11

Realistic supersymmetric models

It should be clear that, without further assumptions, the MSSM is not a tractable framework for SUSY phenomenology: there are just too many free parameters. This is not to say that we cannot do *any* phenomenology with the MSSM. First, assuming only R-parity conservation, we know that all sparticles must decay into other sparticles, until the decay chain terminates in the stable LSP. We have already seen that cosmological considerations require that the LSP cannot have electromagnetic or strong interactions. Since it couples to quarks and electrons only via *weak interactions*, it behaves like a neutrino in that it escapes the experimental apparatus undetected. As a result, the production of SUSY particles in high energy collisions is generically signaled by events with apparent "missing energy and momentum" (carried off by the undetected LSPs). With some mild assumptions of sparticle mass ordering, other relatively robust inferences may also be possible. For instance, if we assume that $\tilde{\mu}_R$ is the only charged sparticle that is accessible at an e^+e^- collider, and that the lightest neutralino is the LSP, we can conclude that

- smuons will be pair produced in e^+e^- collisions with cross sections that are fixed in terms of $m_{\tilde{\mu}_R}$ by *known* couplings to the photon and the Z-boson;
- both smuons will dominantly decay via $\tilde{\mu}_R \to \mu \widetilde{Z}_1$.

Smuon pair production will thus be signaled by a calculable rate for missing energy events with acolinear muon pairs. We will see later that this is the way that the current bound on $m_{\tilde{\mu}_R}$ is obtained from experiments at LEP2.

If instead $\widetilde{W}_1$ is the lightest charged sparticle, there are additional complications from the fact that it is a model-dependent mixture of charged gauginos and higgsinos. The decays of other heavier sparticles are sensitively dependent on the sparticle (and Higgs boson) mass ordering, as well as on the sparticle mixing matrices discussed in Chapter 8. The size of the parameter space makes a general analysis of heavy sparticle decay patterns quite intractable. The analysis of

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SUSY loop-induced contributions to low energy processes is also complicated for the same reason. Indeed, as we saw in Chapter 9, phenomenologically well-motivated, but theoretically ad hoc, universality assumptions are invoked for these analyses.

Clearly, the problem is that we do not have any theoretical principle for determining the soft SUSY breaking parameters of the MSSM. We speculate that the MSSM is the low energy approximation to an underlying fundamental theory in which SUSY is spontaneously broken by some as yet unknown dynamics. We hope that experimental data on sparticle properties will guide us to this dynamics once these are discovered. In the absence of such guidance, we adopt a "top-down" approach based on theoretical assumptions about how superpartners of SM particles acquire masses, resulting in different models of supersymmetry.

The first attempts to construct supersymmetric theories of particle physics were based on global supersymmetry, with the supersymmetry broken at the weak scale. As we saw in Chapter 7 these attempts typically run into problems with the tree-level mass sum rule (7.35). These problems can be avoided if

1. supersymmetry is promoted to a local symmetry, in which case the sum rule is modified to (10.66); the term proportional to $m_{3/2}$ on the right-hand side means that the scalar masses can all be shifted up, thereby evading the phenomenologically unacceptable existence of scalars lighter than the fermions.
2. Alternatively, the tree-level sum rule of global supersymmetry can be evaded if superpartners of SM particles get their masses only at the loop level.

Models that exploit both these alternatives have been constructed. A common feature of all realistic supersymmetric models of particle physics is the necessity of assuming a "hidden sector" whose dynamics somehow breaks supersymmetry. This sector is dubbed hidden because it couples only indirectly (and very weakly) to the "observable sector" of SM particles and their superpartners. The details of how supersymmetry is broken in this sector are, as we will see, unimportant for the physics of the observable sector. What is important is "the agent" that couples the hidden and observable sectors, and communicates the effects of SUSY breaking to the superpartners of SM particles, which then acquire soft SUSY breaking masses and couplings. The idea is that supersymmetry is broken at a scale $F = M_{\rm SUSY}^2 \gg M_W^2$ in the hidden sector where the goldstino resides. This sector is assumed to interact with the observable sector only via the exchange of superheavy particles X. The couplings of the goldstino to the observable particles are suppressed by (a power of) $M_{\rm SUSY}/M_X$, and the effective mass gap in the observable sector is $M_{\rm eff} \sim M_{\rm SUSY}^2/M_X$ (or, more generally, $M_{\rm SUSY}^{n+1}/M_X^n, n = 1, 2, \ldots$). It is this gap that is required to be comparable

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to the weak scale, even though the fundamental SUSY breaking scale may be much larger.[1]

The reason that this approach makes any sense at all is that the radiative corrections in the observable sector correspond to the scale $M_{\rm eff}$. The effective potential of the low energy theory (the observable sector) does not contain any terms of $\mathcal{O}(M_{\rm SUSY}^4, M_{\rm SUSY}^3 M_{\rm eff}, M_{\rm SUSY}^2 M_{\rm eff}^2, M_{\rm SUSY} M_{\rm eff}^3)$ which would render the whole approach inconsistent. This was first analyzed by Polchinski and Susskind in a toy model, and later by others in more realistic scenarios.[2]

Supersymmetric models are characterized by the agent that communicates supersymmetry breaking effects in the hidden SUSY breaking sector to the observable world. Since gravity couples universally to energy, gravitational interactions are one obvious choice for mediating SUSY breaking effects. Indeed in a truly supersymmetric world, gravity mediation *must be present*. Whether its effects are swamped by other things is the relevant issue. Of course, gravity-mediated SUSY breaking requires that supersymmetry is local (as it must be if *all* interactions are supersymmmetric), so that these models are based on supergravity. Supersymmetry breaking effects may also be communicated by the usual SM gauge interactions. In these so-called gauge-mediated supersymmetry breaking (GMSB) scenarios, new "messenger fields" that couple directly to the hidden sector, but which also have SM gauge couplings, act as mediators of SUSY breaking effects: MSSM gauginos and sfermions get supersymmetry breaking masses and couplings only at the loop level, thereby evading the tree-level mass sum rule. We will see that both supergravity models as well as GMSB models have the general structure of geometric hierarchy models discussed above: since gravity is the mediator of SUSY breaking in supergravity models, the scale M_X is identified with the Planck scale, and $M_{\rm SUSY} \sim 10^{10}$ GeV. In GMSB scenarios, $M_{\rm SUSY}$ is identified with the mass of messenger fields; if these are relatively light, the underlying SUSY breaking scale can be much smaller than in gravity-mediated scenarios. More recently other mediation mechanisms, which could most naturally occur if the world had additional (compactified) spatial dimensions, have also been considered.

In this chapter, we will introduce the physical ideas behind these various models, focussing on the differences in their phenomenological implications. We emphasize that each of these models is based on untested assumptions about physics at scales well above the weak scale. It may be that these assumptions will prove to be incorrect. The important thing, however, is that the models make characteristic predictions which will be subject to test in experiments at high energy colliders. A

[1] Indeed there were attempts to make realistic globally supersymmetric models based on this idea, with $M_X \gg M_W$ and $n = 2$. These models were dubbed geometric hierarchy models for obvious reasons.

[2] J. Polchinski and L. Susskind, *Phys. Rev.* **D26**, 3661 (1982); L. Hall, J. Lykken and S. Weinberg, *Phys. Rev.* **D27**, 2359 (1983), and references therein.

common feature of all these scenarios is that sfermions with the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers will turn out to have the same mass parameters and the same A-parameters, renormalized at some high scale, so that unwanted flavor-violating effects discussed in Chapter 9 are absent. The reader should keep an eye open for how this comes about in each of these cases.

11.1 Gravity-mediated supersymmetry breaking

We begin our discussion of models by considering the case where gravitational interactions mediate the effects of supersymmetry breaking in the hidden supersymmetry breaking sector to the superpartners of SM particles. We will concentrate on the *form* of the Lagrangian for the low energy theory that is obtained as a result of coupling the hidden and SM sectors via supergravity, but will not write down the most general formulae for the coefficients of the various terms in the resulting low energy theory. These formulae are cumbersome and, in the absence of a compelling high energy theory, not particularly useful for the abstraction of the low energy phenomenology.

11.1.1 Hidden sector origin of soft supersymmetry breaking terms

We begin our construction of supergravity models by grouping the left-chiral supermultiplets of the model $\{\hat{\mathcal{S}}_i\}$ into observable sector fields $\hat{C}_i$ (these include all the MSSM fields) and the "hidden sector" fields $\hat{h}_m$. The $\hat{h}_m$ fields are assumed to be gauge singlets under the observable sector gauge symmetry group, which may be taken to be $G_{\rm SM}$ or a grand unifying group. The observable sector gauge superfields are correspondingly chosen. The superpotential is chosen to be a sum of two independent parts, with no "superpotential couplings" between them:[3]

$$\hat{f}(\hat{\mathcal{S}}_i) = \hat{f}_{\rm o}(\hat{C}_n) + \hat{f}_{\rm h}(\hat{h}_m). \tag{11.1}$$

New gauge interactions (under which the SM particles are singlets), and the associated degrees of freedom may also be present in the hidden sector.

The locally supersymmetric Lagrangian corresponding to these sets of fields can be worked out using the general results of the previous chapter. With our assumptions, (super)gravity is the only coupling between the hidden sector and the observable sector. We assume that the dynamics of the hidden sector somehow breaks supersymmetry. This could be by any of the mechanisms discussed in Chapter 7. The goldstino degrees of freedom are absorbed by the gravitino which obtains

[3] We may imagine that a symmetry forbids superpotential couplings between the hidden and observable sectors if the superpotential is restricted to be polynomial in the fields. Since supergravity is not renormalizable, the division into the two sectors appears to require an alternative explanation.

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a mass $m_{3/2}$. The low energy effective field theory valid below the Planck scale is obtained by taking the limit as $M_P \to \infty$, keeping $m_{3/2}$ fixed. This will turn out to be a renormalizable supersymmetric Yang–Mills theory based on the low energy gauge group together with a slew of soft SUSY breaking (SSB) masses and couplings, with magnitudes $\sim m_{3/2}$. It should be kept in mind that in this framework, higher dimensional, non-renormalizable operators (consistent with low energy symmetries) suppressed by appropriate powers of M_P will also be present. These operators are referred to as "Planck slop" in the literature.

To illustrate this procedure, we will adopt a simple model wherein the observable sector consists of the fields of the MSSM, and the gauge symmetry is $SU(3)_C \times SU(2)_L \times U(1)_Y$. We take the hidden sector to consist of a single left-chiral superfield $\hat{h}$ whose dynamics breaks local supersymmetry. The hidden sector superpotential $\hat{f}_h$ might be the Polonyi superpotential, although we will be somewhat more general than that.

We first work out the scalar potential for the case of the flat Kähler metric, with

$$K(\hat{\mathcal{S}}, \hat{\mathcal{S}}^\dagger) = \hat{h}^\dagger \hat{h} + \sum_n \hat{C}^{\dagger n} \hat{C}_n.$$

We will return to the more general case later. This potential is given by,

$$\begin{aligned} V &= e^{(\mathcal{S}^{i*}\mathcal{S}_i)/M_P^2} \left[\left| \frac{\partial \hat{f}}{\partial \mathcal{S}_i} + \frac{\mathcal{S}^{i*}\hat{f}}{M_P^2} \right|^2 - \frac{3}{M_P^2} |\hat{f}|^2 \right] \\ &= e^{(h^* h + C^{n*} C_n)/M_P^2} \left[\left| \frac{\partial \hat{f}_h}{\partial h} + \frac{h^* \hat{f}}{M_P^2} \right|^2 + \left| \frac{\partial \hat{f}_o}{\partial C_n} + \frac{C^{n*}\hat{f}}{M_P^2} \right|^2 - \frac{3}{M_P^2} |\hat{f}|^2 \right], \end{aligned} \tag{11.2}$$

where a sum over fields is implied. We assume that the F-component of the hidden sector field $\hat{h}$ develops a VEV $\sim m^2$ which breaks local SUSY and, further, that its scalar component h develops a VEV of order M_P as well. The VEVs of the scalar components of the observable sector fields are assumed to be negligible compared to M_P. Accordingly, we parametrize these VEVs by,

$$\langle h \rangle = a M_P, \tag{11.3a}$$

and

$$\langle \hat{f}_h \rangle = b m^2 M_P \quad \text{and} \quad \langle \frac{\partial \hat{f}_h}{\partial h} \rangle = m^2, \tag{11.3b}$$

with a and b being dimensionless coefficients of order 1. For the specific choice of the Polonyi model, $a = \sqrt{3} - 1$ and $b = 1$. The gravitino mass is

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given by,

$$m_{3/2} = \frac{bm^2}{M_P} e^{a^2/2}. \tag{11.3c}$$

The ratio m/M_P is arbitrary at this point.

