEORETICAL FORMULATIONS

2.1 The Material Properties of METE-FG Nanobeams

Assume a magneto-electro-thermo-elastic function-

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ally graded nanobeam exposed to a magnetic potential $\Upsilon(x, z, t)$ and electric potential $\Phi(x, z, t)$, with length L and uniform thickness h, as shown in Fig. 1 [33]. The effective material properties of the METE-FG nanobeam based on the power-law model can be stated in the following form:

$$P = P_2 V_2 + P_1 V_1 \tag{1}$$

In which P_1 and P_2 denote the material properties of the bottom and higher surfaces, respectively. Also V_1 and V_2 are the corresponding volume fractions related by:

$$V_2 = \left(\frac{z}{h} + \frac{1}{2}\right)^p, \quad V_1 = 1 - V_2 \tag{2}$$

Therefore, according to Eqs. (1) and (2), the effective magneto-electro-elastic material properties of the FG beam is defined as:

$$P(z) = (P_2 - P_1)\left(\frac{z}{h} + \frac{1}{2}\right)^p + P_1 \tag{3}$$

It must be noted that, the top surface at $z = +h/2$ of FG nanobeam is assumed $CoFe_2O_4$ rich, whereas the bottom surface ($z = -h/2$) is $BaTiO_3$ rich.

2.2 Nonlocal Elasticity Theory for the Magneto-Electro-Thermo-Elastic Materials

Contrary to the constitutive equation of classical elasticity theory, Eringen's nonlocal theory notes that the stress state at a point inside a body is regarded to be function of strains of all points in the neighbor regions. For a nonlocal magneto-electro-thermo-elastic solid the basic equations with zero body force may be defined as:

$$\sigma_{ij} = \int_V \alpha(|x' - x|, \tau)\left[C_{ijkl}\varepsilon_{kl}(x') - e_{mij}E_m(x') - q_{nij}H_n(x') - C_{ijkl}\alpha_{kl}\Delta T\right] dV(x') \tag{4a}$$

$$D_i = \int_V \alpha(|x' - x|, \tau)\left[e_{ikl}\varepsilon_{kl}(x') + s_{im}E_m(x') + d_{in}H_n(x') - p_i\,\Delta T\right] dV(x') \tag{4b}$$

$$B_i = \int_V \alpha(|x' - x|, \tau)\left[q_{ikl}\varepsilon_{kl}(x') + d_{im}E_m(x') + \chi_{in}H_n(x') - \lambda_i\,\Delta T\right] dV(x') \tag{4c}$$

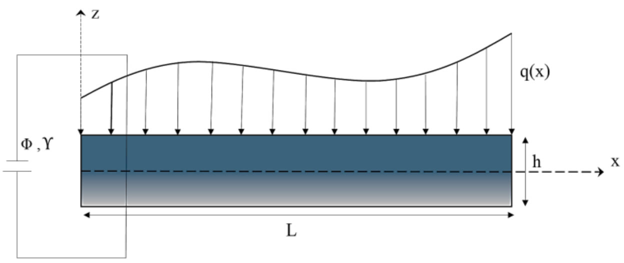

Fig. 1 Configuration of a METE-FG nanobeam.

where σ_{ij}, ε_{ij}, D_i, E_i, B_i and H_i denote the stress, strain, electric displacement, electric field components, magnetic induction and magnetic field and displacement components, respectively; α_{kl} and ΔT are thermal expansion coefficient and temperature change; C_{ijkl}, e_{mij}, s_{im}, q_{nij}, d_{ij}, χ_{ij}, p_i and λ_i are the elastic, piezoelectric, dielectric constants, piezomagnetic, magnetoelectric, magnetic, pyroelectric and pyromagnetic constants, respectively and $\tau = e_0a/l$ is defined as scale coefficient, where e_0 is an experimentally determined material constant and a and l are the internal and external characteristic length of the nanostructures, respectively. Finally it is possible to represent the integral constitutive relations given by Eq. (4) in an equivalent differential form as:

$$\sigma_{ij} - (e_0\,a)^2\nabla^2\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} - e_{mij}\,E_m - q_{nij}\,H_n - C_{ijkl}\,\alpha_{kl}\,\Delta T \tag{5a}$$

$$D_i - (e_0\,a)^2\nabla^2 D_i = e_{ikl}\,\varepsilon_{kl} + s_{im}\,E_m + d_{in}\,H_n - p_i\,\Delta T \tag{5b}$$

$$B_i - (e_0\,a)^2\nabla^2 B_i = q_{ikl}\,\varepsilon_{kl} + d_{im}\,E_m + \chi_{in}\,H_n - \lambda_i\,\Delta T \tag{5c}$$

2.3 Nonlocal Magneto-Electro-Thermo-Elastic FG Nanobeam Model

Based on third order beam theory, the displacement field at any point of the beam are supposed to be in the form:

$$u_x(x, z) = u(x) + z\psi(x) - \alpha z^3\left(\psi + \frac{\partial w}{\partial x}\right) \tag{6a}$$

$$u_z(x, z) = w(x) \tag{6b}$$

in which $\alpha = 4/3h^2$ and u and w are displacement components in the mid-plane along the coordinates x and z, respectively, while ψ denotes the total bending rotation of the cross-section. To satisfy Maxwell's equation, the distribution of electric and magnetic potential along the thickness direction is supposed to change as a combination of a cosine and linear variation as follows:

$$\Phi(x, z, t) = -\cos(\xi z)\,\phi(x, t) + \frac{2z}{h}V \tag{7a}$$

$$\Upsilon(x, z, t) = -\cos(\xi z)\gamma(x, t) + \frac{2z}{h}\Omega \tag{7b}$$

where $\xi = \pi/h$. Also, V and Ω are the initial external electric voltage and magnetic potential applied to the FG nanobeam. The non-zero strains can be stated as:

$$\varepsilon_{xx} = \varepsilon_{xx}^{(0)} + z\varepsilon_{xx}^{(1)} + z^3\varepsilon_{xx}^{(3)} \tag{8a}$$

$$\gamma_{xz} = \gamma_{xz}^{(0)} + z^2\gamma_{xz}^{(2)} \tag{8b}$$

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where

$$\varepsilon_{xx}^{(0)}=\frac{\partial u}{\partial x},\ \varepsilon_{xx}^{(1)}=\frac{\partial \psi}{\partial x},\ \varepsilon_{xx}^{(3)}=-\alpha\left(\frac{\partial \psi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right) \quad (9a)$$

$$\gamma_{xz}^{(0)}=\frac{\partial w}{\partial x}+\psi,\ \gamma_{xz}^{(2)}=-\beta\left(\frac{\partial w}{\partial x}+\psi\right) \quad (9b)$$

