the pressure to the energy density at the centre of such systems will also hold in the present case of rotating systems. A consequence of this is that it is not possible to find a distribution function which gives rise to uniformly rotating systems which have an isotropic pressure and a uniform energy density.

The derivation given previously\(^1\) for the form of the distribution function for truncated Maxwell-Juttner distributions results in an identical expression for the distribution function of a uniformly rotating system with a truncated Maxwell-Juttner distribution, namely,

\[
F(m, E) = g(m)H(\beta m - E) \exp(-\lambda \beta^{-1}m_0^{-1}E),
\]

where \(g\) depends upon the distribution in proper mass in the system, \(H\) is the Heaviside unit function, and \(\lambda\) and \(m_0\) are constants. Investigations have been initiated into the problem of solving the field equations for the energy-momentum tensor corresponding to this distribution function.

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**Finite-amplitude Adiabatic Oscillations of Super-massive Stars**

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As shown recently by Y. Osaki\(^2\) super-massive stars with mass \(M < 3.5 \times 10^4 M_\odot\) can, in the absence of rotation, reach the hydrogen-burning main sequence before the onset of general relativistic instability. Such objects are then pulsationally unstable.\(^3\) A considerable simplification is introduced if one considers only very massive stars, for which the relative amplitude of the fundamental mode of oscillation is practically constant. This sets a lower limit of \(10^4 M_\odot\) to the mass that can be considered. The upper limit is also reduced to \(2 \times 10^5 M_\odot\) if one neglects the relativistic correction. One necessary step in the study of non-linear oscillations of massive stars is to derive an equation for the pulsations. The relativistic correction could be taken into account in the following way.

It was shown by McVittie\(^4\) that the equations of continuity and motion of a fluid acted upon by its gravitational self-attraction, admit the following expressions for the density \(\rho\) and pressure \(p\).

\[
\rho = \frac{1}{f^3} \left( k_\xi + 2h \right)
\]

\[
p = P(t) - \frac{f^2}{f^3} \left( \int k \xi_d + h \xi \right) - \frac{8\pi G}{f^4} \left( \int \frac{h^3}{\xi} \xi_d + \frac{h^3}{4} \right)
\]

if the radial velocity \(q\) has the form

\[
q = \frac{f_t}{f} r
\]

\(f(t)\) being an arbitrary function of the time, in this case the radius of the star, \(P(t)\) an arbitrary function of the time and \(h\) an arbitrary function of \(\xi = r/f\).

It can be shown from direct integration of the equations of motion for small oscillations that such velocity distribution is nearly correct for stars of very large mass.

The previous equations hold only in the Newtonian case. The equation of motion in the post-Newtonian approximation can be written\(^4\)

\[
r = -\frac{1}{\rho} \frac{dP}{dr} = \frac{GM_r}{r^2} \left[ 1 + \frac{\rho + 2GM_r}{c^2} + \frac{4\pi \rho \sigma^2}{c^2 M_r} \right]
\]

and as shown by Kaplan and Lupanov\(^5\) this equation takes roughly the same form as in the Newtonian case if we replace the constant of gravitation \(G\) by

\[
\bar{G} = G \left( 1 + \frac{4\rho_0}{\rho_0 c^2} \right)
\]

\(\rho_0\) and \(\rho_0\) being the pressure and density at the centre of the star.

The energy equation, as given by Fowler\(^4\) can be written

\[
\frac{dp}{dr} = \frac{G_1}{\rho_0 + p/c^2} \frac{\rho_0 c^2}{2} dr
\]

where, for very massive stars \(G_1 = 4/3 + \beta/6\), \(\beta\) being the ratio of gas pressure to total pressure. Substituting the values of \(\rho\) and \(p\) from (1) and (2) in (6), integrating over the whole star and introducing the new variable \(y = f/f_0\), \(f_0\) being the equilibrium ratio, one obtains the following differential equation

\[
q(y'')' + (2 + \beta/2)y'y' + A \beta \frac{4\pi G \rho_0}{y^2} y' G
\]

\[+ 3G_1 q \left( y'y'D + 8\pi \bar{G} \rho_0 \frac{y'}{y^2} - E \right) = 0,
\]

where

\[
q = \frac{\rho_0}{\rho_0 c^2}
\]

\[
A = \int_{0}^{\xi_1} \left( H(1) - H(\xi) \right) 4\pi \xi^2 d\xi
\]

\[
C = \int_{0}^{\xi_1} \left( J(1) - J(\xi) \right) 4\pi \xi^2 d\xi
\]

\[
D = \int_{0}^{\xi_1} \theta \left( H(1) - H(\xi) \right) 4\pi \xi^2 d\xi
\]

\[
E = \int_{0}^{\xi_1} \theta \left( J(1) - J(\xi) \right) 4\pi \xi^2 d\xi
\]

where \(\xi = \xi/\xi_1\), \(\xi_1\) being found in tables of Emden functions, \(\theta\) is the solution of the Emden equation, and

\[
H(\xi) = -\frac{1}{\xi^2} \left( \frac{d\theta}{d\xi} \xi + \theta \right)
\]
\[ J(\xi) = -\frac{1}{2\xi^3} \frac{\theta^{n+1}}{n+1} \]

\( n \) being the polytropic index, in this case \( n = 3 \).

Keeping in mind that for super-massive stars \( \beta \) is very small indeed, one can integrate equation (7) and obtain the following differential equation:

\[ \gamma^2 y'' + \Sigma_0 1n y = 0 \]  

where \( \Sigma_0 = (C\beta - 8gE) \frac{4\pi G\rho_c}{A} \)

It can be shown that, when \( q = 0 \) — i.e. when relativistic corrections can be neglected — equation (8) reduces to the one given by Osaki\(^1\) using energy considerations. Equation (8) would enable us to investigate the adiabatic oscillations of super-massive stars in the mass-range \( 10^4 \) to \( 3.5 \times 10^5M_\odot \). When \( \Sigma_0 < 0 \), gravitational collapse, due to general relativistic effects, sets in.