The next step in the calculation of the effective scalar potential of the "light" observable sector fields valid at an energy scale $Q \ll M_P$ is to evaluate the scalar potential with the "heavy" hidden sector field (whose quanta cannot be excited at this low energy scale) set to its VEV. Finally, we take the flat space limit, $M_P \to \infty$ while keeping $m_{3/2}$ fixed. The effective scalar potential reduces to,

$$V_{\text{eff}} = m^4 e^{a^2} \left[(1+ab)^2 - 3b^2\right] + \left|\frac{\partial \tilde{f}_o}{\partial C_n}\right|^2 + V_{\text{ssb}}, \tag{11.4a}$$

where $\tilde{f}_o$ is the rescaled scalar superpotential $\tilde{f}_o = e^{a^2/2}\hat{f}_o$,

$$V_{\text{ssb}} = \sum_n \left[1 + \left(a + \frac{1}{b}\right)^2 - 3\right] m_{3/2}^2 |C_n|^2 \\ + m_{3/2} \sum_n \left[\frac{\partial \tilde{f}_o}{\partial C_n} C_n + A\tilde{f}_o + \quad \text{h.c.}\right], \tag{11.4b}$$

and $A = a\left(\frac{1}{b} + a\right) - 3$.

The first term of V_{eff} in (11.4a) is the cosmological constant, which may be fine-tuned to zero by adopting $(1+ab)^2 = 3b^2$. The second term of V_{eff} is the "usual" superpotential contribution to the scalar potential, as in theories with global SUSY, where $\tilde{f}_o$ is now identified as the superpotential of the effective theory. The term V_{ssb} evidently contains various soft SUSY breaking terms. The first of these are mass terms for the scalar components of the visible sector superfields: if the cosmological constant is fine-tuned to zero, they are *all* given by $m_{\text{scalar}} = m_{3/2}$ in this simple case, and the desired universality is obtained. We can choose m^2 so that $m_{3/2} \sim M_{\text{weak}}$, the size required for low energy supersymmetry to stabilize the electroweak scale. The smallness of the ratio m/M_P still needs to be explained. The remaining terms in V_{ssb} correspond to bilinear and trilinear soft SUSY breaking terms, and are also of order $m_{3/2}$. These correspond to the b and **a** terms in (8.10) of Chapter 8. Notice that the **c** terms discussed in Chapter 8 are absent; this is because of our simple choice of the Kähler potential.

Before turning to the case of the general Kähler metric, we note that SUSY breaking in the hidden sector may also lead to soft SUSY breaking gaugino masses in the observable sector. This can clearly be seen from the first term in $\mathcal{L}_{F,\text{Int}}^G$ in (10.58b):

$$\mathcal{L}_F^G \ni \frac{1}{4} e^{G/2} \frac{\partial f_{AB}^*}{\partial \mathcal{S}^{j*}} (G^{-1})_k^j G^k \bar{\lambda}_A \lambda_B. \tag{11.5}$$

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In order to obtain non-zero gaugino masses, the gauge kinetic function must be a non-trivial function of hidden sector fields, and SUSY must be broken; i.e. $\langle G^k \rangle \neq 0$. Since $\langle G^k \rangle \sim m^2$, and $\partial f^*_{AB}/\partial \mathcal{S}^{j*} \sim 1/M_\mathrm{P}$, we expect that the resulting gaugino mass $m_{1/2} \sim m^2/M_\mathrm{P} \sim m_{3/2}$, and is also of order the weak scale. Clearly, without any assumptions about unification of gauge interactions, we will obtain *independent* masses for $SU(3)_\mathrm{C}$, $SU(2)_\mathrm{L}$, and $U(1)_\mathrm{Y}$ gauginos.

We should keep in mind that the soft SUSY breaking effective Lagrangian for the low energy theory that we have just obtained will still obtain radiative corrections from the interactions in this *low energy* theory. Although this low energy theory may be weakly coupled, these radiative corrections would be expected to depend on $k = \frac{g^2}{8\pi^2}\ln(M_\mathrm{P}/m_{3/2})$, where g is a typical coupling in the low energy theory (it could be one of the gauge couplings.) Since k is not small, it is important to sum these logarithms. This is done by using "running parameters" obtained by solving the renormalization group equations discussed in Chapter 9. In other words, the parameters in the "low energy theory" should be regarded as renormalized at some high scale $\sim M_\mathrm{P}$.

The careful reader will have traced the reason for universal masses in (11.4b) to our assumption of a flat Kähler metric. More general forms of the Kähler potential lead to non-universal SSB masses and couplings as can be seen from the exercise at the end of this section. This was first pointed out by Soni and Weldon.[4] Rather than list the rather complicated expressions for these parameters, we will just note several important features:

1. Although universality is not a generic feature of supergravity models, the scale of SSB masses and couplings is generally still set by $m_{3/2}$.
2. The trilinear **a** terms, in general, are not proportional to the corresponding superpotential Yukawa couplings.
3. Trilinear **c** terms are possible, but are suppressed by higher powers of M_P.

Regarding the first point, we should mention that even if we do arrange for a model with universal scalar masses at tree level (for instance, by the minimal choice for the Kähler potential), loop corrections will in general spoil the degeneracy. Indeed, assuming universal scalar masses is tantamount to assuming a $U(n)$ symmetry amongst the n observable sector superfields. However, this symmetry is explicitly broken by superpotential Yukawa couplings in the observable sector. Moreover, there is no theoretical argument for such a symmetry. Thus, while it might be possible to accommodate universality by making technical assumptions of the underlying physics, it seems fair to say that universality is not a generic feature of supergravity

[4] S. Soni and H. A. Weldon, *Phys. Lett.* **126B**, 215 (1983). Convenient general expressions for the soft SUSY breaking parameters are given by A. Brignole *et al.* in *Perspectives on Supersymmetry*, edited by G. Kane, World Scientific (1998).

models. This has provided motivation for the construction of alternative scenarios where the scalar mass degeneracies needed for solving the SUSY flavor and CP problems occur for different reasons.

Exercise (Non-universal scalar masses) *Show that the low energy effective potential leads to non-universal scalar mass terms if the Kähler potential has the form,*

$$K(\hat{\mathcal{S}}, \hat{\mathcal{S}}^\dagger) = \hat{h}^\dagger \hat{h} + \sum_n \tilde{K}_n \left(\frac{\hat{h}\hat{h}^\dagger}{M_{\rm P}^2} \right) \hat{C}^{\dagger n} \hat{C}_n,$$

where $\tilde{K}_n$ are dimensionless functions of the hidden sector field. Remember that for this Kähler potential the kinetic energy terms of the scalar components of $\hat{C}_n$ do not have the canonical form, so that these fields have to be rescaled to obtain canonical kinetic energies. Notice that this redefinition implies that we would have obtained universal mass terms if K_n had just been some constants rather than field-dependent functions.

Check also the form of supersymmetry breaking trilinear interactions in the low energy theory. Are the A terms universal?

Exercise *We found that* **c***-type trilinears were absent in the minimal model. Convince yourself that this type of SUSY trilinears can arise if one allows trilinear terms in the Kähler potential. Remember that these terms will always be suppressed by powers of $M_{\rm P}$.*

Although the details of the hidden sector are unimportant for low energy phenomenology, it is gratifying to see such a hidden sector is present in many theoretical frameworks. For instance, in heterotic string models, there is a natural, built-in hidden sector comprised of the dilaton field S, arising from the gravitational sector of the theory, and the moduli fields T_i, which parametrize the size and shape of the compactification. If the auxiliary fields of these multiplets provide the seeds for SUSY breaking, then the resulting effective theory below the Planck scale may be just a four-dimensional supersymmetric gauge theory with weak scale soft SUSY breaking terms.

11.1.2 Why is the μ parameter small?

We have just seen that supergravity models provide a rationale for why the scale of SUSY breaking parameters of the MSSM is much smaller than $M_{\rm P}$. On dimensional grounds we would expect though that the supersymmetry conserving μ parameter

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to be of order $M_{\rm P}$ rather than $m_{3/2}$, which would destroy the mechanism for electroweak symmetry breaking in SUSY models. This is known as the μ problem.

Supergravity models provide an elegant mechanism for generating the μ term with the right magnitude, provided that we assume that μ is forbidden by a symmetry that is violated only by interactions with the hidden sector.[5] Although it would then be absent in the tree-level superpotential of the observable sector, an effective μ term would develop via the gravitational interactions with the hidden sector. To see this, we note that there is nothing to forbid a (non-renormalizable) term,

$$K(\hat{h}, \hat{H}_u, \hat{H}_d) \ni \frac{\lambda \hat{h}^\dagger \hat{H}_u \hat{H}_d}{M_{\rm P}}, \tag{11.6a}$$

in the Kähler potential, where $\hat{h}$ is a hidden sector left-chiral superfield and $\hat{H}_u$ and $\hat{H}_d$ the Higgs superfields of the MSSM. Since the F-component of $\hat{h}$ develops a VEV $\sim m^2$, the action of the low energy effective theory includes a term,

$$\begin{aligned} \int \mathrm{d}^4x \mathcal{L} &\ni -\frac{1}{4}\int \mathrm{d}^4x \mathrm{d}^4\theta K(\hat{h}, \hat{H}_u, \hat{H}_d) + \quad \text{h.c.} \\ &\sim \frac{m^2\lambda}{M_{\rm P}} \int \mathrm{d}^4x \mathrm{d}^2\theta \hat{H}_u \hat{H}_d + \quad \text{h.c.} \end{aligned} \tag{11.6b}$$

The reader will recognize this as the superpotential μ term of the MSSM, with a magnitude $|\mu| \sim m_{3/2}$ as phenomenologically required.

11.1.3 Supergravity Grand Unification (SUGRA GUTs)

Minimal supergravity (mSUGRA) model

We have already encountered the minimal supergravity (mSUGRA) model in Chapter 9 where we adopted the universality hypothesis for gaugino masses, scalar masses and the various A-parameters at a high scale $Q \sim M_{\rm GUT}$. This scenario can be obtained within the framework with gravity mediated SUSY breaking.[6] The choice of a flat Kähler metric leads to a common mass for all scalars of $m_0^2 = m_{3/2}^2 + V_0/M_{\rm P}^2$, where V_0 is the minimum of the scalar potential. It is this technical assumption of the "minimal" choice of the Kähler potential that the "minimal" in minimal supergravity refers to. In this case though universality is likely to hold closer to $Q \sim M_{\rm P}$. Common gaugino masses at $Q = M_{\rm GUT}$ may arise because of grand unification of gauge interactions. But these may also be obtained by assuming that the gauge kinetic function has the same field dependence on the

[5] G. F. Giudice and A. Masiero, *Phys. Lett.* **206B**, 480 (1988).
[6] A. Chamseddine, R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **49**, 970 (1982); R. Barbieri, S. Ferrara and C. Savoy, *Phys. Lett.* **B 119**, 343 (1982); N. Ohta, *Prog. Theor. Phys.* **70**, 542 (1983); L. Hall, J. Lykken and S. Weinberg, *Phys. Rev.* **D27**, 2359 (1983).

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hidden sector fields, for each factor of gauge symmetry: e.g. $f^a_{AB} = c^a \delta_{AB} f(h_m)$, where a labels the different factors of the gauge group, and c^a are real numbers. The constants c^a disappear from the mass term upon canonical normalization of the gaugino kinetic energy terms.

The fundamental parameters of the model are the set (9.22). We have already seen that radiative EWSB (discussed in Chapter 9) allows us to trade the bilinear SSB parameter B (equivalently B_0) in favor of $\tan\beta$, and also to fix the value of μ^2 to reproduce the experimental value of M_Z. Then, the parameter space of the model is given by

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta, \mathrm{sign}\,(\mu). \tag{11.7}$$

It is common to assume that the universality of SSB parameters holds at M_{GUT} rather than M_{P}.

$SU(5)$ supergravity GUT model

We will assume that the reader is familiar with the motivations for grand unification. Grand unified theories are especially attractive when combined with supergravity. The simplest model, based on $SU(5)$ gauge symmetry,

- allows for unification of the gauge symmetries of the SM into a single Lie group,
- provides a group theoretic explanation for the ad hoc hypercharge assignments of the SM or MSSM fields.

It is usually assumed that supersymmetric $SU(5)$ grand unification is valid at mass scales $Q > M_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV, extending at most to the reduced Planck scale M_{P} where gravitational effects become sizable. Below $Q = M_{\mathrm{GUT}}$, the $SU(5)$ model (with a minimal matter content) breaks down to the MSSM with the usual $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$ gauge symmetry.