And $\beta=4/h^2$. Also, the components of electric and magnetic field (E_x, E_z, H_x, H_z) can be obtained as:

$$E_x=-\Phi_{,x}=\cos(\xi z)\frac{\partial \phi}{\partial x},$$
$$E_z=-\Phi_{,z}=-\xi\sin(\xi z)\,\phi-\frac{2V}{h} \quad (10a)$$

$$H_x=-\Upsilon_{,x}=\cos(\xi z)\frac{\partial \gamma}{\partial x},$$
$$H_z=-\Upsilon_{,z}=-\xi\sin(\xi z)\,\gamma-\frac{2\Omega}{h} \quad (10b)$$

The Hamilton's principle can be stated in the following form to obtain the governing equations:

$$\int_0^t \delta(\Pi_S+\Pi_W)\,dt=0 \quad (11)$$

where Π_S is strain energy and Π_W is work done by external applied forces. The first variation of strain energy Π_S can be calculated as:

$$\delta\Pi_S=\int_0^L\int_{-h/2}^{h/2}(\sigma_{xx}\,\delta\varepsilon_{xx}+\sigma_{xz}\,\delta\gamma_{xz}-D_x\,\delta E_x$$
$$-D_z\,\delta E_z-B_x\,\delta H_x-B_z\,\delta H_z)\,dzdx \quad (12)$$

Substituting Eqs. (8) and (9) into Eq. (12) yields:

$$\delta\Pi_S==\int_0^L(N\delta\varepsilon_{xx}^{(0)}+M\delta\varepsilon_{xx}^{(1)}+P\delta\varepsilon_{xx}^{(3)}$$
$$+Q\delta\gamma_{xz}^{(0)}+R\delta\gamma_{xz}^{(2)})\,dx$$
$$+\int_0^L\int_{-h/2}^{h/2}\left(-D_x\cos(\xi z)\,\delta\left(\frac{\partial\phi}{\partial x}\right)\right.$$
$$+D_z\xi\sin(\xi z)\,\delta\phi-B_x\cos(\xi z)\,\delta\left(\frac{\partial\gamma}{\partial x}\right)$$
$$\left.+B_z\xi\sin(\xi z)\,\delta\gamma\right)dzdx \quad (13)$$

in which N, M and Q are the axial force, bending moment and shear force resultants, respectively. Relations between the stress resultants and stress component used in Eq. (13) are defined as:

$$N=\int_A\sigma_{xx}\,dA,\ M=\int_A\sigma_{xx}z\,dA,\ P=\int_A\sigma_{xx}z^3dA$$
$$Q=\int_A\sigma_{xz}\,dA,\ R=\int_A\sigma_{xz}z^2\,dA \quad (14)$$

The work done due to external electric voltage, Π_W, can be written in the form:

$$\Pi_W=\int_0^L(N_H+N_E+N_T)\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial x}+q\,\delta w+f\,\delta u$$
$$\left.-N\delta\varepsilon_{xx}{}^{(0)}-\hat{M}\frac{\partial\delta\psi}{\partial x}+\alpha P\frac{\partial^2\delta w}{\partial x^2}-\hat{Q}\delta\gamma_{xz}{}^{(0)}\right)dx \quad (15)$$

where $\hat{M}=M-\alpha P$, $\hat{Q}=Q-\beta R$ and $q(x)$ and $f(x)$ are the transverse and axial distributed loads and also N_T, N_H and N_E are the normal forces induced by temperature changes, magnetic potential and electric voltage, respectively which are defined as:

$$N_E=-\int_{-h/2}^{h/2}e_{31}\frac{2V}{h}dz,\ N_H=-\int_{-h/2}^{h/2}q_{31}\frac{2\Omega}{h}dz,$$
$$N_T=\int_{-h/2}^{h/2}c_{11}\alpha_1(T-T_0)\,dz \quad (16)$$

For a magneto-electro-thermo-elastic FGM nanobeam in the one dimensional case, the nonlocal constitutive relations (5a) ~ (5c) may be rewritten as:

$$\sigma_{xx}-(e_0a)^2\frac{\partial^2\sigma_{xx}}{\partial x^2}=c_{11}\,\varepsilon_{xx}-e_{31}E_z-q_{31}H_z-c_{11}\,\alpha_1\Delta T \quad (17)$$

$$\sigma_{xz}-(e_0a)^2\frac{\partial^2\sigma_{xz}}{\partial x^2}=c_{55}\gamma_{xz}-e_{15}E_x-q_{15}H_x \quad (18)$$

$$D_x-(e_0a)^2\frac{\partial^2 D_x}{\partial x^2}=e_{15}\gamma_{xz}+s_{11}E_x+d_{11}H_x \quad (19)$$

$$D_z-(e_0a)^2\frac{\partial^2 D_z}{\partial x^2}=e_{31}\varepsilon_{xx}+s_{33}E_z+d_{33}H_z+p_3\Delta T \quad (20)$$

$$B_x-(e_0a)^2\frac{\partial^2 B_x}{\partial x^2}=q_{15}\gamma_{xz}+d_{11}E_x+\chi_{11}H_x \quad (21)$$

$$B_z-(e_0a)^2\frac{\partial^2 B_z}{\partial x^2}=q_{31}\varepsilon_{xx}+d_{33}E_z+\chi_{33}H_z+\lambda_3\Delta T \quad (22)$$

