THE URCA CONVECTION

Giora Shaviv
Departement of Physics and Astronomy
Tel-Aviv University
Ramat Aviv, Israel

The possible role that β-decays may play in stellar collapse was first discussed by Gamow and Schoenberg (1940, 1941). The above authors proposed this mechanism a mean of extracting quickly the energy content of the star and transporting it outside. In this way they hypothesized that stellar collapse may proceed.

The URCA process is composed of β-decay and inverse β-decay occuring inside the star. Let (A, Z-1) be a β-decay unstable nucleus in vacuum (i.e. on earth), namely

$$(A,\ Z-1) \rightarrow (A,\ Z) + e^{-} + \bar{\nu} \qquad (1)$$

where (A, Z) is a nucleus with Z protons and A nucleons (protons plus neutrons) and ν is the emitted antineutrino. The energetics of the process is seen in Fig. 1. Note that ΔQ must include the rest mass energy of the electron. The transformation of a neutron into a proton leads to a lower energy configuration. The transition can go from the ground state of the nucleus (A, Z-1) to the ground state of the nucleus (A, Z) (arrow 1) or from the ground state to an excited state (arrow 2) if such exists and if the transition is allowed. Only under high temperatures of the order of about MeV is the (A, Z-1) nucleus excited and then transitions from excited states are possible. There is no basic difference between ground state and excited states transitions. The most important properties for convection are: The energy difference between the two states is shared by the two emitted particles, the electron (including its rest mass) and the neutrino. The electron is usually more energetic than the surrounding and hence is slowed down (quickly) and deposits its energy in the surrounding. The neutrino has an extremely small cross-section for interaction with matter (about 10^{-44} cm^{2}) and hence escapes from the star. The energy carried by the neutrino is a net loss to the star. The rate of the decay depends on the density (we shall return to this question) but is usually of order of minutes and longer.

https://doi.org/10.1017/S0252921100112485 Published online by Cambridge University Press

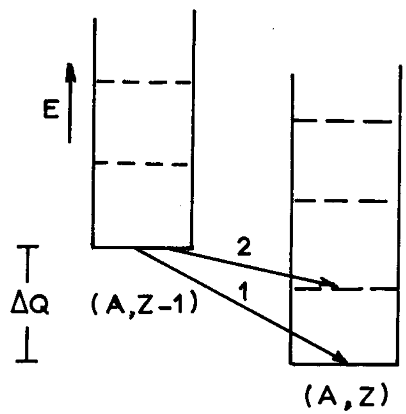

Figure 1.
The energetics of the β-decay.

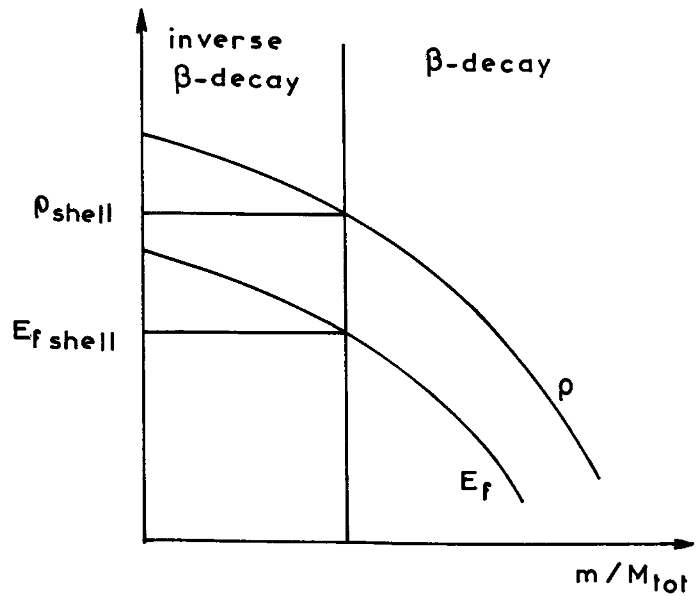

Figure 2.
The density profile in the star
and the URCA shell.

https://doi.org/10.1017/S0252921100112485 Published online by Cambridge University Press

