

Epilogue, or the squirrel who got away?

So, has Monstrous Moonshine been explained? According to most of the fathers of the subject, it hasn't. They consider VOAs in general, and V^{\natural} in particular, to be too complicated to be God-given. The progress, though impressive, has broadened not lessened the fundamental mystery, they would argue.

For what it's worth, I don't completely agree. Explaining away a mystery is a little like grasping a bar of soap in a bathtub, or quenching a child's curiosity. Only extreme measures like pulling the plug, or growing up, ever really work. True progress means displacing the mystery, usually from the particular to the general. Why is the sky blue? Because of how light scatters in gases. Why are Hauptmoduls attached to each $g \in \mathbb{M}$? Because of V^{\natural} . Mystery exists wherever we can ask 'why' – like beauty, it's in the beholder's eye.

Understanding doesn't put an end to questions, it spices them. There's always a horizon, no matter how high you climb, beyond which everything is still hidden.

However, have we really isolated the key conjunction of properties needed for Moonshine to arise? Can we derive the Monster from Monstrous Moonshine? In Section 7.2.4 we make the case for a more direct, topological explanation for Moonshine involving compatible actions of the Virasoro algebra and the braid group \mathcal{B}_3 . In any case, we need a second independent proof of Monstrous Moonshine.

Moonshine is now 'leaving the nest'. We are entering a consolidation phase, tidying up, generalising, simplifying, clarifying, working out more examples, climbing a few metres higher. Important and interesting discoveries will be made in the next few years, and yes, there still is mystery, but no longer does a Moonshiner feel like an illicit distiller: Moonshine is now a day-job!

*Question and Answer in the Mountains*¹

They ask me why I live in the green mountains.
I smile and don't reply; my heart's at ease.
Peach blossoms flow downstream, leaving no trace –
And there are other earths and skies than these.

Li Bai 701 AD

¹ Translated by Vikram Seth, *Three Chinese Poets* (London, Faber and Faber, 1992).