Inserting Eqs. (13) and (15) in Eq. (11) and integrating by parts, and gathering the coefficients of δu, δw, $\delta\psi$, $\delta\phi$ and $\delta\gamma$ the following governing equations are obtained:

$$\frac{\partial N}{\partial x}+f=0 \quad (23)$$

$$\frac{\partial\hat{M}}{\partial x}-\hat{Q}=0 \quad (24)$$

$$\frac{\partial\hat{Q}}{\partial x}+q-(N_H+N_E+N_T)\frac{\partial^2 w}{\partial x^2}+\alpha\frac{\partial^2 P}{\partial x^2}=0 \quad (25)$$

$$\int_{-h/2}^{h/2}\left(\cos(\xi z)\frac{\partial D_x}{\partial x}+\xi\sin(\xi z)D_z\right)dz=0 \quad (26)$$

$$\int_{-h/2}^{h/2}\left(\cos(\xi z)\frac{\partial B_x}{\partial x}+\xi\sin(\xi z)B_z\right)dz=0 \quad (27)$$

By integrating Eqs. (17) ~ (22), over the beam's cross-section area, the force-strain and the moment-strain of the nonlocal third order beam theory can be obtained as follows:

$$N-\mu\frac{\partial^2 N}{\partial x^2}=A_{xx}\frac{\partial u}{\partial x}+B_{xx}\frac{\partial\psi}{\partial x} -\alpha E_{xx}\left(\frac{\partial\psi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+A_{31}^e\phi+A_{31}^m\gamma-N_E-N_H-N_T \quad (28)$$

$$M-\mu\frac{\partial^2 M}{\partial x^2}=B_{xx}\frac{\partial u}{\partial x}+D_{xx}\frac{\partial\psi}{\partial x} -\alpha F_{xx}\left(\frac{\partial\psi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+E_{31}^e\phi+E_{31}^m\gamma \quad (29)$$

$$P-\mu\frac{\partial^2 P}{\partial x^2}=E_{xx}\frac{\partial u}{\partial x}+F_{xx}\frac{\partial\psi}{\partial x}-\alpha H_{xx}\left(\frac{\partial\psi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right) +F_{31}^e\phi+F_{31}^m\gamma \quad (30)$$

$$Q-\mu\frac{\partial^2 Q}{\partial x^2}=(A_{xz}-\beta D_{xz})\left(\frac{\partial w}{\partial x}+\psi\right)-E_{15}^e\frac{\partial\phi}{\partial x}-E_{15}^m\frac{\partial\gamma}{\partial x} \quad (31)$$

$$R-\mu\frac{\partial^2 R}{\partial x^2}=(D_{xz}-\beta F_{xz})\left(\frac{\partial w}{\partial x}+\psi\right)-F_{15}^m\frac{\partial\phi}{\partial x}-F_{15}^m\frac{\partial\gamma}{\partial x} \quad (32)$$

$$\int_{-h/2}^{h/2}\left\{D_x-\mu\frac{\partial^2 D_x}{\partial x^2}\right\}\cos(\xi z)\,dz =(E_{15}^e-\beta F_{15}^e)\left(\frac{\partial w}{\partial x}+\psi\right)+F_{11}^e\frac{\partial\phi}{\partial x}+F_{11}^m\frac{\partial\gamma}{\partial x} \quad (33)$$

$$\int_{-h/2}^{h/2}\left\{D_z-\mu\frac{\partial^2 D_z}{\partial x^2}\right\}\xi\sin(\xi z)\,dz=A_{31}^e\frac{\partial u}{\partial x} +(E_{31}^e-\alpha F_{31}^e)\frac{\partial\psi}{\partial x}-\alpha F_{31}^e\frac{\partial^2 w}{\partial x^2}-F_{33}^e\phi-F_{33}^m\gamma \quad (34)$$

$$\int_{-h/2}^{h/2}\left\{B_x-\mu\frac{\partial^2 B_x}{\partial x^2}\right\}\cos(\xi z)\,dz =(E_{15}^m-\beta F_{15}^m)\left(\frac{\partial w}{\partial x}+\psi\right)+F_{11}^m\frac{\partial\phi}{\partial x}+X_{11}^m\frac{\partial\gamma}{\partial x} \quad (35)$$

$$\int_{-h/2}^{h/2}\left\{B_z-\mu\frac{\partial^2 B_z}{\partial x^2}\right\}\xi\sin(\xi z)\,dz=A_{31}^m\frac{\partial u}{\partial x} +(E_{31}^m-\alpha F_{31}^m)\frac{\partial\psi}{\partial x}-\alpha F_{31}^m\frac{\partial^2 w}{\partial x^2}-F_{33}^m\phi-X_{33}^m\gamma \quad (36)$$

where $\mu=(e_0a)^2$ and quantities used in above equations are defined as:

$$\{A_{xx},B_{xx},D_{xx},E_{xx},F_{xx},H_{xx}\} =\int_{-h/2}^{h/2}c_{11}\{1,z,z^2,z^3,z^4,z^6\}\,dz \quad (37)$$

$$\{A_{xz},D_{xz},F_{xz}\}=\int_{-h/2}^{h/2}c_{55}\{1,z^2,z^4\}\,dz \quad (38)$$

$$\{A_{31}^e,E_{31}^e,F_{31}^e\}=\int_{-h/2}^{h/2}e_{31}\xi\sin(\xi z)\{1,\ z,z^3\}\,dz \quad (39)$$

$$\{E_{15}^e,F_{15}^e\}=\int_{-h/2}^{h/2}e_{15}\cos(\xi z)\{1,\ z^2\}\,dz \quad (40)$$