In the $SU(5)$ model, the $\hat{D}^c$ and $\hat{L}$ superfields are members of a $\bar{\mathbf{5}}$ superfield $\hat{\phi}$, while the $\hat{Q}$, $\hat{U}^c$, and $\hat{E}^c$ superfields occur in the $\mathbf{10}$ representation $\hat{\psi}$. There is a replication of generations. The Higgs sector of the minimal $SU(5)$ model is comprised of three super-multiplets: $\hat{\Sigma}(\mathbf{24})$ which is responsible for breaking $SU(5)$, together with $\hat{\mathcal{H}}_1(\bar{\mathbf{5}})$ and $\hat{\mathcal{H}}_2(\mathbf{5})$ which contain the MSSM Higgs doublet superfields $\hat{H}_d$ and $\hat{H}_u$ respectively.[7] The superpotential is given by,

$$\hat{f} = \mu_\Sigma \mathrm{Tr}\hat{\Sigma}^2 + \frac{1}{6}\lambda_\Sigma \mathrm{Tr}\hat{\Sigma}^3 + \mu_H \hat{\mathcal{H}}_1\hat{\mathcal{H}}_2 + \lambda\hat{\mathcal{H}}_1\hat{\Sigma}\hat{\mathcal{H}}_2 + \frac{1}{4} f_t \epsilon_{ijklm}\hat{\psi}^{ij}\hat{\psi}^{kl}\hat{\mathcal{H}}_2^m + \sqrt{2} f_b \hat{\psi}^{ij}\hat{\phi}_i\hat{\mathcal{H}}_{1j} + \cdots, \tag{11.8a}$$

[7] This is the primary reason why we assigned $\hat{H}_d$ to transform as the $\mathbf{2}^*$ representation of $SU(2)_{\mathrm{L}}$.

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where we neglect the Yukawa couplings of the first two generations, and retain just f_t and f_b, the top and bottom quark Yukawa couplings. The couplings λ and λ_Σ are GUT Higgs sector self-couplings, and μ_Σ and μ_H are superpotential Higgs mass terms. Note that in this model $f_b = f_\tau$ when the $SU(5)$ symmetry is unbroken.

Proton decay is the smoking gun signature of grand unification. In non-supersymmetric GUTs, this occurs via dimension 6 operators involving X and Y gauge bosons. In supersymmetric GUTs, proton decay can also be mediated by color-triplet higgsinos which, being fermions, lead to dimension 5 baryon-number-violating operators which are potentially much more dangerous.[8] Furthermore, higgsino-mediated proton decay depends on Yukawa couplings. As a result, in SUSY GUTs, the proton preferentially decays to kaons rather than to pions as in standard GUTs.

Soft supersymmetry breaking terms are induced by hidden sector local SUSY breaking, and are parametrized by:

$$\begin{aligned}\mathcal{L}_{\rm soft} = & -m^2_{\mathcal{H}_1}|\mathcal{H}_1|^2 - m^2_{\mathcal{H}_2}|\mathcal{H}_2|^2 - m^2_\Sigma {\rm Tr}\{\Sigma^\dagger\Sigma\} - m_5^2|\phi|^2 - m_{10}^2{\rm Tr}\{\psi^\dagger\psi\} \\ & -\frac{1}{2}M_5\bar{\lambda}_\alpha\lambda_\alpha \\ & + \Big[B_\Sigma\mu_\Sigma {\rm Tr}\Sigma^2 + \frac{1}{6}A_{\lambda_\Sigma}\lambda_\Sigma{\rm Tr}\Sigma^3 + B_H\mu_H\mathcal{H}_1\mathcal{H}_2 + A_\lambda\lambda\mathcal{H}_1\Sigma\mathcal{H}_2 \\ & + \frac{1}{4}A_t f_t\epsilon_{ijklm}\psi^{ij}\psi^{kl}\mathcal{H}_2^m + \sqrt{2}A_b f_b\psi^{ij}\phi_i\mathcal{H}_{1j} + \quad {\rm h.c.}\Big]. \end{aligned} \tag{11.8b}$$

The various SSB parameters and the gauge and Yukawa couplings evolve with energy scale according to 15 renormalization group equations (the first two generations are degenerate). Assuming universality at $M_{\rm P}$, one imposes

$$\begin{aligned} m_{10} = m_5 = m_{\mathcal{H}_1} = m_{\mathcal{H}_2} = m_\Sigma \equiv m_0 \\ A_t = A_b = A_\lambda = A_{\lambda_\Sigma} \equiv A_0, \end{aligned} \tag{11.9a}$$

and evolves all the soft masses from $M_{\rm P}$ to $M_{\rm GUT}$. Although there are no large logarithms or couplings, there is substantial evolution due to large group theory factors arising from the fact that the representations contain many particles. The MSSM soft breaking masses at $M_{\rm GUT}$ are specified via

$$\begin{aligned} m_Q^2 = m_U^2 = m_E^2 \equiv m_{10}^2, \quad m_D^2 = m_{\rm L}^2 \equiv m_5^2, \\ m_{H_d}^2 = m_{\mathcal{H}_1}^2, \quad m_{H_u}^2 = m_{\mathcal{H}_2}^2. \end{aligned} \tag{11.10}$$

[8] Indeed, if $\lambda \lesssim 0.7$, triplet higgsinos are too light and one runs into trouble with constraints from proton decay. Some authors have argued that these constraints, in fact, rule out minimal SUSY $SU(5)$. It is clear, however, that the proton decay rate depends on the unknown details of GUT scale physics, and can be altered by complicating the GUT sector. For this reason, we will not consider such constraints here.

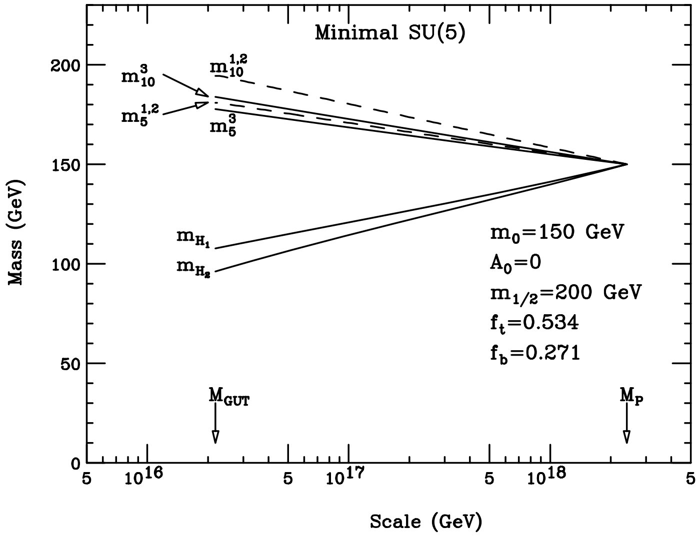

Figure 11.1 Evolution of SSB masses for a typical case study in SUSY $SU(5)$ GUT model with $\tan\beta = 35$, which allows for b-τ Yukawa unification. We choose $\lambda = 0.7$ and $\lambda_\Sigma = 0.1$. Although not shown, A_t and A_b evolve to -88 GeV and -78 GeV at $Q = M_{\rm GUT}$. Reprinted from H. Baer, M. Diáz, P. Quintana and X. Tata, *JHEP* **04**, 016 (2000).

The evolution of SSB masses in SUSY $SU(5)$ is shown in Fig. 11.1. A striking feature is the sizable GUT scale splitting between the Higgs and matter scalar mass parameters, arising from the large λ Yukawa coupling contribution to the running of Higgs boson mass parameters. The masses of the **10** and **5*** multiplets evolve differently, as do those of multiplets of the different generations. In particular, third generation multiplet masses are somewhat suppressed compared to their counterparts of the first two generations owing to Yukawa coupling contributions to the RGE running. Thus, even in an $SU(5)$ SUGRA GUT model, we would expect non-universality. However, scalar masses for multiplets in the first two generations are still highly degenerate, so that FCNCs are suppressed. We have also checked that even if we start with $A_0 = 0$ at $Q = M_{\rm P}$, sizable values of A_t and A_b are generated at $Q = M_{\rm GUT}$. This could have a significant effect on the phenomenology of third generation sparticles.

Exercise *Draw a Feynman diagram involving triplet higgsino exchange for the baryon number violating process,*

$$\tilde{d}\tilde{u} \to \bar{s}\bar{\nu}_\mu .$$

Notice that the squarks in the initial state can be obtained from u and d quarks by exchanging a chargino. This "dressed" diagram mediates the process $du \to \bar{s}\bar{\nu}_\mu$

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which in turn can cause $p \to K^+\bar{\nu}_\mu$. Convince yourself that the amplitude for this process is suppressed only by a single power of $M_{\rm GUT}$ and, hence, is much larger than that of the baryon number violating amplitude mediated by exchange of GUT gauge bosons.

Non-universal gaugino masses

Since there is no reason to expect the gauge kinetic function to be field independent, gaugino masses are a generic prediction of supergravity models (or for that matter any non-renormalizable theory with broken supersymmetry). Moreover, (11.5) shows that the gaugino mass scale in the effective low energy theory is expected to be $\sim m_{3/2}$. Gauge invariance dictates that the gauge kinetic function must transform as the symmetric product of two adjoints under the gauge symmetry. If any of the auxiliary fields that break supersymmetry transform non-trivially under the grand unifying gauge group (but of course as an MSSM gauge singlet), non-universal MSSM gaugino masses are obtained. These may then be parametrized by,

$$\mathcal{L} \supset \frac{\langle F_h \rangle_{AB}}{M_{\rm P}} \bar{\lambda}_A \lambda_B + \cdots \tag{11.11}$$

where the λ_A are the gaugino fields, and F_h is the auxiliary field component of $\hat{h}$ that acquires a SUSY, and possibly GUT symmetry, breaking VEV. It is only in the special case where the fields F_h which break supersymmetry are GUT singlets that universal gaugino masses are obtained.

In the context of $SU(5)$ grand unification, F_h belongs to an $SU(5)$ representation which appears in the symmetric product of two adjoints:

$$(\mathbf{24}\times\mathbf{24})_{\rm symmetric} = \mathbf{1} \oplus \mathbf{24} \oplus \mathbf{75} \oplus \mathbf{200}\,, \tag{11.12}$$

where only $\mathbf{1}$ yields universal masses. If instead F_h transforms as any other irreducible representation that appears in (11.12), the MSSM gaugino mass parameters at $Q = M_{\rm GUT}$, though different, are related by group theory. The weak scale gaugino masses are then obtained by renormalization group evolution, starting from these non-universal values, as discussed in Chapter 9. The relative GUT scale $SU(3)$, $SU(2)$, and $U(1)$ gaugino masses M_3, M_2, and M_1 are listed in Table 11.1 along with the approximate masses after RGE evolution to $Q \sim M_Z$. These scenarios represent the predictive subset of the more general (and less predictive) case of an arbitrary superposition of these representations. The model parameters may be chosen to be,

$$m_0,\ M_3^0,\ A_0,\ \tan\beta,\ \text{and sign}\,(\mu), \tag{11.13}$$

where M_i^0 is the $SU(i)$ gaugino mass at scale $Q = M_{\rm GUT}$. M_2^0 and M_1^0 can be obtained in terms of M_3^0 via Table 11.1. Notice that the nature of the neutralino LSP

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Table 11.1 *Relative gaugino mass parameters at $Q = M_{\rm GUT}$ and their relative values evolved to $Q = M_Z$ in the four possible F_h irreducible representations in $SU(5)$ SUSY GUTS.*

	$M_{\rm GUT}$			M_Z		
F_h	M_3	M_2	M_1	M_3	M_2	M_1
1	1	1	1	~ 6	~ 2	~ 1
24	2	-3	-1	~ 12	~ -6	~ -1
75	1	3	-5	~ 6	~ 6	~ -5
200	1	2	10	~ 6	~ 4	~ 10

as well as the mass gap between the LSP and $\widetilde{Z}_2$ depend on the gauge transformation properties of F_h: as a result, SUSY phenomenology changes significantly in the different scenarios.[9]

$SO(10)$ supergravity GUT models

We saw in Section 9.7 that it is necessary to introduce right-handed neutrino superfields in order to give neutrinos a mass without spoiling the conservation of R-parity. Within the MSSM, or within the $SU(5)$ GUT framework just discussed, these gauge singlet superfields had to be introduced ad hoc. The body of evidence in support of neutrino mass, however, makes the grand unified group $SO(10)$ especially appealing because the minimal $SO(10)$ model contains three generations of matter superfields, with each generation together with a singlet neutrino superfield $\hat{N}^c$ included in the **16**-dimensional spinorial representation $\hat{\psi}_{16}$. Thus, $SO(10)$ allows not only for gauge group unification, but also for unification of matter in each generation into a single irreducible representation. Morover, singlet neutrinos essential for the implementation of the see-saw mechanism occur automatically in this framework. The Higgs bosons $\hat{H}_u$ and $\hat{H}_d$ lie within a **10**-dimensional fundamental representation $\hat{\phi}_{10}$. The superpotential for the model includes the term

$$\hat{f} \ni f\hat{\psi}_{16}\hat{\psi}_{16}\hat{\phi}_{10} + \cdots , \tag{11.14}$$

responsible for quark and lepton masses, with f being the single Yukawa coupling per generation in the GUT scale theory. The ellipsis represents terms including for instance higher dimensional Higgs representations and interactions responsible for the breaking of $SO(10)$.

