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Toroidal flux ropes

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Abstract. We present a method how to describe analytically a magnetic field distribution in the vicinity of a large interplanetary flux rope. The field consists of the pre-existing one and an additional current-free part. This work was supported by INTAS grant 03-51-6206, AV ČR project S1003006, and RFBR grant 03-02-16340.

1. Introduction

The problem is formulated in toroidal coordinates, μ, η, and φ,

$$x = \frac{a \sinh\mu \cos\varphi}{\cosh\mu - \cos\eta}\ , \quad y = \frac{a \sinh\mu \sin\varphi}{\cosh\mu - \cos\eta}\ , \quad z = \frac{a \sin\eta}{\cosh\mu - \cos\eta}\ ,$$

and it is a continuation of our previous work (Vandas et al., 2003). The parameter $a = \sqrt{R_0^2 - r_0^2} = \text{const.}$, where R_0 and r_0 are toroid major and minor radii, respectively.

A toroid with a variable minor radius $r_0(\varphi)$ is defined by $\mu = \mu_0(\varphi)$, $\cosh\mu_0 = R_0/r_0$ through $r_0(\varphi) = r_{00}/2[1+q+(1-q)\cos\varphi]$, where $r_{00} = r_{\max}$ and $q = r_{\min}/r_{\max}$. Normal components of an external field $\boldsymbol{B}^{tot}$ on the surface must be zero:

$$B_\mu^{tot,0} \sinh\mu_0 - B_\varphi^{tot,0} \mu_0' = 0\ . \tag{1.1}$$

Here $B_\mu^{tot} = B_\mu^{amb} + \frac{1}{h\mu}\frac{\partial\Psi}{\partial\mu}$, $B_\varphi^{tot} = B_\varphi^{amb} + \frac{1}{h\varphi}\frac{\partial\Psi}{\partial\varphi}$, h_μ and h_φ are the Lamè coefficients, the prime is a derivative by φ, B^{amb} is a pre-existing ambient field and Ψ is a magnetic scalar potential of an additional field $\boldsymbol{B}^{add} = \text{grad}\,\Psi$,

$$\Psi = B_0 a \sqrt{\cosh\mu - \cos\eta} \sum P_{n-1/2}^m(\cosh\mu) \times$$

$$(\alpha_n^m \cos n\eta \cos m\varphi + \beta_n^m \cos n\eta \sin m\varphi + \gamma_n^m \sin n\eta \cos m\varphi + \beta_n^m \sin n\eta \sin m\varphi)\ . \tag{1.2}$$

By selection of coefficients in (1.2) we can satisfy (1.1). The additional field vanishes at large distances because the Legendre functions $P_{n-1/2}^m(\cosh\mu) \to 0$ when their argument is approaching 1. We have the following relationships:

$$\frac{\partial\Psi}{\partial\mu} = \frac{B_0 a \sinh\mu}{\cosh\mu - \cos\eta} \sqrt{\cosh\mu - \cos\eta} \sum \left[\frac{1}{2} P_{n-1/2}^m(\cosh\mu) + (\cosh\mu - \cos\eta) P_{n-1/2}^{m'}(\cosh\mu)\right]$$

$$\times\,(\alpha_n^m \cos n\eta \cos m\varphi + \beta_n^m \cos n\eta \sin m\varphi + \gamma_n^m \sin n\eta \cos m\varphi + \delta_n^m \sin n\eta \sin m\varphi)\,;$$

$$\frac{\partial\Psi}{\partial\varphi} = B_0 a \sqrt{\cosh\mu - \cos\eta} \sum m P_{n-1/2}^m(\cosh\mu)$$

$$\times\,(-\alpha_n^m \cos n\eta \sin m\varphi + \beta_n^m \cos n\eta \cos m\varphi - \gamma_n^m \sin n\eta \sin m\varphi + \delta_n^m \sin n\eta \cos m\varphi)\,.$$

The prime is a derivative by the function argument.

