This is a wonderful book for expert number theorists or keen amateurs who want to bring their personal computing resources to bear in the search for interesting mathematical objects related to prime number theory. The only real prerequisite is a passionate interest in prime numbers and the closely related topic of integer factorisation. Although to understand the detailed mechanism of the number field sieve, or the finer points of the elliptic curve method, one would benefit from a reasonable grounding in number theory, there are large parts of the work that should, in my opinion, appeal to any determined sixth-former. For example, the book has a delightful chapter near the end, ‘The Ubiquity of Prime Numbers’.

References

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This book traces the history of algebra from its beginnings in ancient Babylon to roughly the end of the nineteenth century. The Babylonian heritage described in the first chapter is known through the hundreds of thousands of clay tablets found in Mesopotamia and first deciphered in the 1930s. It is clear that the Babylonians knew how to find quite large Pythagorean triples (hundreds of years before Pythagoras) and, as early as 2000BC, how to solve linear and quadratic equations. What the authors do not tell us is why the Babylonian civilisation fell: their treatment focuses firmly on the mathematics.

The next significant step is covered in the second chapter. The ancient Greeks civilisation is the first known to have developed the idea of proof, by around 500BC. As is well known, the Greeks developed an axiomatic treatment of geometry and used geometric methods to solve algebraic equations. Eventually, however, the Greeks developed a literal symbolism, exemplified in the works of Diophantus, which are described in some detail in the third chapter.

In Europe, the fall of the ancient empires led to the ‘dark ages’ – a period of nearly a millennium when scholarship barely survived. The algebraic ‘torch’ moved further east with the rise of Islam, to the Arabian empire centred on Baghdad, where al-Khwarizmi flourished in the ninth century. Indeed, the term ‘algebra’ comes from the title of al-Khwarizmi’s book Al-jabr wa’l muqābalah. The Arab scholars had translations of both Greek and Hindu works and it was this eclecticism, together with the conquest of significant parts of Europe that eventually led to the renaissance of mathematics in western Europe. The long period of history from al-Khwarizmi to Leonardo of Pisa and Luca Pacioli is dealt with in chapter four, which also covers the development of algebraic notation from literal to symbolic form that occurred during this era.

From the sixteenth century, European mathematicians began to surpass the ancients. Part of chapter five describes the well-known dispute between Tartaglia and Cardano over the solution of the cubic equations. In other sections we learn about the work of Bombelli and Viète. The remainder of the book is devoted to the modern era, beginning with the eighteenth-century struggle to find a complete proof
fundamental theorem of algebra. From this point onward, the reader’s mathematical knowledge is tested more severely. For example, the authors say that ‘basically [Gauss] constructs the splitting field of the initial polynomial’ in his second proof of the fundamental theorem, and go on to describe the construction in greater detail. Readers lacking a basic knowledge of field theory are likely to be lost at this point.

Naturally the major themes of algebra are dealt with: the work of Lagrange, Ruffini and Abel on the impossibility of solving the general quintic in terms of radicals; the study of cyclotomic equations by Gauss; the beginnings of group theory with Lagrange, Galois and Cayley; the work of Kummer and Dedekind on ideal theory; and the non-commutative algebras of Grassmann and Hamilton. For all these sections, readers need some familiarity with the concepts of modern algebra.

This is a book written for mathematicians that describes a widely accepted version of how algebra developed. Historians will be disappointed that the authors do not discuss the sources or the strength of the historical evidence. For example, much of our knowledge of Greek mathematics is second-hand, based on copies of the great works made by monks down the centuries and on secondary sources such as the commentaries of Proclus on works that are now lost. Unfortunately, little survives of the works of the most sophisticated Greek mathematicians, Archimedes and Apollonius. Viewers of the BBC2 Horizon documentary (shown 14 March, 2002) will have learned more about the strength of the historical record than readers of this book. The documentary described J. L. Heilberg’s 1906 discovery of a palimpsest bearing a faint trace of the only known copy of Archimedes’ Method (a palimpsest is parchment that has had one set of text washed off and another written over the top). The palimpsest was lost during the First World War but was recently rediscovered. Modern methods of scrutiny are revealing just how close he came to developing the calculus.

It would be unfair to dismiss the book for its failure to satisfy serious historians, since the authors clearly aimed all along to appeal to mathematicians. The treatment is genuinely expository and accessible to undergraduates who are studying the corresponding mathematics. When algebra is taught from an axiomatic standpoint, the danger always exists that students will think that it was discovered by starting from the axioms. Books like this serve a useful purpose in making it clear that mathematics starts from concrete problems whose generalisation reveals the underlying theory only gradually. I gladly recommend The beginnings and evolution of algebra as a companion to undergraduate algebra courses.

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This is a reprint of the original edition of Lang’s A first course in calculus, which was first published in 1964. In the foreword, the author explains why he thinks it is worth doing this. He refers, rather elliptically, to ‘an evolution (in) sociological and educational conditions’ which have led to the need for a ‘short, straightforward and clear introduction to the subject’, but he doesn’t say what these changes are. I suspect that this tactfulness is a latent criticism of what has been going on over the last forty years in high-school education in the USA, but there is little point in speculation here. One could, of course, transplant the debate to these shores, and ask what has happened to mathematics education in the UK over the same time period.