$$\{F_{11}^e,F_{33}^e\}=\int_{-h/2}^{h/2}\left\{s_{11}\cos^2(\xi z)\ ,\ s_{33}\xi^2\sin^2(\xi z)\right\}dz \quad (41)$$

$$\{A_{31}^m,E_{31}^m,F_{31}^m\}=\int_{-h/2}^{h/2}q_{31}\xi\sin(\xi z)\{1,\ z,z^3\}\,dz \quad (42)$$

$$\{E_{15}^m,F_{15}^m\}=\int_{-h/2}^{h/2}q_{15}\cos(\xi z)\{1,\ z^2\}\,dz \quad (43)$$

$$\{F_{11}^m,F_{33}^m\}=\int_{-h/2}^{h/2}\left\{d_{11}\cos^2(\xi z),\ d_{33}\xi^2\sin^2(\xi z)\right\}dz \quad (44)$$

$$\{X_{11}^m,X_{33}^m\}=\int_{-h/2}^{h/2}\left\{\chi_{11}\cos^2(\xi z)\ ,\ \chi_{33}\xi^2\sin^2(\xi z)\right\}dz \quad (45)$$

The explicit relation of the nonlocal normal force, bending moment and shear force can be derived by substituting for their second derivative from Eqs. (23) ~ (25) into Eqs. (28) ~ (31) as follows:

$$N=A_{xx}\frac{\partial u}{\partial x}+K_{xx}\frac{\partial\psi}{\partial x}-\alpha E_{xx}\frac{\partial^2 w}{\partial x^2}+A_{31}^e\phi +A_{31}^m\gamma+\mu\left(-\frac{\partial f}{\partial x}\right)-N_E-N_H-N_T \quad (46)$$

$$\hat{M}=K_{xx}\frac{\partial u}{\partial x}+I_{xx}\frac{\partial\psi}{\partial x}-\alpha J_{xx}\left(\frac{\partial\psi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right) +(E_{31}^e-\alpha F_{31}^e)\,\phi+(E_{31}^m-\alpha F_{31}^m)\,\gamma +\mu\left(-\alpha\frac{\partial^2 P}{\partial x^2}-q+\frac{\partial}{\partial x}(N_E+N_H+N_T)\frac{\partial w}{\partial x}\right) \quad (47)$$

$$\hat{Q}=\bar{A}_{xz}\left(\frac{\partial w}{\partial x}+\psi\right)-(E_{15}^e-\beta F_{15}^e)\frac{\partial\phi}{\partial x} +\mu\left((N_H+N_E+N_T)\frac{\partial^3 w}{\partial x^3}-\alpha\frac{\partial^3 P}{\partial x^3}-\frac{\partial q}{\partial x}-(E_{15}^m-\beta F_{15}^m)\frac{\partial\gamma}{\partial x}\right) \quad (48)$$

where

$$K_{xx}=B_{xx}-\alpha E_{xx}\ ,\ I_{xx}=D_{xx}-\alpha F_{xx}, \quad J_{xx}=F_{xx}-\alpha H_{xx} \quad (49)$$

and

$$\bar{A}_{xz}=A^*_{xz}-\beta I^*_{xz}\ ,\ A^*_{xz}=A_{xz}-\beta D_{xz}\ , \quad I^*_{xz}=D_{xz}-\beta F_{xz} \quad (50)$$

Finally, based on third-order beam theory, the nonlocal equations of motion for a magneto-electro-elastic FG nanobeam can be obtained by substituting for N, $\hat{M}$ and $\hat{Q}$ from Eqs. (46) ~ (48) into Eqs. (23) ~ (25) as follows:

$$A_{xx}\frac{\partial^2 u}{\partial x^2}+K_{xx}\frac{\partial^2 \psi}{\partial x^2}-\alpha E_{xx}\frac{\partial^3 w}{\partial x^3}+A_{31}^{e}\frac{\partial \phi}{\partial x}+A_{31}^{m}\frac{\partial \Upsilon}{\partial x}+\mu\left(-\frac{\partial^2 f}{\partial x^2}\right)+f=0 \tag{51}$$

$$K_{xx}\frac{\partial^2 u}{\partial x^2}+I_{xx}\frac{\partial^2 \psi}{\partial x^2}-\alpha J_{xx}\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^3 w}{\partial x^3}\right)-\overline{A}_{xz}\left(\psi+\frac{\partial w}{\partial x}\right)+(E_{31}^{e}-\alpha F_{31}^{e})\frac{\partial \phi}{\partial x}+(E_{31}^{m}-\alpha F_{31}^{m})\frac{\partial \gamma}{\partial x}+(E_{15}^{e}-\beta F_{15}^{e})\frac{\partial \phi}{\partial x}+(E_{15}^{m}-\beta F_{15}^{m})\frac{\partial \gamma}{\partial x}=0 \tag{52}$$

$$\overline{A}_{xz}\left(\frac{\partial \psi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+\mu\left((N_E+N_H+N_T)\frac{\partial^4 w}{\partial x^4}-\frac{\partial^2 q}{\partial x^2}\right)-(N_E+N_H+N_T)\frac{\partial^2 w}{\partial x^2}-(E_{15}^{e}-\beta F_{15}^{e})\frac{\partial^2 \phi}{\partial x^2}-(E_{15}^{m}-\beta F_{15}^{m})\frac{\partial^2 \gamma}{\partial x^2}+q+\alpha\left(E_{xx}\frac{\partial^3 u}{\partial x^3}+J_{xx}\frac{\partial^3 \psi}{\partial x^3}-\alpha H_{xx}\frac{\partial^4 w}{\partial x^4}+F_{31}^{e}\frac{\partial^2 \phi}{\partial x^2}+F_{31}^{m}\frac{\partial^2 \gamma}{\partial x^2}\right)=0 \tag{53}$$