One can rewrite (1.1) in the form:

$$-\frac{\sinh\mu_0}{\sqrt{\cosh\mu_0-\cos\eta}}(B_\mu^{\mathrm{amb},0}\sinh\mu_0 - B_\varphi^{\mathrm{amb},0}\mu_0') =$$

$$B_0\sum(\alpha_n^m r_n^m\cos n\eta+\beta_n^m s_n^m\cos n\eta+\gamma_n^m r_n^m\sin n\eta+\delta_n^m s_n^m\sin n\eta)\ , \tag{1.3}$$

where

$$r_n^m = p_n^m\cos m\varphi + q_n^m\sin m\varphi\ ,$$

$$s_n^m = p_n^m\sin m\varphi - q_n^m\cos m\varphi\ ,$$

$$p_n^m=\sinh^3\mu_0[\frac{1}{2}P_{n-1/2}^m(\cosh\mu_0)+(\cosh\mu_0-\cos\eta)P_{n-1/2}^{m\prime}(\cosh\mu_0)]\ ,$$

$$q_n^m=(\cosh\mu_0-\cos\eta)\mu_0' m P_{n-1/2}^m(\cosh\mu_0)\ .$$

Writing (1.3) for a number of points on the surface we have a system of linear equations from the least squares method for coefficients $\alpha_n^m, \beta_n^m, \gamma_n^m, \delta_n^m$.

The components are

$$B_\mu^{\mathrm{add}}=B_0\sinh\mu\sqrt{\cosh\mu-\cos\eta}\sum[\frac{1}{2}P_{n-1/2}^m(\cosh\mu)+(\cosh\mu-\cos\eta)P_{n-1/2}^{m\prime}(\cosh\mu)]$$

$$\times(\alpha_n^m\cos n\eta\cos m\varphi+\beta_n^m\cos n\eta\sin m\varphi+\gamma_n^m\sin n\eta\cos m\varphi+\delta_n^m\sin n\eta\sin m\varphi)\ ,$$

$$B_\eta^{\mathrm{add}}=B_0\sqrt{\cosh\mu-\cos\eta}\sum P_{n-1/2}^m(\cosh\mu)$$

$$\times[\frac{1}{2}\sin\eta(\alpha_n^m\cos n\eta\cos m\varphi+\beta_n^m\cos n\eta\sin m\varphi+\gamma_n^m\sin n\eta\cos m\varphi+\delta_n^m\sin n\eta\sin m\varphi)$$

$$+n(\cosh\mu-\cos\eta)(-\alpha_n^m\sin n\eta\cos m\varphi-\beta_n^m\sin n\eta\sin m\varphi+\gamma_n^m\cos n\eta\cos m\varphi+\delta_n^m\cos n\eta\sin m\varphi)]\ ,$$

$$B_\varphi^{\mathrm{add}}=B_0\frac{(\cosh\mu-\cos\eta)^{3/2}}{\sinh\mu}\sum mP_{n-1/2}^m(\cosh\mu)$$

$$\times(-\alpha_n^m\cos n\eta\sin m\varphi+\beta_n^m\cos n\eta\cos m\varphi-\gamma_n^m\sin n\eta\sin m\varphi+\delta_n^m\sin n\eta\cos m\varphi)\ .$$

On Figure 1 a modified field is shown.

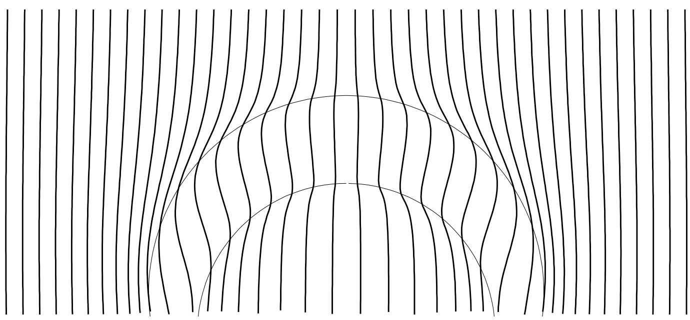

Figure 1. A modified uniform field around a toroid with a variable minor radius.

References

M. Vandas, E. P. Romashets, and S. Watari 2003, Potential magnetic fields around flux ropes. *Astron. Astrophys.* **412**, 281–292.

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