The matter in the star is practically fully ionized at the relevant densities (above 10^6 gm/cm^3) and the electrons are degenerate. The fermi energy, which is the average energy needed to add an electron at thermal equilibrium, is a monotonic function of the density (at constant temperature $\varepsilon_f c \rho^{1/3}$ at the relevant densities). At a density of about 5×10^5 gm/cm^3 the fermi energy is about 1/2 MeV, namely it equals the rest mass energy of the electron. As the density continues to increase a moment will come at which the degenerate electrons, which are pushed to higher and higher energy levels will be energetic enough to cause the inverse process, namely :

$$(A,Z) + e^- \rightarrow (A,Z-1) + \nu . \qquad (2)$$

Here ν is the antineutrino which has the same nuclear properties as the neutrino (cross section $\sim 10^{-44}$ cm^2) and escapes from the star, in general, so long as the densities are below nuclear ones. At $\rho=\rho_{shell}$ when the inverse process (2) occurs we get the URCA shell. The name URCA was given by Gamow and Schoenberg after the famous Casino in South America where the gambler was bound to lose his money. The URCA shell is quite narrow, i.e. the process occurs for $\Delta\rho << \rho_{shell}$. At densities $\rho < \rho_{shell}$ the dominant process is β-decay while for $\rho > \rho_{shell}$ inverse β-decay is dominant. The location of the URCA shell is given by the condition

$$\varepsilon_f = \Delta Q - m_e c^2 \qquad (3)$$

where ΔQ is the energy difference between the two nuclei.

Tsuruta and Cameron (1969) considered the effect of density variations on the rate of URCA losses. The picture of Gamow and Schoenberg is static. As the star contracts the location $\rho=\rho_{shell}$ advances outward and gives rise to neutrino (and energy) losses. In the case of Tsuruta and Cameron the same nucleus can oscillate (by means of general stellar vibrations) around $\rho=\rho_{shell}$. When the nucleus (A,Z) moves to a higher density it absorbs an electron and emits an anti-neutrino, when the product nucleus (A,Z-1) moves to the low density, it finds that the phase space of the sea of electrons around it is free and it β-decays releasing the electron and neutrino. The sum of the two processes can therefore be written schematically as:

$$(A,Z) \rightarrow (A,Z) + \nu + \bar{\nu} \qquad (4)$$

Paczynski (1972) considered the effect of convection on the URCA process. The fundamental problem and motivation was the difficulty in the theory of neutron star formation. There are some statistical arguments (Gunn and Ostriker 1971) that hint at the fact that stars in the range 4-10 $M_\odot$ should be the progenitors of pulsars. However, when models of such stars are calculated it is found that due to the particular behaviour of stars with nuclear shell burnings, the carbon ignites at high densities $\sim 10^9 - 10^{10}$ gm/cm^3 and low temperatures. The ignition of the 2C $\rightarrow$ Mg reaction under

https://doi.org/10.1017/S0252921100112485 Published online by Cambridge University Press

these conditions is dominated by the corrections to reaction rates due to the high density and gives rise to detonation. Numerical calculations by various people have shown that no remnant is left. On the other hand, if carbon ignition is not achieved with a violent reaction rate, e.g. if a fast and efficient cooling mechanism would be available, then the collapse could be delayed to still higher density and the outcome of the explosion is a neutron star. The fast and concentrated energy production by the carbon gives rise to convection. Paczynski considered the effect this convection has on the URCA pairs. The estimates of Paczynski, based on mixing lengh theory and some properties of stellar convective cores, yielded the following result

$$L_\nu \propto T_c^{170} \tag{5}$$

where L_ν is the neutrino luminosity and T_c the central temperature.
This very high temperature sensitivity follows from straightforward application of the expressions derived by Tsuruta and Cameron (1970) for stellar vibrations to convective cores. The oscillations considered are very fast compared to the typical β-decay times and hence the nuclei will be out of equilibrium (and not in equilibrium as assumed by Tsuruta and Cameron (1970) and Paczynski (1973)). Next, one has to conserve the number of decaying nuclei. Suppose a convective core inside which an URCA shell exists is given and a nucleus inverse-β decays far away from the URCA shell on the high density side. This nucleus cannot (practically) β decay before it crosses the URCA shell to the low density side. Hence the energy loss must be evaluated first per cycle and not immediately per unit time.

The extreme sensitivity of the neutrino losses to temperature (much more than the nuclear reactions energy production) led Paczynski to the conclusion that the URCA neutrinos can cool the star sufficiently fast and control carbon burning. Consequently he assumed stable carbon burning which delayed the collapse of the star and yielded the desired conditions for the formation of pulsars (Paczynski 1973).

A new development came when Bruenn (1973) showed that an accurate calculation shows that the final outcome of the URCA pair may be heating and not cooling. Consider first the β-decay. Assume that convection turn-over time scale is short compared to β-decay rate (a good assuption). The β-unstable nucleus will therefore be carried by the convection way past the URCA shell to regions of low density. The escaping neutrino causes of course an energy loss but the emitted electron may (since the electrons are emitted with a certain spectrum of energies) have energy well above the average energy of the electrons in the medium (ε_f at that place). The fast electron is slowed down and transfers its extra energy to the medium i.e. it heats the medium.