$$(E_{15}^{e}-\beta F_{15}^{e})\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial \psi}{\partial x}\right)+F_{11}^{e}\frac{\partial^2 \phi}{\partial x^2}+F_{11}^{m}\frac{\partial^2 \gamma}{\partial x^2}+A_{31}^{e}\frac{\partial u}{\partial x}+(E_{31}^{e}-\alpha F_{31}^{e})\frac{\partial \psi}{\partial x}-\alpha F_{31}^{e}\frac{\partial^2 w}{\partial x^2}-F_{33}^{e}\phi-F_{33}^{m}\gamma=0 \tag{54}$$

$$(E_{15}^{m}-\beta F_{15}^{m})\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial \psi}{\partial x}\right)+F_{11}^{m}\frac{\partial^2 \phi}{\partial x^2}+X_{11}^{m}\frac{\partial^2 \gamma}{\partial x^2}+A_{31}^{m}\frac{\partial u}{\partial x}+(E_{31}^{m}-\alpha F_{31}^{m})\frac{\partial \psi}{\partial x}-\alpha F_{31}^{m}\frac{\partial^2 w}{\partial x^2}-F_{33}^{m}\phi-X_{33}^{m}\gamma=0 \tag{55}$$

3. SOLUTION PROCEDURE

Here, on the basis of the Navier method, an analytical solution of the governing equations for buckling of a simply supported magneto-electro-elastic FG nanobeam is presented. To satisfy governing equations of motion, the displacement variables are adopted to be of the form:

$$(u,\psi)=\sum_{n=1}^{\infty}(U_n,\Psi_n)\cos\left(\frac{n\pi}{L}x\right) \tag{56}$$

$$(w,\phi,\gamma)=\sum_{n=1}^{\infty}(W_n,\Phi_n,\Upsilon_n)\sin\left(\frac{n\pi}{L}x\right) \tag{57}$$

where U_n, W_n, Ψ_n, Φ_n and Υ_n are the unknown Fourier coefficients. Using Eqs. (56) ~ (57) the analytical solution can be obtained from the following equations:

$$[K]_{5*5}\{d\}=0 \tag{58}$$

where

$$k_{1,1}=-A_{xx}\left(\frac{n\pi}{L}\right)^2,\quad k_{1,2}=-K_{xx}\left(\frac{n\pi}{L}\right)^2,$$

$$k_{1,3}=\alpha E_{xx}\left(\frac{n\pi}{L}\right)^3,$$

$$k_{1,4}=-A_{31}^{e}\left(\frac{n\pi}{L}\right),\quad k_{1,5}=-A_{31}^{m}\left(\frac{n\pi}{L}\right),$$

$$k_{2,2}=-I_{xx}\left(\frac{n\pi}{L}\right)^2+\alpha J_{xx}\left(\frac{n\pi}{L}\right)^2-\overline{A}_{xz},$$

$$k_{2,3}=-\overline{A}_{xz}\left(\frac{n\pi}{L}\right)+J_{xx}\left(\frac{n\pi}{L}\right)^3,$$

$$k_{2,4}=-\left((E_{15}^{e}-\beta F_{15}^{e})+(E_{31}^{e}-\alpha F_{31}^{e})\right)\left(\frac{n\pi}{L}\right),$$

$$k_{2,5}=-\left((E_{15}^{m}-\beta F_{15}^{m})+(E_{31}^{m}-\alpha F_{31}^{m})\right)\left(\frac{n\pi}{L}\right),$$

$$k_{3,5}=-\left((E_{15}^{m}-\beta F_{15}^{m})-\alpha F_{31}^{m}\right)\left(\frac{n\pi}{L}\right)^2$$

$$k_{3,3}=(N_H+N_E+N_T)\left(\frac{n\pi}{L}\right)^2\left(1+\mu\left(\frac{n\pi}{L}\right)^2\right)-\overline{A}_{xz}\left(\frac{n\pi}{L}\right)^2-\alpha^2 H_{xx}\left(\frac{n\pi}{L}\right)^4$$

$$k_{3,4}=-\left((E_{15}^{e}-\beta F_{15}^{e})-\alpha F_{31}^{e}\right)\left(\frac{n\pi}{L}\right)^2,$$

$$k_{4,4}=-\left(F_{11}^{e}\left(\frac{n\pi}{L}\right)^2+F_{33}^{e}\right),$$

$$k_{5,5}=-\left(F_{11}^{m}\left(\frac{n\pi}{L}\right)^2+F_{33}^{m}\right),\quad d=\{U_n,\Psi_n,W_n,\Phi_n,\Upsilon_n\}^T$$

4. TYPES OF THERMAL LOADING

4.1 Uniform Temperature Rise (UTR)

For a FG nanobeam at reference temperature T_0 the temperature is uniformly raised to a final value T which the temperature change is $\Delta T = T - T_0$.

4.2 Linear Temperature Rise (LTR)

For a FG nanobeam for which the beam thickness is thin enough, the temperature distribution is assumed to be varied linearly through the thickness as follows [25]:

$$T=T_1+\Delta T\left(\frac{1}{2}+\frac{z}{h}\right) \tag{59}$$

where the buckling temperature difference is $\Delta T = T_2 - T_1$ and T_2 and T_1 are the temperature of the top surface and the bottom surface, respectively.

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4.3 Heat Conduction (HC)

The one-dimensional temperature distribution through-the-thickness can be obtained by solving the steady-state heat conduction equation with the boundary conditions on bottom and top surfaces of the beam across the thickness [25]:

$$-\frac{d}{dz}\left(\kappa(z,T)\frac{dT}{dz}\right)=0\,,\quad T\left(\frac{h}{2}\right)=T_2\,,\ T\left(-\frac{h}{2}\right)=T_1 \tag{60}$$

where κ is heat conductivity coefficient. The solution of above equation is:

$$T=T_1+\Delta T\frac{\int_{-\frac{h}{2}}^{z}\frac{1}{\kappa(z,T)}dz}{\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{1}{\kappa(z,T)}dz} \tag{61}$$

4.4 Sinusoidal Temperature Rise (STR)

The temperature field when METE-FG nanobeam is subjected to sinusoidal temperature rise across the thickness can be defined as [34]:

$$T=T_1+\Delta T\left(1-\cos\frac{\pi}{2}\left(\frac{1}{2}+\frac{z}{h}\right)\right) \tag{62}$$

5. RESULTS AND DISCUSSION

The influences of different thermal environments, magnetic and electric fields on the buckling of METE-FG nanobeams with material properties listed in Table 1 are evaluated in the present study.