[9] G. Anderson *et al.*, *Phys. Rev.* **D61**, 095005 (2000).

The unification of Yukawa couplings of each generation means that the third generation neutrino Yukawa coupling f_ν is large: $f_\nu = f_t$. The third generation neutrino Yukawa coupling f_ν splits $\tilde{\tau}_{\rm L}$ and $\tilde{\nu}_\tau$ masses from their first and second generation cousins, in the same way that tau Yukawa couplings split the staus from other slepton masses. In some models, this splitting is potentially measurable in linear collider experiments.

The soft SUSY breaking terms will include a common mass m_{16} for all matter scalars and a mass m_{10} for the Higgs scalars, along with a universal gaugino mass $m_{1/2}$, and common trilinear and bilinear SSB masses A_0 and B. Motivated by apparent gauge coupling unification in the MSSM, it is common to assume that $SO(10)$ breaks directly to the gauge group $SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}$ at $Q = M_{\rm GUT} = 2 \times 10^{16}$ GeV, though $SO(10)$ could well have broken to $SU(5)$ at a yet higher scale.

A novel feature arises because the rank of $SO(10)$ (the rank is the largest number of mutually commuting generators) is one higher than that of the MSSM gauge group. This effectively means that $SO(10)$ includes a (broken) $U(1)_X$ factor that is not a low energy symmetry. Naively, one would suppose that if the $U(1)_X$ breaking scale M_X is sufficiently large, $U(1)_X$ would be negligible for TeV scale physics. To see that this is not the case, let us consider a simple toy model, where $U(1)_X$ is broken by VEVs of a pair of MSSM gauge singlet fields Φ and $\bar{\Phi}$ with $U(1)_X$ charges +1 and -1, respectively. If we denote the scalar components of the MSSM fields by $\mathcal{S}_i$ and their $U(1)_X$ charges by x_i, we can write the scalar potential that determines $\langle\Phi\rangle$ and $\langle\bar{\Phi}\rangle$ as,

$$V = V_{\rm symm}(\Phi, \bar{\Phi}) + m^2|\Phi|^2 + \overline{m}^2|\bar{\Phi}|^2 + \frac{g_X^2}{2}\left[|\Phi|^2 - |\bar{\Phi}|^2 + x_i|\mathcal{S}_i|^2\right]^2. \quad (11.15)$$

The term $V_{\rm symm}$ comes from the superpotential for the heavy fields and is chosen to be symmetric under $\Phi \leftrightarrow \bar{\Phi}$. The next two terms are SSB masses for the heavy fields Φ and $\bar{\Phi}$: they are of order of the weak scale, but otherwise unrelated. The last term, which is the usual $U(1)_X$ D-term contribution to the potential, forces the minimum to be along the nearly D-flat direction $\langle\Phi\rangle \approx \langle\bar{\Phi}\rangle$. However, if $m \neq \overline{m}$, the minimum of the potential deviates from the D-flat direction by,

$$\langle\Phi\rangle^2 - \langle\bar{\Phi}\rangle^2 \simeq \frac{\overline{m}^2 - m^2}{2g_X^2}. \quad (11.16)$$

The last term in (11.15) then shows that the MSSM scalars $\mathcal{S}_i$ receive an additional contribution to the mass proportional to their $U(1)_X$ charge,

$$\Delta m_i^2 = \frac{x_i}{2} \times (\overline{m}^2 - m^2), \quad (11.17)$$

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which by (11.16) is of order the weak scale. Thus $U(1)_X$ leaves its imprint on the MSSM sfermion mass spectrum even if M_X is very large.

Returning to the $SO(10)$ model, the scalar field squared mass parameters at $Q = M_{\rm GUT}$ are then given by

$$\begin{aligned} m_Q^2 &= m_E^2 = m_U^2 = m_{16}^2 + M_D^2 \\ m_D^2 &= m_L^2 = m_{16}^2 - 3M_D^2 \\ m_{H_{u,d}}^2 &= m_{10}^2 \mp 2M_D^2 \\ m_N^2 &= m_{16}^2 + 5M_D^2, \end{aligned} \tag{11.18}$$

where M_D^2 parametrizes the magnitude of the $U(1)_X$ D-terms just discussed, and can, owing to our ignorance of the gauge symmetry breaking mechanism, be taken as a free parameter of order of the weak scale, with either positive or negative values. Thus, the model is characterized by the following free parameters

$$m_{16},\ m_{10},\ M_D^2,\ m_{1/2},\ A_0,\ \text{sign}\,(\mu). \tag{11.19}$$

Since

$$\frac{m_t}{m_b} \sim \frac{f_t v_u}{f_b v_d},$$

solutions with unification of Yukawa couplings are possible only for large values of $\tan\beta$. This argument is only qualitative because radiative corrections to m_b are very important. In practice, the value of $\tan\beta$ is restricted by the requirement of Yukawa coupling unification, and so is tightly constrained to a narrow range around $\tan\beta \sim 50$–55.

Inverted hierarchy models

A phenomenologically interesting class of models referred to as *inverted mass hierarchy* (IMH) models, have the matter sfermion mass order inverted from the order of the corresponding fermions. Specifically, scalars of the first and second generation are expected to have masses at the multi-TeV scale so that a decoupling solution to the SUSY flavor and CP problems may be invoked. Because these sparticles have very tiny couplings to the Higgs sector, they do not lead to unnaturally large fine-tuning. On the other hand, third generation sfermions (which have large couplings to the Higgs sector) are expected to be in the sub-TeV mass range to accommodate constraints from naturalness.

One class of IMH models has the inverted mass hierarchy generated *radiatively*. In this case, all the scalars begin with multi-TeV masses at the GUT scale, while gaugino masses are in the sub-TeV range. In models with Yukawa unification and

$SO(10)$-like GUT scale boundary conditions of

$$4A_0^2 = 2m_{10}^2 = m_{16}^2, \tag{11.20}$$

the large third generation Yukawa coupling acts to drive third generation and Higgs scalars to sub-TeV values, while leaving multi-TeV first and second generation scalars. A positive D-term contribution with $M_D \sim m_{16}/3$ is needed for radiative symmetry breaking.[10] When fully implemented, including constraints from radiative EWSB, it turns out that first and second generation scalars as heavy as 2–3 TeV can be allowed. This is sufficient to suppress many CP-violating processes, but is not enough to fully suppress FCNCs. Such a model might be viable if it is coupled with a partial degeneracy solution for first and second generation scalars. Yukawa coupling unification is also possible for first and second generation scalar masses ∼8–10 TeV, but third generation sfermions then have masses around 3–5 TeV.

A second class of IMH models arises if one assumes an IMH already in place at the GUT scale. This may be possible in non-minimal gravity-mediated models. In evaluating sparticle mass spectra from these GUT scale IMH models, it is crucial to use two-loop RGEs. The form of the two-loop RGEs for SSB masses is given by

$$\frac{dm_i^2}{dt} = \frac{1}{16\pi^2}\beta_{m_i^2}^{(1)} + \frac{1}{(16\pi^2)^2}\beta_{m_i^2}^{(2)}, \tag{11.21}$$

where $t = \ln Q$, $i = Q_j,\ U_j, D_j, L_j$, and E_j, and $j = 1$–3 is a generation index. The one-loop β-function for the evolution of (the initially sub-TeV) third generation scalar masses depends only on these scalar masses and the (also sub-TeV) gaugino masses. Two-loop terms are formally suppressed relative to one-loop terms by the square of a coupling constant as well as an additional loop factor of $16\pi^2$. However, these two-loop terms include contributions from *all* scalars. Specifically, the two-loop β functions include,

$$\beta_{m_i^2}^{(2)} \ni a_i g_3^2\sigma_3 + b_i g_2^2\sigma_2 + c_i g_1^2\sigma_1, \tag{11.22}$$

where

$$\sigma_1 = \frac{1}{5}g_1^2\left\{3(m_{H_u}^2 + m_{H_d}^2) + Tr[\mathbf{m}_Q^2 + 3\mathbf{m}_L^2 + 8\mathbf{m}_U^2 + 2\mathbf{m}_D^2 + 6\mathbf{m}_E^2]\right\},$$
$$\sigma_2 = g_2^2\left\{m_{H_u}^2 + m_{H_d}^2 + Tr[3\mathbf{m}_Q^2 + \mathbf{m}_L^2]\right\}, \quad \text{and}$$
$$\sigma_3 = g_3^2 Tr[2\mathbf{m}_Q^2 + \mathbf{m}_U^2 + \mathbf{m}_D^2],$$

and the $\mathbf{m}_i^2$ are squared mass matrices in generation space. The numerical coefficients a_i, b_i, and c_i are related to the quantum numbers of the scalar fields, but are

[10] Such a D-term reduces $m_{H_u}^2$ relative to $m_{H_d}^2$, which facilitates EWSB. Indeed, slightly better Yukawa coupling unification is obtained if the D-term splitting is applied to just the Higgs scalars rather than to all the sparticles, but a qualitatively similar hierarchy is obtained.

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all positive quantities. Thus, incorporation of multi-TeV masses for the first and second generation scalars leads to an overall positive, *possibly dominant*, contribution to the slope of SSB mass trajectories versus energy scale. Although formally a two-loop effect, the smallness of the couplings is compensated by the much larger values of masses of the first two generations of scalars. In running from $M_{\rm GUT}$ to $M_{\rm weak}$, this results in an overall *reduction* of scalar masses, and is most important for the sub-TeV third generation scalar masses which may be driven tachyonic. That this not occur then constrains the size of the hierarchy. For values of SSB masses which fall short of these constraints, a sort of see-saw effect amongst scalar masses occurs: the higher the value of first and second generation scalar masses, the larger will be the two-loop suppression of third generation and Higgs scalar masses. In this class of models, first and second generation scalars with masses of order 10–15 TeV may co-exist with sub-TeV third generation scalars, thus giving a very large suppression to both FCNC and CP-violating processes.

11.2 Anomaly-mediated SUSY breaking

In supergravity models, MSSM soft SUSY breaking parameters are thought to arise from tree-level gravitational interactions of observable sector superfields with gauge singlet hidden sector fields that can acquire a Planck scale VEV. It was subsequently recognized that there is an additional one-loop contribution to SSB parameters that is always present when SUSY is broken.[11] Usually this latter contribution, which originates in the super-Weyl anomaly (and is, therefore, called the anomaly-mediated supersymmetry breaking (AMSB) contribution), only makes a loop suppressed correction to the leading tree-level SSB parameters, so that the pattern of sparticle masses is qualitatively unchanged from what we have described in the last section. However, in models without SM gauge singlet superfields that can acquire a Planck scale VEV, the usual supergravity contribution to gaugino masses is suppressed by an additional factor $M_{\rm SUSY}/M_{\rm P}$ relative to $m_{\frac{3}{2}} = M_{\rm SUSY}^2/M_{\rm P}$, and the anomaly-mediated contribution can dominate. Extra dimensional theories potentially offer an alternative way to suppress supergravity couplings between the observable sector and the hidden sector (goldstino) field which lead to tree-level MSSM SSB parameters $\sim m_{\frac{3}{2}}$: these supergravity contributions may be exponentially suppressed if the SUSY breaking and visible sectors reside on different branes that are "sufficiently separated" in a higher dimensional space.[12] In this case, the suppression is the result of geometry and not a symmetry, though then one has to wonder about the dynamics that results in such a geometry. Moreover, it has been

[11] L. Randall and R. Sundrum, *Nucl. Phys.* **B557**, 79 (1999); G. Giudice *et al.*, *JHEP* **12**, 027 (1998).
[12] The term brane means a lower dimensional spatial slice of the entire space.

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argued that while it is possible to find models where AMSB terms may dominate, their construction appears to require more than just spatial separation between the observable sector and SUSY breaking branes.[13]

A derivation of the AMSB contribution to SSB parameters would require techniques beyond those that we have developed. We will, therefore, simply list the relevant results and proceed to discuss their implications. Before doing so, we note that these contributions are determined just by the super-conformal anomaly. Since anomalies depend only on the low energy theory, the AMSB contributions to SSB parameters are insensitive to (unknown) physics at the high scale. These contributions, which can be written in terms of the β-functions and anomalous dimensions of the theory with *unbroken* supersymmetry, can be explicitly checked to be invariant under renormalization group evolution, consistent with their insensitivity to ultra-violet physics.[14]

The AMSB contribution to the gaugino mass is given by,

$$M_i = \frac{\beta_{g_i}}{g_i} m_{\frac{3}{2}}, \tag{11.23}$$

where β_{g_i} is the corresponding beta function, defined by $\beta_{g_i} \equiv \mathrm{d}g_i/\mathrm{d}\ln\mu$. The gaugino masses are not universal, but given by the ratios of the respective β-functions.

