As an example in the use of (2), we can consider one of the sides of the quadrilateral to be the line at infinity. Let ABC be a triangle, H its orthocentre, and u the line at infinity. Let AB, BC, CA cut u in P, P_1, E. Then A, B, C, P, P_1, E are the six vertices of the quadrilateral, and their joins with H form a pencil in involution. HP and HC are at right angles; so also are HA and HP_1. Thus the pencil from H is rectangular, and consequently HE and HB are at right angles, *i.e.* CA and HB are at right angles—another proof of the concurrency of the altitudes of a triangle.

G. PHILIP.

Note on Right-Angled Triangles.—It frequently happens in teaching that one wishes to draw a right-angled triangle with rational sides, and at the same time, for the sake of variety if nothing else, one desires to avoid the hackneyed one whose sides are 3, 4, 5.

The following simple rules, easily remembered, enable one to do this with little trouble.

1. If we select any odd number for one side, the approximate halves of its square are the other two sides.

(By "approximate halves" is meant the actual halves diminished and increased respectively by $\frac{1}{2}$.)

e.g. $13, 84, 85 \; (i.e.\ 13, \frac{169}{2} - \frac{1}{2}, \frac{169}{2} + \frac{1}{2}).$

2. Choose any three consecutive numbers. The middle number is the first side. Half the product of the other two is the second side, while the third side is the second side $+1$.

e.g. from 10, 11, 12 we derive the sides 11, 60, 61

$$\left(i.e.\ 11, \frac{10 \times 12}{2},\ 60 + 1.\right)$$

Similarly, 12, $71\frac{1}{2}$, $72\frac{1}{2}$, or 24, 143, 145 are got from 11, 12, 13.

If one wishes a triangle where the differences between the hypotenuse and the longer side is greater than 1, one may adopt the following method, of which (2) is obviously a particular case.

Write down any 3 numbers in A.P. The middle number is the first side. Divide the product of the other two by twice the common difference. This gives the second side, and this, increased by the common difference, yields the third side.

As in (2), multiplication, to avoid fractions, may be necessary.

e.g. 6, 11, 16 gives the sides $11, \frac{96}{10}, \frac{146}{10}$

$$\left(i.e. \quad 11, \frac{6 \times 16}{2 \times 5}, \frac{96}{10} + 5\right), \text{ or } 55, 48, 73.$$

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https://doi.org/10.1017/S1757748900000888 Published online by Cambridge University Press