Due to the reason that the present results are the first published results for the METE-FG nanobeams, the present results are verified by results of nonlocal FG Reddy beams presented by Rahmani and Jandaghian [23] and the results are presented in Table 2. The beam geometry has the following dimensions: L (length) = 10000nm and h (thickness) = varied. Also, it is supposed that the temperature rise in lower surface to reference temperature T_0 of the beam is $T_1 - T_0 = 5$K.

The influences of different parameters such as various temperature rise (UTR, LTR, HC, and STR), magnetic potential (Ω), external electric voltage (V), nonlocal parameter (μ) and gradient index on critical buckling temperature are tabulated in Table 3. For all magnetic potentials and external voltages due to the softening influence of nonlocal parameter and gradient index on the beam structure increasing the their values leads to reduction in ΔT_{cr}. Also, positive/negative values of magnetic potential/electric voltage produce larger buckling temperature (ΔT_{cr}) than negative/positive ones. Moreover, it is observable that positive values of magnetic potential show an increasing influence on critical buckling temperatures, whereas the negative ones have a reducing impact. This is due to the reason that compressive and tensile in-plane forces are generated in the nanobeam when positive and negative magnetic potentials are applied, respectively.

Table 1 Magneto-electro-thermo-elastic coefficients [33,35].

Properties	$BaTiO_3$	$CoFe_2O_4$
c_{11} (GPa)	166	286
c_{55}	43	45.3
e_{31} (Cm^{-2})	–4.4	0
e_{15}	11.6	0
q_{31} (N/Am)	0	580.3
q_{15}	0	550
s_{11} ($10^{-9}C^2m^{-2}N^{-1}$)	11.2	0.08
s_{33}	12.6	0.093
χ_{11} ($10^{-6}Ns^2C^{-2}/2$)	5	–590
χ_{33}	10	157
$d_{11} = q_{33}$	0	0
α_1 (10^{-6}1/K)	15.7	10
κ (W/mK)	3.2	2.5

Table 2 Comparison of the non-dimensional buckling load for a S-S FG nanobeam with various power-law index ($L/h = 20$).

p	Nonlocal parameter							
	$\mu = 1$		$\mu = 2$		$\mu = 3$		$\mu = 4$	
	RBT [23]	Present	RBT [23]	Present	RBT [23]	Present	RBT [23]	Present
0	8.9258	8.925759	8.1900	8.190046	7.5663	7.566381	7.0309	7.030978
0.1	9.7778	9.777865	8.9719	8.971916	8.2887	8.288712	7.7021	7.702196
0.2	10.3898	10.389845	9.5334	9.533453	8.8074	8.807489	8.1842	8.184264
0.5	11.4944	11.494448	10.5470	10.547009	9.7438	9.743863	9.0543	9.054379
1	12.3709	12.370918	11.3512	11.351234	10.4869	10.486847	9.7447	9.744790
2	13.1748	13.174885	12.0889	12.088934	11.1683	11.168372	10.3781	10.378089
5	14.2363	14.236343	13.0629	13.062900	12.0682	12.068171	11.2142	11.214218

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Table 3 Variation of the critical buckling temperature of S-S FG nanobeam under various temperature rise for various nonlocal parameter, magnetic potential and electric voltage (L/h = 10).

μ (nm^2)			Ω = −0.05			Ω = 0			Ω = +0.05		
			p = 0.2	p = 1	p = 5	p = 0.2	p = 1	p = 5	p = 0.2	p = 1	p = 5
0	V = −5	UTR	689.586	610.029	589.889	706.266	620.220	593.471	722.946	630.411	597.052
		LTR	1374.09	1192.34	1144.42	1407.57	1212.43	1151.43	1441.05	1232.51	1158.44
		HC	1350.18	1145.61	1112.08	1383.08	1164.91	1118.89	1415.97	1184.20	1125.70
		STR	1894.62	1635.13	1555.60	1940.78	1662.67	1565.12	1986.94	1690.21	1574.65
	V = 0	UTR	687.056	602.302	576.310	703.736	612.493	579.892	720.417	622.684	583.474
		LTR	1369.02	1177.11	1117.86	1402.50	1197.20	1124.86	1435.98	1217.28	1131.87
		HC	1345.19	1130.98	1086.26	1378.09	1150.27	1093.07	1410.99	1169.57	1099.88
		STR	1887.62	1614.24	1519.48	1933.78	1641.79	1529.01	1979.94	1669.33	1538.54
	V = +5	UTR	684.527	594.574	562.732	701.207	604.766	566.314	717.887	614.957	569.895
		LTR	1363.94	1161.89	1091.29	1397.42	1181.97	1098.29	1430.90	1202.05	1105.30
		HC	1340.20	1116.35	1060.45	1373.10	1135.64	1067.26	1406.00	1154.94	1074.07
		STR	1880.62	1593.36	1483.37	1926.78	1620.90	1492.90	1972.94	1648.44	1502.42
2	V = −5	UTR	509.058	494.293	590.254	519.249	497.875	606.934	529.440	501.456	509.058
		LTR	1141.23	993.358	957.376	1174.71	1013.44	964.384	1208.20	1033.53	971.392
		HC	954.424	930.319	1154.27	973.720	937.129	1187.17	993.017	943.939	954.424
		STR	1573.55	1362.25	1301.35	1619.71	1389.79	1310.87	1665.88	1417.33	1320.40
	V = 0	UTR	501.331	480.714	587.724	511.522	484.296	604.405	21.7130	487.878	501.331
		LTR	1136.16	978.130	930.807	1169.64	998.214	937.815	1203.12	1018.30	944.823
		HC	1116.38	939.792	904.501	1149.28	959.089	911.311	1182.18	978.386	918.121
		STR	1566.55	1341.36	1265.23	1612.71	1368.91	1274.76	1658.88	1396.45	1284.28
	V = +5	UTR	493.604	467.136	585.195	503.795	470.717	601.875	513.986	474.299	493.604
		LTR	1131.08	962.902	904.238	1164.56	982.986	911.246	1198.04	1003.07	918.254
		HC	1111.40	925.161	878.683	1144.29	944.458	885.493	1177.19	963.754	892.303
		STR	1559.55	1320.48	1229.12	1605.71	1348.02	1238.64	1651.88	1375.57	1248.17