The anomaly-mediated contribution to the scalar mass parameter is given by,

$$m_{\tilde{f}}^2 = -\frac{1}{4}\left\{\frac{\mathrm{d}\gamma}{\mathrm{d}g}\beta_g + \frac{\mathrm{d}\gamma}{\mathrm{d}f}\beta_f\right\} m_{\frac{3}{2}}^2, \tag{11.24}$$

where β_f is the β-function for the corresponding superpotential Yukawa coupling, and $\gamma = \partial \ln Z/\partial \ln \mu$, with Z the wave function renormalization constant. Finally, the anomaly-mediated contribution to the trilinear SUSY breaking scalar coupling is given by,

$$A_f = \frac{\beta_f}{f} m_{\frac{3}{2}}. \tag{11.25}$$

The following features of the AMSB contributions to the SSB parameters are worth noting.

1. AMSB contributions to gaugino and sfermion masses as well as A-parameters are all of the same scale, $m_{3/2}/16\pi^2$. Requiring this to be the weak scale puts the gravitino mass in a cosmologically safe range.[15]

[13] See A. Anisimov, M. Dine, M. Graesser and S. Thomas, *Phys. Rev.* **D65**, 105011 (2002).
[14] Indeed the AMSB expressions for scalar masses and A-parameters were first obtained via this route. See I. Jack, D.R.T. Jones and A. Pickering, *Phys. Lett.* **B426**, 73 (1998); L. Avdeev, D. Kazakov and I. Kondrashuk, *Nucl. Phys.* **B510**, 289 (1998).
[15] See e.g. S. Weinberg, *The Quantum Theory of Fields, Vol. III*, p 198, Cambridge University Press (2000).

2. Since Yukawa interactions are negligible for the first two generations, the anomaly-mediated contributions to the masses of the corresponding matter scalars with the same gauge quantum numbers are essentially equal. This solves the SUSY flavor problem if the AMSB contribution is dominant. Indeed the ultra-violet insensitivity of the AMSB scenario guarantees that no flavor violation results from high scale physics as long as AMSB contributions dominate.
3. The anomaly contribution turns out to be negative for sleptons, necessitating additional sources for the squared masses of scalars. Since the masses are insensitive to high scale physics, we cannot ameliorate this within this framework by adding new fields at the high scale. There are several proposals in the literature, but phenomenologically it suffices to add a universal contribution m_0^2 (which, of course, preserves the desired degeneracy between the first two generations of scalars) to Eq. (11.24), and regard m_0 as an additional parameter. It is assumed that the ad hoc introduction of m_0^2 in Eq. (11.24) does not affect the other parameters. This is referred to as the minimal AMSB model which we now examine.

11.2.1 The minimal AMSB (mAMSB) model

As we have just mentioned, the mAMSB model is *defined* by assuming that gaugino masses and A-parameters are given by (11.23) and (11.25), respectively while the expression for SSB scalar masses is amended by the addition of a (sufficiently large) universal mass parameter m_0^2 to make slepton masses positive. It is assumed that the AMSB mass relations hold at $Q = M_{\rm GUT}$, and weak scale parameters are obtained from these via RGE evolution.

At one-loop level, with the field content of the MSSM at low energy, gaugino masses are given by,

$$M_1 = \frac{33}{5}\frac{g_1^2}{16\pi^2}m_{3/2}, \tag{11.26a}$$

$$M_2 = \frac{g_2^2}{16\pi^2}m_{3/2}, \quad \text{and} \tag{11.26b}$$

$$M_3 = -3\frac{g_3^2}{16\pi^2}m_{3/2}. \tag{11.26c}$$

Notice the differing sign on the gluino mass term. This has implications for the sign of the SUSY contribution to the anomalous magnetic moment of the muon. Third generation scalar masses are given by

$$m_{U_3}^2 = \left(-\frac{88}{25}g_1^4 + 8g_3^4 + 2f_t\hat{\beta}_{f_t}\right)\frac{m_{3/2}^2}{(16\pi^2)^2} + m_0^2, \tag{11.27a}$$

$$m_{D_3}^2 = \left(-\frac{22}{25}g_1^4 + 8g_3^4 + 2f_b\hat{\beta}_{f_b}\right)\frac{m_{3/2}^2}{(16\pi^2)^2} + m_0^2, \tag{11.27b}$$

$$m_{Q_3}^2 = \left(-\frac{11}{50}g_1^4 - \frac{3}{2}g_2^4 + 8g_3^4 + f_t\hat{\beta}_{f_t} + f_b\hat{\beta}_{f_b}\right)\frac{m_{3/2}^2}{(16\pi^2)^2} + m_0^2, \tag{11.27c}$$

$$m_{L_3}^2 = \left(-\frac{99}{50}g_1^4 - \frac{3}{2}g_2^4 + f_\tau\hat{\beta}_{f_\tau}\right)\frac{m_{3/2}^2}{(16\pi^2)^2} + m_0^2, \tag{11.27d}$$

$$m_{E_3}^2 = \left(-\frac{198}{25}g_1^4 + 2f_\tau\hat{\beta}_{f_\tau}\right)\frac{m_{3/2}^2}{(16\pi^2)^2} + m_0^2, \tag{11.27e}$$

$$m_{H_u}^2 = \left(-\frac{99}{50}g_1^4 - \frac{3}{2}g_2^4 + 3f_t\hat{\beta}_{f_t}\right)\frac{m_{3/2}^2}{(16\pi^2)^2} + m_0^2, \tag{11.27f}$$

$$m_{H_d}^2 = \left(-\frac{99}{50}g_1^4 - \frac{3}{2}g_2^4 + 3f_b\hat{\beta}_{f_b} + f_\tau\hat{\beta}_{f_\tau}\right)\frac{m_{3/2}^2}{(16\pi^2)^2} + m_0^2. \tag{11.27g}$$

The A-parameters are given by,

$$A_t = \frac{\hat{\beta}_{f_t}}{f_t}\frac{m_{3/2}}{16\pi^2}, \tag{11.28a}$$

$$A_b = \frac{\hat{\beta}_{f_b}}{f_b}\frac{m_{3/2}}{16\pi^2}, \quad \text{and} \tag{11.28b}$$

$$A_\tau = \frac{\hat{\beta}_{f_\tau}}{f_\tau}\frac{m_{3/2}}{16\pi^2}. \tag{11.28c}$$

The quantities $\hat{\beta}_{f_i}$ that enter the expressions for scalar masses and A-parameters are given by,

$$\hat{\beta}_{f_t} = 16\pi^2\beta_t = f_t\left(-\frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 + 6f_t^2 + f_b^2\right), \tag{11.29a}$$

$$\begin{aligned}\hat{\beta}_{f_b} &= 16\pi^2\beta_b \\ &= f_b\left(-\frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 + f_t^2 + 6f_b^2 + f_\tau^2\right),\end{aligned} \tag{11.29b}$$

$$\hat{\beta}_{f_\tau} = 16\pi^2\beta_\tau = f_\tau\left(-\frac{9}{5}g_1^2 - 3g_2^2 + 3f_b^2 + 4f_\tau^2\right). \tag{11.29c}$$

The first two generations of squark and slepton masses are given by the corresponding formulae above with the Yukawa couplings set to zero. Eq. (11.26a)–(11.28c) serve as RGE boundary conditions at $Q = M_{\rm GUT}$. We evolve the MSSM parameters to the weak scale and, as usual obtain B and μ^2 in accord with the constraint from radiative electroweak symmetry breaking. The model is, therefore, characterized by the parameter set,

$$m_0,\ m_{3/2},\ \tan\beta,\ \text{and sign}\ (\mu). \tag{11.30}$$

The most notable feature of this framework is the hierarchy of gaugino masses. The gluino is (as in models with unified gaugino mass parameters) much heavier than the electroweak gauginos, but the novel feature is that $M_1/M_2 \sim 3$, so that the wino is lighter than the bino. Ignoring gaugino–higgsino mixing, the charged and neutral components of the $SU(2)$ gauginos would be degenerate: it is important to include radiative corrections to decide which of these is the LSP. Happily, these make the neutralino lighter than the chargino (else the model would be in trouble with cosmology). The near degeneracy of the chargino and the wino LSP have implications for particle phenomenology as well as cosmology. In particular, for the evaluation of the relic neutralino density, charginos and neutralinos coexist at the neutralino decoupling temperature, and co-annihilation effects are very important.

In Table 11.2, we show sparticle masses in the minimal AMSB model for two values of m_0, with other parameters being the same. Note that the parameter $m_{3/2}$ should be selected typically above 30–35 TeV to evade constraints from LEP experiments. From the spectra in the table, we see that for the smaller value of m_0, sleptons can be very light, though for very large values of m_0 they will be degenerate with squarks. We observe several characteristics of the AMSB spectrum. Most notable is that the $\widetilde{W}_1$ and $\widetilde{Z}_1$ are nearly degenerate in mass, so that in addition to the usual leptonic decay modes $\widetilde{W}_1 \to \widetilde{Z}_1 \ell\nu$, the only other kinematically allowed (and in these cases dominant) decay of the chargino is $\widetilde{W}_1^\pm \to \widetilde{Z}_1\pi^\pm$. The chargino has a very small width, corresponding to a lifetime $\sim 1.5 \times 10^{-9}$ s, so that it would be expected to travel a significant fraction of a meter before decaying. We also see that the $\tilde{\ell}_{\rm L}$ and $\tilde{\ell}_{\rm R}$ are nearly mass degenerate. This degeneracy, which seems fortuitous, is much tighter than expected in the mSUGRA framework.

In the minimal AMSB framework, $m_{\widetilde{W}_1} - m_{\widetilde{Z}_1}$ is typically bigger than 160 MeV, so that $\widetilde{W}_1 \to \widetilde{Z}_1\pi$ is always allowed and the chargino decays within the detector. The chargino would then manifest itself only as missing energy, unless the decay length is a few tens of centimeters, so that the chargino track can be established in the detector. The track would then seem to disappear since the presence of the soft pion would be very difficult to detect. Some parameter regions with $m_{\widetilde{W}_1} - m_{\widetilde{Z}_1} < m_{\pi^\pm}$ may be possible; in this case, the chargino would mainly decay via $\widetilde{W}_1 \to \widetilde{Z}_1 e\nu$ and its decay length (depending on the mass difference) may then be larger than several meters. It would then show up via a search for long-lived charged exotics.

Table 11.2 *Model parameters and weak scale sparticle masses in GeV for two minimal anomaly-mediated SUSY breaking case studies.*

parameter	$AMSB(200)$	$AMSB(500)$
m_0	200	500
$m_{3/2}$	35,000	35,000
$\tan\beta$	5	5
μ	> 0	> 0
$m_{\tilde{g}}$	804	818
$m_{\tilde{u}_L}$	775	894
$m_{\tilde{t}_1}$	542	611
$m_{\tilde{b}_1}$	683	774
$m_{\tilde{\ell}_L}$	149	481
$m_{\tilde{\ell}_R}$	136	477
$m_{\tilde{\tau}_1}$	118	471
$m_{\tilde{\tau}_2}$	160	484
$m_{\widetilde{W}_1}$	109	110
$m_{\widetilde{Z}_2}$	313	316
$m_{\widetilde{W}_1} - m_{\widetilde{Z}_1}$	0.171	0.172
m_h	114	113
m_A	658	813
μ	634	643
θ_τ	0.96	0.98
θ_b	0.08	0.05

Exercise *Verify by explicit computation that the one-loop expressions for the gaugino masses and A-parameters are scale invariant. For the hypercharge gaugino mass, for example, this means that*

$$M_1(Q) = \frac{33}{5}\frac{g_1(Q)^2}{16\pi^2}m_{3/2}$$

is true at **all scales***, not just as a boundary condition. Thus you need to verify that,*

$$Q\frac{dM_1}{dQ} = \frac{33}{5}\frac{m_{3/2}}{16\pi^2}Q\frac{dg_1^2}{dQ},$$

etc. are consistent with the RGEs of the MSSM listed in Section 9.2.2 together with the RGEs for gauge and Yukawa couplings.

Verify also that the expressions (11.27a)–(11.27g) for scalar masses are similarly scale invariant only if $m_0^2 = 0$.

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11.2.2 *D*-term improved AMSB model

While the addition of a common term m_0^2 to all scalar squared masses solves the tachyonic slepton mass problem, it destroys the scale invariance of the soft parameters with respect to renormalization group evolution, which renders the predictions of AMSB models insensitive to high scale physics. Indeed a variety of ways have been suggested to solve the tachyon mass problem, many of which do not maintain the scale invariance of the AMSB soft SUSY breaking parameters. Instead of studying all these various alternatives, we will focus on a modification of the AMSB relation that preserves this scale invariance.

