SPIEGEL, E. and O'DONNELL, C. J. *Incidence Algebras* (Pure and Applied Mathematics Vol. 206, Marcel Dekker, 1997), ix + 335 pp., 0 8247 0036 8, US$125.

This is a monograph on a rather specialised topic. In Chapter 1 the basic definitions are given. An incidence algebra $I(X, R)$ is defined over a locally finite partially ordered set X and a commutative ring R as the set of mappings from $X \times X$ to R with the restriction that $f(x, y) = 0$ if x is not less than or equal to y; the usual matrix operations of addition, multiplication and multiplication by scalars in R are used to make this set into an associative algebra over R. Examples are given to show where these structures can arise in combinatorial or geometric settings as well as in algebra.

Chapter 2 gives a good account of the Möbius Algebra leading to the Polya-de Bruijn Theorem. Chapter 3 leads from generating functions and incidence coalgebras to incidence Hopf algebras. In Chapter 4 radicals of the incidence algebra are considered. The Jacobson radical is related to that for R as expected. It is shown that the upper and lower nil radicals coincide. Chapters 5 and 6 are devoted respectively to maximal ideals and to prime ideals. In Chapter 7 the isomorphism problem is considered. If F is a field then $I(X, F)$ and $I(Y, F)$ are isomorphic if and only if X and Y are isomorphic. For a ring R somewhat weaker conclusions are drawn but the full result is shown to hold in certain cases. Further ring theoretic properties are considered in the final chapter, with special attention being given to rings of quotients and polynomial identities.

The monograph is well written. The definitions and results are clearly presented. There is a full list of references and at the end of each chapter there is a brief history of the material and a clear attribution to the original developers. Personally I found the combinatorial applications rather than the algebra to be of the greatest interest.

A. D. SANDS

KISSIN, E. and SHULMAN, V. *Representations on Krein spaces and derivations of C^*-algebras* (Pitman Monographs and Surveys in Pure and Applied Mathematics Vol. 89, Longman, 1997), iii + 602 pp., 0 582 23157 4, £85.

Krein spaces first appeared in physics papers in the early 1940s, when Dirac and Pauli proposed their use in quantum field theory. However, the first mathematical paper about operators on Krein spaces, written by Pontryagin and published in 1944, was based on a suggestion of Sobolev arising from a problem in classical mechanics. Subsequently the theory was developed by M. G. Krein, Naimark, Phillips and others, and it has found assorted applications to topics in quantum field theory, differential equations, complex function theory and operator theory.

A Krein space is a particular type of (infinite-dimensional) indefinite inner product space. Any orthogonal decomposition $H = H_- \oplus H_+$ of a Hilbert space H produces a Krein space, and every Krein space arises in this way. However, this correspondence is not bijective, because the same Krein space arises from many different choices of definite inner product and decomposition of H. The theory of Krein spaces is concerned with aspects of the indefinite inner product which are independent of these choices; for example, the dimensions of H_- and H_+ satisfy this. An operator which is symmetric with respect to the indefinite inner product on a Krein space is said to be "J-symmetric". The theory of J-symmetric operators has some common features with the usual theory of self-adjoint operators, but it is considerably more intricate and some of its aspects are completely different from the self-adjoint case. These aspects of Krein spaces have been described in earlier books, for example *Indefinite inner product spaces* by J. Bognár (Springer-Verlag, 1974) and *Linear operators in spaces with indefinite metric* by T. Y. Azizov and I. S. Iokhvidov (Wiley, 1989).

The book under review concentrates more on the general theory of J-symmetric operator algebras and representations of *-algebras on Krein spaces, which are the indefinite analogues of C^*-algebras and their representations on Hilbert space. Many of the results here have been known for more than twenty years, but they have been scattered in the literature. The authors present these results in a coherent fashion, making reference to the modern theory of C^*-algebras where relevant, and they include some new results in representation theory. The theory of representations on general Krein spaces is quite weak, but it is much stronger in the case of those Krein spaces, known as Π_κ-spaces or Pontryagin spaces, where H_- or H_+ is finite-dimensional.

The final one-third of the book is devoted to relatively recent applications to the study of *-derivations of C^*-algebras. Let $\mathcal{A}$ be a C^*-algebra of operators on a Hilbert space H, and δ be a closed, densely-defined (unbounded) *-derivation from $\mathcal{A}$ into $\mathcal{B}(H)$. One seeks closed operators S on H (with good properties) which implement δ in the sense that

$$\delta(A)x = SAx - ASx \quad (x \in \mathrm{Dom}(S), A \in \mathrm{Dom}(\delta)).$$

It is easy to construct a J-symmetric representation of the Banach *-algebra $\mathrm{Dom}(\delta)$ on a Krein space H' in such a way that closed operators S implementing δ correspond to closed subspaces of H' invariant under the representation, and moreover Hilbert space properties of S correspond neatly to Krein space properties of the subspace. However, H' is not a Π_κ-space, so the results obtained from representation theory are quite weak. One can obtain stronger results if one already has a skew-symmetric operator S which implements δ and one of the deficiency indices of S is finite, a situation which is not unusual in practice. Then there is a representation of $\mathrm{Dom}(\delta)$ on a Π_κ-space, and the stronger representation theory can be used to show that δ can also be implemented by extensions of S with better Hilbert space properties. Moreover, these constructions enable index theory of semigroups of *-endomorphisms of $\mathcal{B}(H)$, as developed by Powers and Arveson, to be derived from the representation theory of Krein spaces.

Since little material on representations on Krein spaces or implementation of derivations has previously appeared in books, this book will be of great interest to those specialising in Krein spaces, and also those specialising in C^*-algebras with interests in derivations. A novice who wishes to master Krein spaces may prefer to start with a more leisurely introduction to the basic geometry and operator theory, and a reader who has not already studied the basic theory of C^*-algebras may not appreciate Chapter 6 and some earlier sections. The background material summarised in the first section varies enormously in level of difficulty, but much of the book can be read without knowledge of the more demanding topics. It is the nature of this subject that the proofs require verification of many routine properties, and the authors have used fine judgment to steer a course between too much detail and too little detail. The text has been written carefully, with only a few typographical errors. Indeed, this book succeeds in describing a subject which was well worth writing about.

C. J. K. BATTY

OLLERENSHAW, K. and BRÉE, D. *Most-perfect pandiagonal magic squares: their construction and enumeration* (Institute of Mathematics and its Applications, Southend-on-Sea, 1998), xiii + 152 pp., 0 905091 06 X, £19.50.

This is a marvellous book. It is very readable, carefully planned, and contains fascinating material.

Magic squares have a long history, and well-known constructions exist for squares of all sizes greater than two. However, the enumeration problem of counting the different magic squares of a particular type has so far remained unresolved. The authors consider a special type of magic