A BIVISCIOUS MODIFIED BINGHAM MODEL OF SNOW AVALANCHE MOTION

by

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ABSTRACT

A modified Bingham numerical model is developed and tested for the simulation of the motion of snow avalanches. This two-dimensional, incompressible model takes the form of a two-viscosity system in which a large viscosity is employed in the low stress regions of the flow and a smaller viscosity is used in the high stress regions. The model involves three parameters: the two viscosities, and the value of the stress for the transition between the two flow regimes. A simple no-slip boundary condition is used at the interface between the flowing snow and the stationary snow surface. Model parameters are evaluated by simulating the motion of the leading edge of the flowing snow, velocity versus depth information, and debris distribution of small snow test experiments.

INTRODUCTION

A two-dimensional linear viscous model has been used to simulate the mechanics of flowing snow (Dent and Lang 1980). It was found that for small test slides of less than 20 m s⁻¹ the model, with several modifications, provided reasonable simulation. One modification to the linear viscous model that was found necessary was the inclusion of a friction boundary layer between the flowing snow and the stationary snow surface. This friction condition introduced a second parameter into the model. By adjusting the friction coefficient and the viscosity coefficient, the model was able to simulate the snow tests.

A major failure of the linear viscous model for flowing snow was its response to low stresses. In the modeling, it was necessary to halt the computations at the point where the leading edge of the flow fell below an arbitrarily small velocity. The continuation of the calculations beyond that point would have eventually allowed the fluid to deform until the depth of the material was reduced to zero and the horizontal dimension had become infinite. This motion exemplifies one obvious difference between flowing snow and a linear viscous fluid. This is a simplified version of the general Bingham equation that can be found in Malvern (1969), for example. In that form the equations must be expressed in a manner that is frame-invariant. This requires the yield stress \( \tau_0 \) to be expressed as a function of the stress invariants. The detailed development of this and subsequent equations in a general two-dimensional form can be found in Dent (unpublished). Well-known materials of the Bingham type include paints, greases, concrete, and toothpaste.

In addition to modeling the locking property of flowing snow, a Bingham model contains the necessary stress-deformation components to model the boundary layer that was treated as a friction force in the linear viscous model. In part, this is due to the additional parameter \( \tau_0 \) involved in the Bingham equations. But also the very nature of the Bingham model, being physically more accurate, allows a more realistic representation of the motion, and, as will be seen by the results, provides a very good fit to the data.

BIVISCIOUS MODEL

The implementation of the Bingham model proved to be a difficult task. Of primary importance to this model is the location of the yield surface that separates the two flow regimes. On one side of this surface, the material is locked and behaves as a rigid body. On the other side, the constitutive equation, when substituted into Cauchy's equations,
The general two-dimensional biviscous constitutive equation was implemented by using the framework of a numerical code to solve the two-dimensional incompressible Navier-Stokes equations (Malvern 1970). The flow was assumed to be incompressible, which simplifies both the analysis and the resulting equations. This assumption is dubious at best, but little data are available to check it. The resulting code utilizes a marker and cell method and finite differences the governing equations. Stresses are calculated at the cell nodes using the constitutive equation and the kinematic flow field. These stresses are then used in a finite-difference approximation to the momentum balance equation to determine the advanced time flow field. The exact implementation of this procedure is again detailed in the thesis by Dent (unpublished).

**DIVISCIOUS MODELING RESULTS**

The biviscous model was used to simulate the tests on flowing snow described in Dent and Lang (1980, 1982). These tests decelerated 2.2 m³ of snow from 18 m s⁻¹ to rest, on a level runout of packed snow. Data on the position of the leading edge of the snow, velocity versus depth, and final distribution of debris were collected. Also, qualitative information on the mechanics of the flowing snow was gathered.

Since the flow entered the runout area from an essentially friction-free polyethylene surface, it was allowed that the initial configuration would be a mass of material moving at constant speed on a horizontal friction-free surface. The initial velocity of this material was taken to be 17 m s⁻¹, which was derived from the initial slope of the curve relating position to time. The spatial dimensions of this material were determined from film footage taken of the test and are illustrated in Figure 2.

The numerical modeling commenced with the flow of the material off the frictionless surface onto a surface employing a no-slip boundary condition. The computational grid consisted of an area 23 m long and 50 cm high. The horizontal dimension was divided into 140 cells, each 0.20 m long, and the vertical dimension divided into 10 cells each 0.05 m high. This proved to be about the minimum grid size that was economically feasible. A smaller cell size was tried, but little overall variance from results of a similar test on the 0.20 x 0.05 m grid. A larger-celled grid was, however, deemed inappropriate since the boundary layer at the bottom of the flow was of the order of 5 cm. Cells with vertical dimensions larger than 5 cm would be unable to resolve this layer. The horizontal dimension was then chosen to provide reasonable resolution in that direction and to maintain an aspect ratio between the cell dimensions of no more than 9 to 1.