The key observation is that additional contributions to soft SUSY breaking scalar masses that arise from Fayet–Iliopoulos D-terms, introduced in Section 6.5.1, automatically preserve this scale invariance property, as long as the charges of the corresponding $U(1)$ symmetries have no mixed anomalies with the MSSM gauge group.[16] In other words, as long as the extra contributions to scalar mass squared parameters take the form,

$$\delta m_i^2 = m_0^2 \sum_a k_a Y_{ai},$$

where Y_a are the generators of (mixed anomaly-free) $U(1)$ symmetries, and k_a are constants (one k_a for each such $U(1)$ factor), the scale invariance of AMSB scalar masses is maintained.[17] Moreover, since this invariance holds for arbitrarily small values of the corresponding "gauge coupling", it is not necessary for these $U(1)$s to survive as gauge symmetries of the low energy theory for this mechanism to work: i.e. global $U(1)$ symmetries of the low energy superpotential are sufficient. Notice that these D-term contributions to scalar mass parameters are the same for all sparticles with the same gauge quantum numbers so that flavor-changing neutral current constraints are satisfied.

The MSSM symmetries already include the hypercharge $U(1)$. Unfortunately, the corresponding D-term contributions cannot solve the slepton mass problem since the superfields $\hat{L}$ and $\hat{E}^c$ have opposite signs of hypercharge: the hypercharge D-term can make only one of $m^2(\tilde{\ell}_{\rm L})$ or $m^2(\tilde{\ell}_{\rm R})$ positive, but not both. We need at least one other D-term. Assuming that lepton flavor is not separately conserved by the superpotential (i.e neutrinos mix), there are only two independent anomaly-free $U(1)$ symmetries in the MSSM. These are the usual hypercharge symmetry, and $U(1)_{B-L}$ (or combinations thereof). Their D-term contributions to sparticle masses (at the weak scale) can thus be parametrized in terms of two parameters, D_Y and

[16] See I. Jack and D. R. T. Jones, *Phys. Lett.* **B482**, 167 (2000).
[17] Another possibility that also preserves the scale invariance has been proposed by I. Jack and D. R. T. Jones, *Phys. Lett.* **B491**, 151 (2000).

D_{B-L}, as[18]

$$\delta m_U^2 = -\frac{4}{3}D_Y - \frac{1}{3}D_{B-L}, \tag{11.31a}$$

$$\delta m_D^2 = \frac{2}{3}D_Y - \frac{1}{3}D_{B-L}, \tag{11.31b}$$

$$\delta m_Q^2 = \frac{1}{3}D_Y + \frac{1}{3}D_{B-L}, \tag{11.31c}$$

$$\delta m_E^2 = 2D_Y + D_{B-L}, \tag{11.31d}$$

$$\delta m_L^2 = -D_Y - D_{B-L}, \tag{11.31e}$$

$$\delta m_{H_u}^2 = D_Y, \tag{11.31f}$$

$$\delta m_{H_d}^2 = -D_Y. \tag{11.31g}$$

The values of D_Y and D_{B-L} must be of order of the weak scale squared. The value of D_{B-L} may possibly be the only imprint of the additional $U(1)$ symmetry. A necessary (but not sufficient) condition for a viable spectrum is,

$$0 < D_Y < -D_{B-L} < 2D_Y.$$

To summarize, the negative slepton mass problem can be solved maintaining the attractive ultra-violet insensitivity characteristic of the AMSB framework if there is an additional source of SUSY breaking that results in non-vanishing D-terms of a $U(1)$ symmetry with charges that are free of any mixed anomalies with the MSSM gauge group factors.[19]

11.3 Gauge-mediated SUSY breaking

As the name indicates, in gauge-mediated SUSY breaking (GMSB), SM gauge interactions communicate the effects of SUSY breaking to the superpartners of SM particles.[20] In addition to the fields of the SUSY breaking and the observable sectors that we have already discussed, there is a third set of fields that has both SM gauge interactions, as well as couplings to the hidden sector: these couplings may originate in the superpotential, or in new gauge interactions with the hidden sector (under which SM particles are neutral). Through these couplings, SUSY breaking effects are first felt by the fields in the new sector, and then

[18] The reader can easily check that this parametrization is equivalent to that in the original paper of I. Jack and D. R. T. Jones, with their parameters ζ_1 and ζ_2 given by $\zeta_1 = 2D_Y + \frac{8}{11}D_{B-L}$ and $\zeta_2 = \frac{1}{11}D_{B-L}$.

[19] For an explicit model that realizes this scenario, see N. Arkani-Hamed, D. E. Kaplan, H. Murayama and Y. Nomura, *JHEP* **0102**, 041 (2001).

[20] Interest in this picture was rekindled by M. Dine and A. Nelson, *Phys. Rev.* **D48**, 1277 (1993) and M. Dine, A. Nelson, Y. Nir and Y. Shirman, *Phys. Rev.* **D53**, 2658 (1996); see also references therein.

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communicated to the observable sector by SM gauge interactions. The third sector that links the SUSY breaking and observable fields is referred to as the "messenger sector".

At tree level, SUSY is unbroken in the MSSM sector. MSSM sparticles feel SUSY breaking effects only via their couplings to messenger particles in loops, and so evade the fatal tree-level mass sum rule. These loop effects, which involve the usual SM gauge couplings, again lead to SSB masses of the geometric hierarchy form,

$$m_i \propto \frac{g_i^2}{16\pi^2}\frac{\langle F_S\rangle}{M}$$

where $\langle F_S\rangle$ is the *induced* SUSY breaking VEV of some (elementary or composite) gauge singlet superfield in the messenger sector, M is the messenger sector mass scale, g_i is the SM gauge coupling constant for the corresponding sparticle, and $16\pi^2$ is a loop factor.[21] We thus conclude that colored superpartners are heavier than their uncolored counterparts and, likewise, uncolored particles that have just hypercharge gauge interactions are lighter than their cousins which also couple to $SU(2)_\mathrm{L}$. Such a spectrum is the hallmark of the GMSB scenario.

The induced SUSY breaking scale in the messenger sector should be distinguished from the corresponding scale $\langle F\rangle$ in the SUSY breaking sector. If the sectors are perturbatively coupled, we would expect $\langle F_S\rangle < \langle F\rangle$, while if they are strongly coupled, $\langle F_S\rangle \sim \langle F\rangle$. The gravitino mass, however, is determined by the fundamental SUSY breaking scale $\langle F\rangle$, and by (10.67a) is,

$$m_{3/2} = \frac{\langle F\rangle}{\sqrt{3}M_\mathrm{P}}.$$

We note that the SSB masses of MSSM superpartners are suppressed by just the messenger mass scale M, and not M_P as in gravity-mediated scenarios. If $M \ll M_\mathrm{P}$, the underlying scale of SUSY breaking can be much lower in GMSB models as compared to gravity-mediated scenarios.[22] In this case of low energy SUSY breaking the gravitino mass, which is suppressed relative to other sparticle masses by a factor $\sim M/M_\mathrm{P}$, may be very small. Indeed, the gravitino may be the LSP. Since the lightest MSSM particle can now decay to the gravitino, the phenomenological implications of such a scenario may differ dramatically from corresponding expectations in mSUGRA.

[21] If the sparticle has coupling to more than one factor of $SU(3)_\mathrm{C} \times SU(2)_\mathrm{L} \times U(1)_\mathrm{Y}$, there will be one such contribution for each coupling.

[22] If SUSY is local, there will be a gravity mediated contribution also, but this is negligible compared to the corresponding gauge-mediated contribution.

11.3.1 The minimal GMSB model

The messenger sector is assumed to consist of n_5 vector-like multiplets of messenger lepton and messenger quark superfields that carry the $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers,

$$\hat{\ell} \sim (\mathbf{1}, \mathbf{2}, 1) \qquad \hat{\ell}' \sim (\mathbf{1}, \mathbf{2}^*, -1)$$
$$\hat{q} \sim (\mathbf{3}, \mathbf{1}, -\frac{2}{3}) \qquad \hat{q}' \sim (\mathbf{3}^*, \mathbf{1}, \frac{2}{3}), \tag{11.32a}$$

coupled via the superpotential,

$$\hat{f}_M = \lambda_\ell \hat{\mathcal{S}} \hat{\ell}' \hat{\ell} + \lambda_q \hat{\mathcal{S}} \hat{q}' \hat{q}. \tag{11.32b}$$

Here $\hat{\mathcal{S}}$ is a gauge singlet field that also couples to the SUSY breaking sector. We assume that this coupling induces a VEV for both its scalar and its auxiliary component. Notice that the messenger sector forms complete vector multiplets of $SU(5)$. This ensures that the apparent unification of gauge couplings is not altered by their inclusion.

It is straightforward to see that the messenger quarks (and likewise, messenger leptons) combine to form a Dirac quark ($SU(2)$ doublet lepton) with a mass $m_{q_M} = \lambda_q \langle \mathcal{S} \rangle$ ($m_{\ell_M} = \lambda_\ell \langle \mathcal{S} \rangle$) where $\langle \mathcal{S} \rangle$ is the VEV of the scalar component of the singlet field $\hat{\mathcal{S}}$. In addition to this supersymmetric mass contribution, the scalar partners of the messenger quarks (leptons) acquire a SUSY breaking mass from the VEV $\langle F_S \rangle$ of the auxiliary component of $\hat{\mathcal{S}}$ that mix scalar components of $\hat{q}$ and $\hat{q}'$ ($\hat{\ell}$ and $\hat{\ell}'$). Diagonalizing the messenger squark and slepton mass matrices, we find that these acquire masses,

$$m^2_{\tilde{\ell}_M} = |\lambda_\ell \langle \mathcal{S} \rangle|^2 \pm |\lambda_\ell \langle F_S \rangle|, \tag{11.33a}$$
$$m^2_{\tilde{q}_M} = |\lambda_q \langle \mathcal{S} \rangle|^2 \pm |\lambda_q \langle F_S \rangle|. \tag{11.33b}$$

Notice that $\langle F_S \rangle / \lambda_i \langle S \rangle^2$ cannot be arbitrarily large – otherwise the messengers will be too light or even tachyonic. We will denote the messenger mass scale by $M \equiv \lambda \langle S \rangle$, where $\lambda \simeq \lambda_\ell \simeq \lambda_q$. If $\langle F_S \rangle \to 0$, we recover a supersymmetric spectrum in the messenger sector.

Exercise *Using the master formula, compute the mass spectrum of the messenger quarks. Note that the supersymmetry breaking contribution to messenger squark masses comes from the F-term of the superpotential,*

$$\lambda_q \hat{\mathcal{S}} \hat{q} \hat{q}' \big|_F \ni (\lambda_q \tilde{q} \tilde{q}' + \quad \text{h.c.}) \langle F_s \rangle.$$

Combine this with the supersymmetric contribution to messenger squark masses to obtain their masses.

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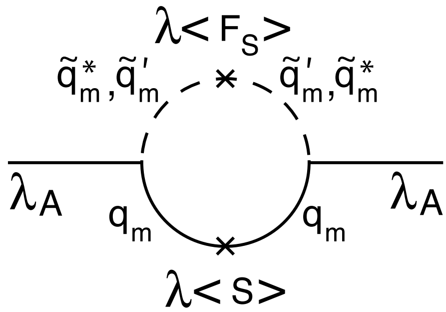

Figure 11.2 Diagram leading to gluino and hypercharge gaugino masses in GMSB models. A similar diagram with messenger leptons and messenger sleptons in the loop will also contribute to the $SU(2)$ gaugino mass. Messenger leptons and sleptons also contribute to the hypercharge gaugino mass. The dashed line denotes messenger sfermions while the solid line denotes the messenger fermion. A contribution arises only from the messenger fermion mass term indicated by the cross on the fermion line. The cross on the sfermion line indicates the SSB mixing term between the sfermions.

It is now possible to compute the SSB mass parameters induced in the visible sector via gauge interactions with messenger sector fields. Gauginos obtain masses from one-loop diagrams including messenger fields as indicated in Fig. 11.2. In the approximation $\langle F_S \rangle \ll \lambda \langle S \rangle^2$ (i.e. the SUSY breaking scale is smaller than the messenger mass scale), the gaugino for gauge group i gets a mass

$$M_i = \frac{\alpha_i}{4\pi} n_5 \Lambda \tag{11.34}$$

where

$$\Lambda = \frac{\langle F_S \rangle}{\langle S \rangle}. \tag{11.35}$$

The factor n_5 arises because each messenger generation makes the same contribution to the gaugino mass.

MSSM scalars do not couple directly to the messenger sector, so that their squared masses are induced only via two-loop diagrams such as the ones depicted in Fig. 11.3. Now, the squared mass scales with the number of messenger multiplets, and in the same approximation as in (11.34) we obtain,

$$m_i^2 = 2n_5 \Lambda^2 [C_1^i (\frac{\alpha_1}{4\pi})^2 + C_2^i (\frac{\alpha_2}{4\pi})^2 + C_3^i (\frac{\alpha_3}{4\pi})^2]. \tag{11.36a}$$

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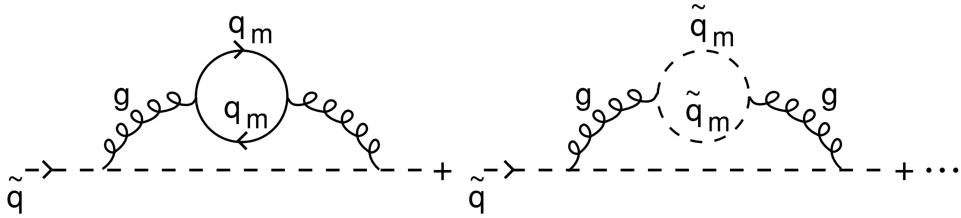

Figure 11.3 Examples of two-loop Feynman diagrams leading to scalar masses in GMSB models.