The influence of thermal loading type on the variations of the critical temperature of METE-FG nanobeams versus power-law index at L/h = 10, μ = 2, V = +5 and Ω = +0.1 is plotted in Fig. 2. As one can see for all kind of thermal loads the critical buckling temperature decreases when the gradient index rises, especially for lower values of graient index. Also, comparing the results of these four temperature fields reveals that sinusoidal temperature rise (STR) provides larger values of ΔT_{cr} than UTR, LTR and HC, while UTR presents the lower values for critical temperature. Also, the results predicted according to linear temperature rise and heat conduction are close together, but more exactly the HC results are a little less than those of LTR.

To illustrate the influence of the small scale parameter on the thermal buckling responses, Fig. 3 presents the variations of the critical temperature difference of METE-FG nanobeams for various magnetic potentials at slenderness ratio L/h = 10, p = 1 and V = +5. It is clearly observable that, for all kinds of thermal loadings the nonlocal parameter diminishes the rigidity of nanostructures so that it reduces the critical buckling temperatures.

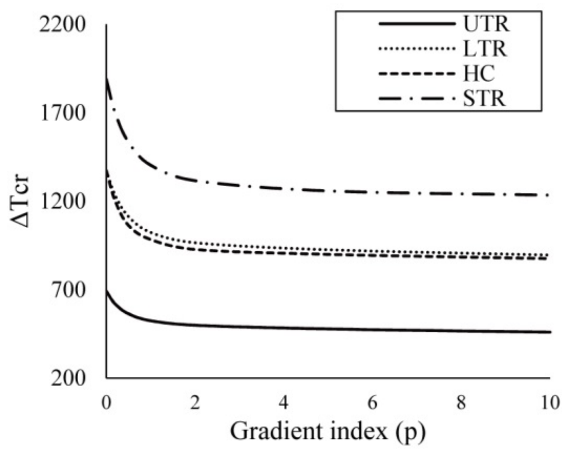

Fig. 2 Effect of thermal loading on the critical buckling temperature of the S-S FG nanobeam with respect to gradient index (L/h = 10, μ = 2, V = +5, Ω = +0.1).

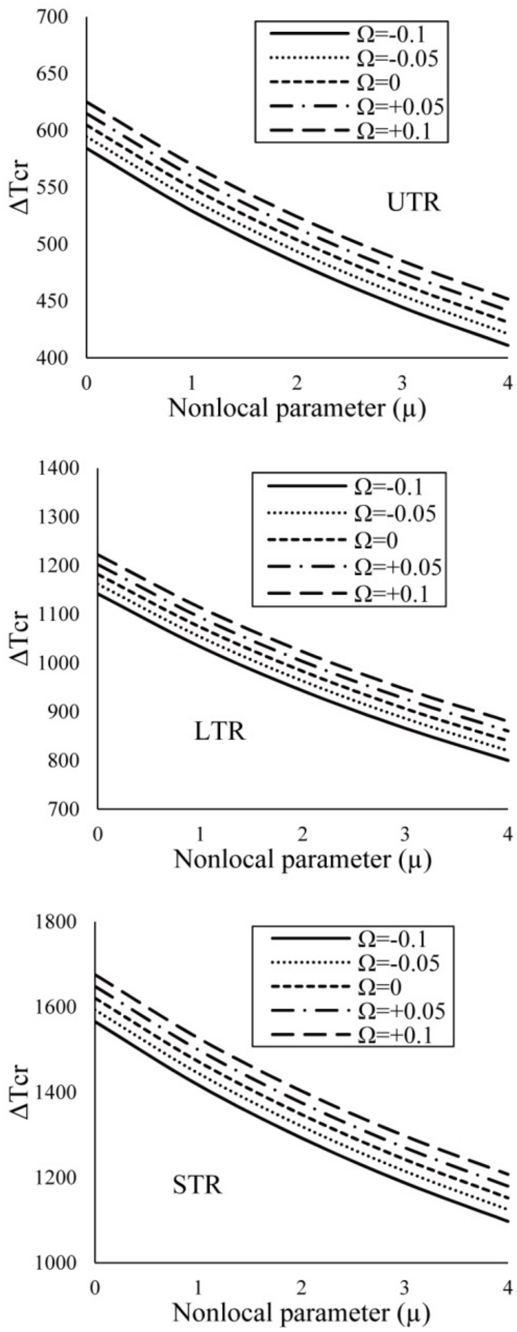

Fig. 3 Effect of nonlocal parameter on the critical buckling temperature of the METE-FG nanobeam (L/h = 10, V = +5, p = 1).

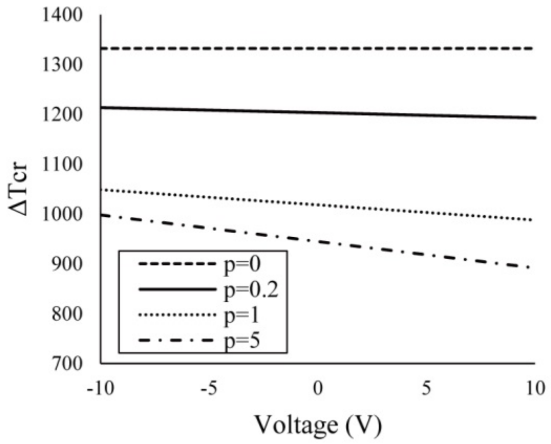

Fig. 4 Effect of material composition and electric voltage on the critical buckling temperature of the S-S FG nanobeam under LTR (L/h = 10, $\mu = 2$, Ω = +0.05).