The three-program modeling parameters, \( \tau_0 \), \( \nu \), and \( \nu' \), were then adjusted so that the computed flow conformed to the observed motion of the test. It was found that the parameters \( \tau_0 \) and \( \nu \) were principally responsible for the motion of the leading edge and the distance of total runout. However, many different combinations of \( \tau_0 \) and \( \nu \) produced the same runout. Flow velocities were not large enough to provide definite distinctions between combinations of these parameters. Figure 3 shows several one-
Dent and Lang: Bingham model of avalanche motion

Fig. 3. Constitutive relations that modeled the motion of the leading edge of snow.

dimensional equivalent constitutive relations involving combinations of $\tau_0$ and $v$ that gave good results for leading-edge motion.

The velocity profile measured in the window test provided another criterion to be satisfied by the numerical simulation. It was found that the computer-generated velocity profile was also principally a function of the parameters $\tau_0$ and $v$. As $\tau_0$ was increased, and $v$ decreased, to maintain the same leading-edge characteristics, the velocity profile became sharper, with larger gradients near the surface and smaller gradients above. Conversely, combinations of small $\tau_0$ and large $v$ produced gradients more closely resembling the parabolic shape expected for pure viscous fluids. Matching the shape of the velocity gradient provided the necessary information to define the two parameters $\tau_0$ and $v$ uniquely. These values were found to be: for $\tau_0$, expressed in units of stress per unit density, 2.20 m$^2$/s$^2$; and for $v$, the kinematic viscosity, 0.002 m$^2$/s. It was also noted that these values provided the best comparisons of leading edge versus time with the experimental snow test. This comparison is illustrated in Figure 4. Figure 5 shows examples of the velocity gradient calculated by the computer model, corresponding to the location of the data acquired in the snow tests. Also shown is the profile found from the snow tests, which is plotted on a velocity scale twice that of the other plots because the velocities measured from behind the window were about half those measured for the motion at the center of the flow. It is believed that this is due primarily to the boundary drag exerted on the edge of the flowing snow. The velocities of the flow measured from the window were about 7.0 m s$^{-1}$, at the leading edge. Meanwhile, at the center of the flow, the leading edge was found to be moving at nearly 16 m s$^{-1}$.

The magnitude of the third parameter $v'$ was found to have very little effect on the motion of the leading edge. The velocity profile, however, was affected by this parameter, though small adjustments of $\tau_0$ and $v$ could be made to compensate. It was also found that $v'$ had a pronounced effect, with $\tau_0$ and $v$, on the final distribution of debris. As $v'$ incre-

Fig. 4. Position of the leading edge versus time: comparison between experiment and computer model.

Fig. 5. Velocity profile comparison between snow test and computer model. Calculations for various combinations of model parameters.

Fig. 6. Final depth profile of debris; comparison between experiment and computer model.
ased, deformation in the upper regions of the flow decreased. This resulted in less total deformation of the initial flow configuration. A value of $v'$ equal to 0.10 m$^2$s$^{-1}$, combined with the previously specified values of $\tau_0$ and $v$, provided the best comparison of final depth profiles of the debris. This result is plotted in Figure 6, as well as the results of a simulation with $v = 0.20$ m$^2$s$^{-1}$. Figure 7 shows a full time series of particle plots for this simulation. In these plots, the friction-free surface extends from the left boundary to the 8.00 m mark. From there onward the surface is no-slip. The vertical dimension (labeled depth) is plotted on a scale exaggerated by a factor of 4 over the horizontal scale.