Here, C_i are quadratic Casimirs given by,

$$C_1^i = \frac{3}{5} Y_i^2,$$
$$C_2^i = \begin{cases} 3/4 \text{ for doublets} \\ 0 \text{ for singlets} \end{cases} \tag{11.36b}$$
$$C_3^i = \begin{cases} 4/3 \text{ for triplets} \\ 0 \text{ for singlets.} \end{cases}$$

Note that the unknown messenger sector Yukawa couplings drop out from (11.34) and (11.36a). These formulae are rather general, in that if the messengers can be grouped into a **10** or **10*** of $SU(5)$, then $n_5 \to n_{10}$ where $n_{10} = 3$ for each set of **10** and **10*** messenger fields. The value of n_5 (and hence n_{10}) cannot be too large since the gauge couplings will then diverge in their running from the weak to the GUT scale, and perturbative unification will be spoiled. Typically, $n_5 \leq 4$ is a valid choice, though if the messenger scale is large higher values of n_5 are allowed. As the parameter n_5 increases, the gaugino masses increase at a greater rate than the scalars, since $M_i \propto n_5$, while $m_i \propto \sqrt{n_5}$.

We note that these formulae for gaugino and scalar masses are simply modified by multiplication by threshold functions if our approximation $x \equiv \langle F_S \rangle / \lambda \langle S \rangle^2 \ll 1$ in which we have written them ceases to be valid.[23] Our formulae for the gaugino (scalar) masses are good approximations for x as large as 0.9 (0.5).

We see from (11.36a) that scalars with the same gauge quantum numbers will receive identical masses. This gives a natural explanation for the scalar mass degeneracy needed to solve the SUSY flavor problem, and provides strong motivation for this class of models.[24] We see also the characteristic pattern of sparticle masses

[23] S. Martin, *Phys. Rev.* **D55**, 3177 (1997).

[24] We remark, however, that since messenger field $\hat{\ell}$ and $\hat{H}_u$ carry the same gauge quantum numbers, in any Yukawa coupling involving h_u, we can replace the Higgs field by a messenger slepton. This would lead to flavor violation unless such couplings are forbidden by a global symmetry, or the messenger scale is sufficiently large. Thus it is really the squark loop contributions to FCNC effects that are naturally suppressed in these scenarios.

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noted earlier. Squark and gluino mass parameters are much larger than those for sleptons and Higgs scalars, and $\tilde{\ell}_L$ are considerably heavier than $\tilde{\ell}_R$. Notice also that (11.34) leads to the one-loop GUT relation between gaugino masses, but for very different reasons.

Exercise (The gravitino mass) *Explore the range of the gravitino mass in this framework. To do so, write*

$$m_{3/2} = \frac{\langle F \rangle}{\lambda \langle F_S \rangle} \times \frac{\Lambda M}{\sqrt{3} M_P} \equiv C_{grav} \frac{\Lambda M}{\sqrt{3} M_P}, \tag{11.37}$$

where λ is the messenger sector Yukawa coupling, taken to be common for messenger quarks and leptons. Since we want sparticles at the weak scale ~ 100 GeV, we must have Λ to be few tens of TeV. For given values of Λ and M, the gravitino is lightest when $\langle F_S \rangle$ is close to the fundamental SUSY breaking scale and when the messenger scale is not very different from Λ. Show that for reasonable values of parameters the gravitino mass may be in the eV range. How heavy can it be?

Like scalar masses, the **a**-terms only arise via two-loop diagrams. Remember, however, that it is the *squared* scalar masses that arise at two loops, so that scalar mass parameters have the same order of magnitude as gaugino masses. In comparison to this, **a**-terms which are suppressed by an extra loop factor, are small. As an approximation, they are frequently taken to be

$$\mathbf{a}_u = \mathbf{a}_d = \mathbf{a}_e = 0. \tag{11.38}$$

It should be remembered that the formulae (11.34), (11.36a), and (11.38) for the MSSM parameters in GMSB models hold at the scale $Q \sim M$ where the heavy messenger fields are integrated out. As for SUGRA models, these parameters must be evolved to the weak scale for the extraction of phenomenology using the MSSM.

The bilinear b term is also generated at two loops and so is tiny. In principle, this means that the requirement of radiative EWSB should fix $\tan\beta$ since the weak scale $B\mu$ is fixed by the condition $b_0 = 0$. This is not what is usually done in practice. The reason is that it is difficult to generate μ in these scenarios. The rationale then is any modification to the model that allows for μ affects the Higgs sector and so will presumably also affect the b term. In practice, therefore, μ and b are treated as free weak scale parameters: as usual μ^2 is fixed to reproduce M_Z^2, and b_0 is traded in for $\tan\beta$. There is one difference in the radiative symmetry breaking mechanism from gravity-mediated models that seems worth mentioning. In mSUGRA, $m_{H_u}^2$ turns negative because of the large logarithm that arises due to the disparity

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between the GUT and weak scales. In the GMSB scenario $m_{H_u}^2$ turns negative even if the messenger scale is close to the weak scale because the colored squarks are much heavier than Higgs scalars, i.e. large t-squark masses drive $m_{H_u}^2$ to negative values.

We have already noted that if the messenger scale $M \ll M_\mathrm{P}$, the gravitino may be very light. But if gravitinos couple with gravitational strength, why do we care? The point is that since gravitinos get masses via the super-Higgs mechanism the couplings of their longitudinal components (essentially the goldstinos) are enhanced by a factor $E/m_{3/2}$ in exactly the same way that longitudinal W couplings are enhanced by a factor of E/M_W. In other words, "the effective dimensionless coupling" of longitudinal gravitinos to a particle–sparticle pair is $\sim E/M_\mathrm{P} \times E/m_{3/2}$, where the first factor is the usual coupling of gravity to energy and the second factor the enhancement just discussed. It is easy to check that for $E \sim 100$ GeV and $m_{3/2} \sim 1$ eV, this coupling is $\sim 10^{-6}$. Dimensional analysis gives the lifetime of a 100 GeV particle decaying via this coupling as $\sim 10^{-12}$ seconds! *Thus interactions of very light longitudinal gravitinos may be relevant for particle physics, and even for collider phenomenology.* We will return to this in later chapters.

If gravitinos are light, sparticles can decay via $\tilde{p} \to p\tilde{G}$ with a decay rate that depends on the gravitino mass. It is more convenient to use C_grav introduced in (11.37) to parametrize this decay rate. Notice that, by construction, $C_\mathrm{grav} \geq 1$. The parameter space of GMSB models can thus be specified by,

$$\Lambda,\ M,\ n_5,\ \tan\beta,\ \mathrm{sign}\,(\mu),\ C_\mathrm{grav}. \tag{11.39}$$

For a given number n_5 of messenger multiplets, the mass scale of MSSM superpartners is set by Λ. The second entry, M ($M > \Lambda$) is the mass scale associated with the messenger fields, and specifies the scale at which the mass formulae (11.34) and (11.36a) as well as $\mathbf{a} = 0$, hold. The SSB parameters relevant for phenomenology are then obtained by evolving these from M to the weak scale where radiative EWSB determines the magnitude but not the sign of μ. MSSM sparticle masses are, therefore, only logarithmically sensitive to M, and, of course, independent of C_grav. Increasing C_grav only increases the lifetime of sparticles which decay mainly to the gravitino, but does not affect MSSM sparticle masses.

An example of the renormalization group evolution that fixes the sparticle spectrum is shown in Fig. 11.4. While the gaugino masses are related as in mSUGRA, sfermion masses are very different. In particular, we have $m_{\tilde{q}} \gg m_{\tilde{e}_\mathrm{L}} \sim m_{\tilde{e}_\mathrm{R}}$.

For GMSB models, the parameter Λ should be $\sim$ 10–150 TeV in order for sparticles to obtain masses of order of the weak scale. The messenger scale $M \geq \Lambda$. If the SUSY breaking scale is small so that the gravitino is the LSP, GMSB phenomenology may differ dramatically from phenomenology of models with a weak scale gravitino and a neutralino LSP. If the gravitino is the LSP and other

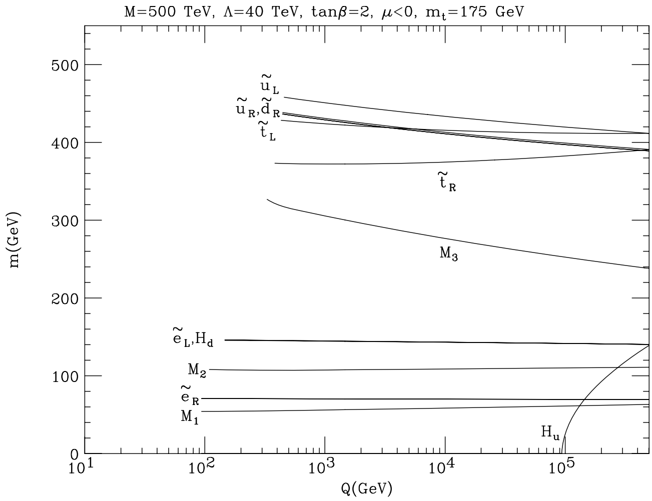

Figure 11.4 Renormalization group trajectories for the soft SUSY breaking masses versus renormalization scale Q from the messenger scale ($M = 500$ TeV) to the weak scale. In this example, we take $\Lambda = 40$ TeV, $n_5 = 1$, $\tan\beta = 2$, $\mu < 0$, and $m_t = 175$ GeV. We will see later that this scenario is excluded by lower bounds on both the selectron as well as h masses. The point of this figure is only to illustrate the RG evolution, for which the LEP exclusion is not relevant. Reprinted with permission from H. Baer, M. Brhlik, C.-H. Chen and X. Tata, *Phys. Rev* **D55**, 4463 (1997), copyright (1997) by the American Physical Society.

sparticles can decay to it in a lifetime short compared to the age of the Universe, then the cosmological considerations that require the lightest MSSM sparticle to be only weakly interacting no longer apply, and this next-to-lightest SUSY particle (NLSP) may be charged. Typically, the NLSP is the lightest neutralino or the lighter stau (which would be very close in mass to $\tilde{e}_R$ and $\tilde{\mu}_R$ for small to moderate values of $\tan\beta$).

In the case of a neutralino NLSP, collider phenomenology is most different when the gravitino is very light, so that the NLSP decays *inside the experimental apparatus*.[25] The main decay modes for a neutralino NLSP are $\widetilde{Z}_1 \to \gamma\tilde{G}$, $Z\tilde{G}$ or $h\tilde{G}$. For a stau NLSP, the decay mode would be $\tilde{\tau}_1 \to \tau\tilde{G}$. Heavier sparticles cascade decay to the NLSP which subsequently decays into the gravitino LSP, and SUSY event topologies are very sensitive to the nature of the NLSP.

Since sparticle masses are only weakly dependent on the messenger scale M, the $\Lambda - \tan\beta$ plane provides a convenient panorama for displaying the various phenomenological possibilities. These are illustrated in Fig. 11.5 where we show

[25] Of course, heavier sparticles can then also decay into gravitinos but, as we will see in Chapter 13, the branching fractions for these decays are negligible.

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this plane for values of n_5 ranging from 1–4. The gray region is excluded because electroweak symmetry is not correctly broken, while the various shaded regions are excluded by constraints from LEP experiments. In the region labeled 1, the neutralino is the NLSP and decays into the gravitino. In region 2, $m_{\tilde{\tau}_1} < m_{\widetilde{Z}_1}$, with all other sleptons heavier than $\widetilde{Z}_1$, so that cascade decays terminate in $\tilde{\tau}_1$ (which decays via $\tilde{\tau}_1 \to \tau\tilde{G}$), except very close to the boundary between regions 1 and 2 where $\widetilde{Z}_1 \to \tilde{\tau}_1\tau$ is forbidden. In regions 3 and 4, in addition to $\widetilde{Z}_1 \to \tilde{\tau}_1\tau$, the decays $\widetilde{Z}_1 \to \tilde{\ell}_R\ell$ ($\ell = e, \mu$) are also allowed. In region 3, however, $m_{\tilde{\ell}_1} < m_{\tilde{\tau}_1} + m_\tau$, while just the opposite is the case in region 4. We will discuss the implications of this in Chapter 13.