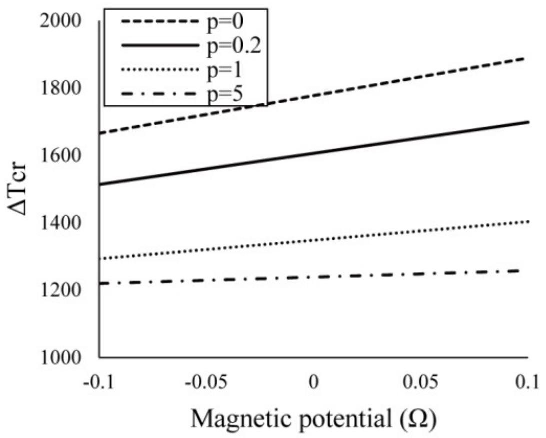

Fig. 5 Effect of material composition on the critical buckling temperature of the S-S FG nanobeam under STR versus magnetic potential (L/h = 10, $\mu = 2$, V = +5).

Hence, the nonlocal beam model estimates lower values of critical buckling temperature than local beam model. Therefore, nonlocality has a major role on the stability of nanostructures in thermal loads.

Figures 4 and 5 present the variation of critical buckling temperature versus electric voltage and magnetic potential, respectively for different gradient index at L/h = 10 and $\mu = 2$. It is observable that for all gradient indexes by increasing magnetic potential/electric voltage from its negative values to its positive values the critical buckling temperature increase/decrease. Also, the impact of magnetic field on the lower values of gradient index is more significant. But, an opposite behavior is observed for electric field.

Moreover, when the gradient index is set to zero (p = 0) the electric field has no influence on the ΔT_{cr} due to the reason that piezoelectric coefficient (e_{31}) of $CoFe_2O_4$ is equal to zero.

Finally, Fig. 6 shows the influence of slenderness ratio (L/h) on the critical buckling temperature for various types of thermal loading when μ = 2(nm)^2, V = +5 and Ω = +0.1. It is found that slenderness ratio has a remarkable influence on the thermal buckling responses of METE-FG nanobeams. Therefore, an increase in beam thickness (lower slenderness ratios) results in increment in the critical buckling temperature. The reason is that when the beam becomes thicker, its buckling is postponed and it can endure higher temperatures.

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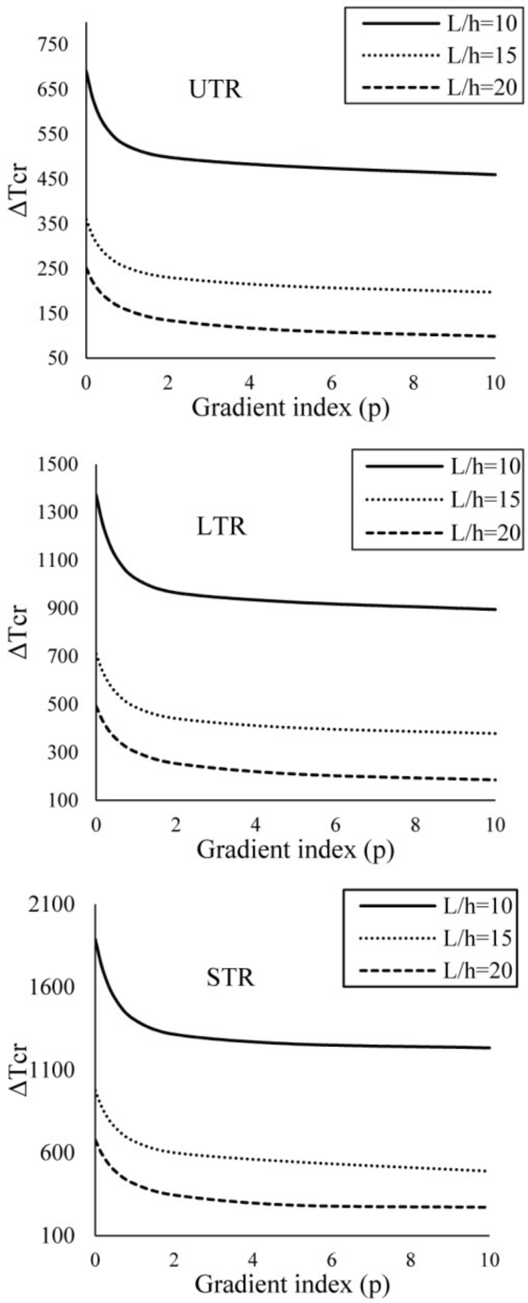

Fig. 6 Effect of slenderness ratio on the critical buckling temperature of the S-S FG nanobeam with respect to gradient index (μ = 2, V = +5, Ω = +0.1).

6. CONCLUSIONS

Thermal buckling of magneto-electro-thermo-elastic FG nanobeam is investigated using a higher order beam theory. Material properties of METE-FG nanobeam change gradually in thickness direction based on power-law model. To consider the influences of small sizes, Eringen's nonlocal elasticity theory is adopted. The notability of different parameters such as thermal loadings, magnetic and electric fileds, power-law index, nonlocal parameter and slenderness ratio on the critical buckling temperatures of METE-FG nanobeams is explored.

It is observed that for all kinds of thermal loadings an increase in the gradient index and nonlocal parameter leads to reduction in critical buckling temperatures due to their softening effect on the beam structure. Moreover, depending on the sign and magnitude of magnetic potential and electric voltage the critical buckling temperatures of METE-FG nanobeam experience both increasing and decreasing trends.

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