As can be seen from examining Figures 4, 5 and 6, the modeling results, with $\tau_0/p = 2.2$ m$^2$s$^{-2}$, $v = 0.002$ m$^2$s$^{-1}$, and $v' = 0.10$ m$^2$s$^{-1}$, model closely those of the snow experiment. Moreover these parameters form a unique set in which variation in one parameter will degrade the modeling results, whatever adjustments may be made in the other two coefficients. Additional validity to the values of these parameters is obtained from other experiments. The work of Maeno and Nishimura (1979) and of Maeno and others (1980) on snow suspended by air to form a fluidized bed, produced measurements of kinematic viscosity of the order of 0.001 m$^2$s$^{-1}$ for incompletely fluidized snow. Bucher and Roche (1946), in measuring the frictional resistance of hard wet snow for speeds between 0.2 and 2.4 m s$^{-1}$, found that the linear fit to their data yielded a constant of proportionality of 475 (N-S)/m. If it is assumed that there was a 2 mm layer of granulated snow of density 300 kg m$^{-3}$ between the sliding surfaces and that the velocity gradient was linear in this region, then the viscosity in this layer would be about 0.003 m$^2$s$^{-1}$. Similar tests by Dent and Lang (1982), using hard sintered snow over the velocity gradient range 50 to 300 m s$^{-2}$ yielded a viscosity coefficient of 0.004 m$^2$s$^{-1}$ and a $\tau_0/p$ value of 1.8 m$^2$s$^{-2}$. These values are for a very narrow range of slow speeds and probably differ at higher speeds, but do serve as order of magnitude values.

A last observation is that the tangential boundary condition used in the modeling at the bottom boundary was the no-slip condition. The quality of the modeling results lends credence to the hypothesis that this boundary condition is appropriate for flowing snow.

In carrying out the computer modeling the time-step between calculation cycles was chosen such that the maximum distance traveled by any part of the fluid was less than 0.1 of a cell dimension. Using the cell dimensions previously described and this time-step criterion, no numerical instabilities were encountered for the range of parameters involved in this modeling. To generate the results exhibited in Figure 7, each modeling run required about 1000 calculation cycles, taking, for the 1400 cell computational system, about 30 min of CPU time on the system used.

CONCLUSIONS

For snow flow in the speed range $< 20$ m s$^{-1}$ the biviscous model has provided satisfactory results. The overall motion of the snow as depicted in the motion of the leading edge and the final distribution of the depth of the debris were well simulated. In addition, details noted in the snow tests were reproduced by the computer model. Quantitatively, the velocity as a function of depth was accurately modeled. Qualitatively, the formation of the boundary layer can be seen in the time-sequence particle plots in Figure 7. The particles near the front of the flow at the bottom are retarded as the upper part of the flow proceeds over them. These particles are seen to be left in a layer along the bottom boundary, just as the dye placed originally in the front of the flow in the snow tests was seen to be distributed as a layer over the entire runout area (Dent and Lang 1982). Examination of the motion of the marker particles in the upper portions of the flow shows that little deformation is taking place in this region. This motion is confirmed by observations (Dent and Lang...
Another aspect of the flow seen in both the snow tests and the computer modeling is the surging motion of the leading edge. Although it is not shown clearly in the particle plots, the front of the flow mass was continually breaking over the slower moving flow near the surface. This motion showed up most strikingly by monitoring the velocity at the leading edge. It was found that this velocity was not a smooth function of time but exhibited large variations around the average decaying velocity. Figure 8 is a plot generated by the computer at the time of execution showing the speed of the leading edge versus time. The speed plot shows this surging motion clearly. This motion was also seen when reducing the velocity of the leading edge in the tests of snow flow from 16 mm film. It showed up as anomalous measurements of the velocity of the leading edge at sporadic times in the flow. It could also be seen viewing the motion-picture film, as surging or jetting of the leading edge, much like the motion of water waves sloshing on a beach after breaking.

At speeds above 20 m s⁻¹, much conjecture still exists as to the behavior of flowing snow. Mellor (1968), Perla (1980), and others have speculated that the flowing snow enters a turbulent flow regime at high speeds. This transition point must be a function of speed and type of snow in the avalanche. So far, there is no documentation on when avalanching exists as to the behavior of flowing snow. However, for those avalanches with a central core moving flow near the surface. This motion showed up as anomalous measurements of the velocity of the leading edge at sporadic times in the flow. It could also be seen viewing the motion-picture film, as surging or jetting of the leading edge, much like the motion of water waves sloshing on a beach after breaking.

The excellent internal consistency shown by the biviscous formulation of the tests on flowing snow (Dent and Lang 1982), the branching formulation, and the model may be generalized to additional flow laws. It is a simple matter to generalize from a biviscous formulation to a tri-viscous formulation or a flow law that involves more viscosities. In this way the velocity-squared forces could also be approximated.

The computer simulation methodology, particularly the multi-viscosity approach, has proved to be suitable for solving problems concerned with flowing snow. The excellent internal consistency shown by the biviscous modeling of the tests on flowing snow inspires a great deal of confidence in the method. In addition, the ease with which the model may be generalized to include more complex constitutive laws indicates a very good prospect for its use as additional information about the mechanics of the flowing material is learned.