The lifetimes for NLSP decay depend on $C_{\rm grav}$ and range from essentially instantaneous to very long. NLSPs produced in collider detectors may have a long lifetime, and decay with a displaced vertex, or possibly even decay outside the detector. In the latter case, a neutralino NLSP would escape undetected as in gravity-mediated models. A stau (or charged slepton) NLSP would behave as a stable charged particle in the apparatus, and leave an ionizing track which may be detectable. NLSP decays will be considered in more detail in Chapter 13.

11.3.2 Non-minimal GMSB models

While the minimal GMSB framework leads to strong correlations between various sparticle masses, it is possible to conceive of extensions where the correlations are relaxed. Examples of things that have been considered include:

- Additional interactions needed to generate μ and b parameters may split the SSB mass parameters of the Higgs and lepton doublets at $Q = M$, even though these have the same gauge quantum numbers.
- Allowing incomplete messenger representations can effectively result in different numbers of messengers n_{5_i} for each factor of the low energy gauge group.
- If the hypercharge D-term has a non-vanishing VEV in the messenger sector, there would be additional contributions to the scalar masses that may be parametrized by $\delta m^2_{\tilde{f}} = Y_{\tilde{f}} K_{\rm Y}$, where $K_{\rm Y}$ is the D-term VEV with the gauge coupling absorbed into it.

We mention these variations to make the reader aware that although the minimal GMSB framework is well motivated and constrained, the implications that we have drawn from it are based on a number of unstated assumptions about physics at the messenger scale and beyond.

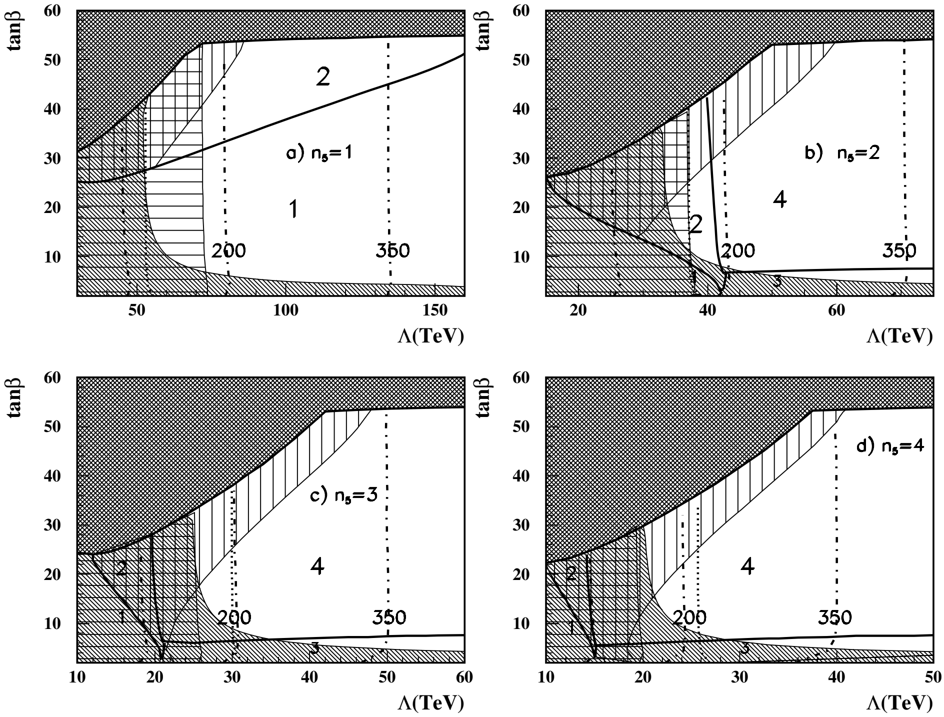

Figure 11.5 The four regions of the $\Lambda - \tan\beta$ parameter plane of the mGMSB model discussed in the text. The heavy solid lines denote the boundaries between these regions. The gray region is excluded because electroweak symmetry is not correctly broken. The shaded regions are excluded by various constraints from LEP experiments: $m_{\tilde{\tau}} > 76$ GeV (vertical shading), $m_{\widetilde{Z}_1} > 95$ GeV (horizontal shading), and $m_h > 110$ GeV (diagonal shading). The dot-dashed contours are where the chargino mass is 100, 200 or 350 GeV, while the dotted line is the contour of $m_{\tilde{e}_R} = 100$ GeV. We thank Dr. Yili Wang for supplying this figure which appears in her doctoral dissertation.

11.4 Gaugino-mediated SUSY breaking

Gaugino-mediated SUSY breaking is a model based on extra dimensions that provides a novel solution to the SUSY flavor problem.[26] Within this framework, chiral supermultiplets of the observable sector reside on one brane whereas the SUSY breaking sector is confined to a different, spatially separated brane. Gravity and gauge superfields, which propagate in the bulk, directly couple to fields on both the branes. As a result of their direct coupling to the SUSY breaking sector, gauginos acquire a mass. Direct couplings between the observable and SUSY breaking

[26] D. E. Kaplan, G. D. Kribs and M. Schmaltz, *Phys. Rev.* **D62**, 035010 (2000); Z. Chacko *et al.*, *JHEP* **01**, 003 (2000); M. Schmaltz and W. Skiba, *Phys. Rev.* **D62**, 095004 (2000) and **D62**, 095005 (2000).

sectors are exponentially suppressed, and MSSM scalars dominantly acquire SUSY breaking masses via their interactions with gauginos (or gravity) which directly feel the effects of SUSY breaking. As a result, scalar SSB mass parameters are suppressed relative to gaugino masses, and may be neglected in the first approximation. The same is true for the A- and B-parameters.

In a specific realization, to preserve the success of the unification of gauge couplings, it is assumed that there is grand unification (either $SU(5)$ or $SO(10)$) and, further, that the compactification scale M_c, below which there are no Kaluza–Klein excitations, is larger than M_{GUT}. Furthermore, since the construction ensures flavor-blind interactions for just light bulk fields, we require that the scale $M_c \lesssim M_{Planck}/10$ in order to sufficiently suppress other flavor-violating scalar couplings from heavy bulk fields that would be generically present. Based on the discussion in the last paragraph, the boundary conditions for the soft SUSY breaking parameters of the MSSM are taken to be $m_0 = A_0 = B_0 = 0$ at the scale M_c. The condition $B_0 = 0$ fixes $\tan\beta$. In both $SU(5)$ and $SO(10)$ models, this value of $\tan\beta$ is found to be too small to be compatible with the unification of bottom and tau Yukawa couplings in the MSSM, which requires $\tan\beta \geq 30$. For this reason, and because the value of $B_0\mu_0$ may also depend on how the μ problem is solved, we will ignore the $B_0 = 0$ constraint and, as usual, choose $\tan\beta$ instead of B_0 as a free parameter.[27] The MSSM parameters can then be obtained from the parameter set,

$$m_{1/2},\ \ M_c,\ \ \tan\beta,\ \ \text{and sign}\ (\mu) \tag{11.40}$$

where it is the grand unification assumption that leads to a universal gaugino mass above $Q = M_{GUT}$, and $|\mu|$ is fixed assuming radiative EWSB. The gravitino can be made heavier than gauginos and, as in the mSUGRA framework, is irrelevant for collider phenomenology. The LSP may be the stau or the lightest neutralino, though cosmological considerations exclude the former (unless R-parity is not conserved).

For illustration we choose the GUT group to be $SU(5)$. This model is then a special case of our earlier discussion of $SU(5)$, except that the SSB parameters now "unify" at the scale M_c (rather than M_P) where they take on values specific to the model. The unification of the τ and b Yukawa couplings constrain $\tan\beta \sim 30$–50. In Fig. 11.6, we show the evolution of the various SSB parameters of the MSSM, starting with the inoMSB boundary conditions. Here, the unified gaugino mass is taken to be 300 GeV at $Q = M_{GUT}$. The compactification scale is taken to be $M_c = 10^{18}$ GeV. We see that although these start from zero, RG evolution results in GUT scale scalar masses and A-parameters that are not negligible compared to $m_{1/2}$: although there is no large logarithm, large group theory coefficients are the cause

[27] It is also possible that Higgs fields reside in the bulk, in which case they would directly feel SUSY breaking effects, resulting in a non-vanishing value for B_0 as well as other SSB parameters in the Higgs sector. Such scenarios are, of course, less predictive than the minimal one that we consider here.

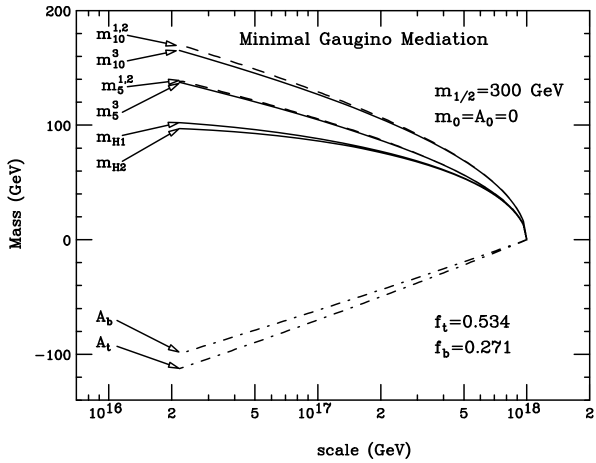

Figure 11.6 Renormalization group evolution of soft SUSY breaking $SU(5)$ masses versus scale in the minimal gaugino mediation model. We take $\tan\beta = 35$ and $\mu < 0$ to achieve $b - \tau$ Yukawa coupling unification. Reprinted from H. Baer, M. Diáz, P. Quintana and X. Tata, *JHEP* **04**, 016 (2000).

of this sizable renormalization group evolution. While the inter-generation splitting is small, the splittings between the **5** and the **10** dimensional matter multiplets, as well as between these and the Higgs multiplets, is substantial.

In Table 11.3 we show a sample spectrum for this model. We choose $m_{1/2} =$ 300 GeV, $\tan\beta = 35$, and other parameters as in Fig. 11.6. The spectrum is not unlike that in the mSUGRA framework with small m_0, so that sleptons are relatively light and squarks are lighter than the gluino.

11.5 An afterword

The reader will have noticed that we have not constructed a complete supersymmetric model in the sense of the SM. Instead, we have assumed that SUSY is broken in some sector, and discussed several mechanisms for how this is communicated to MSSM superpartners. As mentioned at the start of this chapter, MSSM phenomenology depends more upon this messenger mechanism and not so much upon the dynamics of SUSY breaking.

This is not to say that the question of SUSY breaking is not important. Indeed, a complete model must address the μ problem, and at the same time generate $b \equiv B\mu$ and other SSB parameters so that (8.19b), with radiative corrections included, yields the correct value of M_Z, and a sparticle spectrum consistent with experimental constraints. The value of $\tan\beta$ as given by (8.19a) would then be a prediction. We stress that these EWSB conditions (8.19a) and (8.19b) only depend on our assumption of the MSSM field content of the low energy theory, and therefore

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Table 11.3 *Input and output parameters for the Minimal Gaugino Mediation model case study described in the text. Mass parameters are in GeV units.*

parameter	scale	value
m_0	M_c	0
A_0	M_c	0
$m_{1/2}$	M_{GUT}	300
g_5	M_{GUT}	0.717
f_t	M_{GUT}	0.534
$f_b = f_\tau$	M_{GUT}	0.271
λ	M_{GUT}	1
λ'	M_{GUT}	0.1
$\tan\beta$	M_{weak}	35
μ	M_{weak}	< 0
$m_{\tilde{g}}$	M_{weak}	737.2
$m_{\tilde{u}_L}$	M_{weak}	668.5
$m_{\tilde{d}_R}$	M_{weak}	633.1
$m_{\tilde{t}_1}$	M_{weak}	482.8
$m_{\tilde{b}_1}$	M_{weak}	541.5
$m_{\tilde{\ell}_L}$	M_{weak}	258.6
$m_{\tilde{\ell}_R}$	M_{weak}	210.0
$m_{\tilde{\tau}_1}$	M_{weak}	143.3
$m_{\widetilde{W}_1}$	M_{weak}	240.2
$m_{\widetilde{Z}_2}$	M_{weak}	240.0
$m_{\widetilde{Z}_1}$	M_{weak}	124.8
m_h	M_{weak}	115.6
m_A	M_{weak}	311.2
μ	M_{weak}	-411.5

should be valid as long as the underlying fundamental theory reduces to the MSSM at low energy. This is not to say that every high energy theory will necessarily lead to an acceptable model. For instance, while there is an elegant mechanism for generating μ in gravity-mediated SUSY breaking scenarios (where the SUSY breaking scale is large), it is not straightforward (see Section 11.3.2) to generate acceptable values for both μ and b in GMSB scenarios with a low SUSY breaking scale. We circumvent the complications associated with the underlying mechanism of SUSY breaking and the associated μ problem because whatever the underlying physics is, it must be consistent with the EWSB conditions (8.19a) and (8.19b) as long as the low energy theory is the MSSM. Fortunately, TeV scale phenomenology depends more on how SUSY breaking is felt by weak scale superpartners and not so much on the underlying dynamics of SUSY breaking.

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