The same situation occurs when the (A,Z) nucleus is transferred by the convection to the high density region. Again, the inverse β-decay process is slow compared to convection velocities and the inverse process may occur at densities for which ε_f

is much greater than the energy difference between the nuclei. Since the energy difference is fixed, so will be the energy of the absorbed electron. Under these conditions the absorbed electron will come from deep in the sea of electrons. Thus a "hole" is created below the fermi level. The subsequent thermalization of the distribution, namely the relaxation to a new thermodynamic equilibrium with a smaller number of electrons will convert some of the energy difference, $\varepsilon_f - \varepsilon_{th} > 0$ (the difference between the average energy and the energy of the absorbed electron) into thermal energy of the whole sea of electrons. Said in other words, a high energy electron may jump into the hole and give its extra energy to the rest. This particular behaviour of β decays is well known to nuclear physicists, namely if you put a β unstable nucleus in a container the radioactive decay heats the medium in spite of the fact that the neutrino escapes.

The basic argument raised by Bruenn can be demonstrated in the following way. Let E be the internal energy of the matter in which the species Ni β decays. We have from the first law :

$$dE = \left(\frac{\partial E}{\partial V}\right)_{T,N_i} dV + \left(\frac{\partial E}{\partial T}\right)_{V,N_i} dT + \sum_i \left(\frac{\partial E}{\partial N_i}\right)_{V,T,N} DN_i$$

$$= - \langle\varepsilon_\nu\rangle - pdV \qquad (6)$$

where the heat lost from the unit mass considered is replaced by $\langle\varepsilon_\nu\rangle$ the average energy of the emitted neutrinos. Bruenn assumes the process to occur at constant volume. This is not the case in stars. The fast pressure equilibrium will give rise to compression in the case of electron capture (there are fewer particles) and to expansion in electron emission. In spite of this neglect, the qualitative result is correct. The change in temperature due to the electron capture is therefore given by

$$dT = \left[- \langle\varepsilon_\nu\rangle - \left(\frac{\partial E}{\partial N(A,Z)}\right)_{V,T} + \left(\frac{\partial E}{\partial N\ (A,Z+1)}\right)_{V,T} + \left(\frac{\partial E}{\partial N_e}\right)_{V,T} \right] / \left(\frac{\partial E}{\partial T}\right)_{V,N_i} \qquad (7)$$

The sum of the second and third terms on the right hand side is exactly ΔQ. Since we consider only very degenerate matter for which $kT \ll \varepsilon_f$, we can approximate the thermodynamic derivatives with those evaluated at T=0. Hence one gets:

$$\left(\frac{\partial E}{\partial N_e}\right)_{V,T} = \left(\frac{\partial E}{\partial N_e}\right)_{V,S} = \varepsilon_f + m_e c^2 \qquad (8)$$

https://doi.org/10.1017/S0252921100112485 Published online by Cambridge University Press

where m_ec^2 is the rest mass energy of the electron. One finally gets:

$$dT = (- <\varepsilon_\nu> + \Delta\varepsilon) \;/\; \left(\frac{\partial E}{\partial T}\right)_{V,N_i} \qquad \frac{\partial E}{\partial T} > 0 \tag{9}$$

Where $\Delta\varepsilon = \varepsilon_f - \varepsilon_{th}$ is the difference between the electron fermi energy and the electron capture threshold and $\varepsilon_{th} = -m_ec^2 - \Delta Q$. Cooling will occur only if $<\varepsilon_\nu> > \Delta\varepsilon$. The calculations by Bruenn have shown that for $T < T_{neut}(\rho)$ the e-capture will result in heating and vice versa. The effect occurs at $\rho \sim 10^9$ gm cm^{-3}, for $T \sim 10^9$ °K, namely higher than the carbon ignition temperatures and hence Bruenn concluded that the URCA process cannot stabilize the carbon burning and the story was back the beginning. Figure 3, taken from Regev (1975) demonstrates the basic result. Close to the URCA shell the electrons have very little extra energy and the cooling dominates, however, outside a very narrow strip $\Delta\rho << \rho_{shell}$ heating dominates and if a convective core extends over a sufficiently large density gradient the total heating may overcome the total cooling and convection may have the opposite effect: heating instead of cooling.

The pendulum swung in the opposite direction after Couch and Arnett (1974) introduced the idea of a cycle. Consider a given mass element moved up and down by the convective currents. The energy balance at the high density side is

$$\Delta E^{ec} = \mu^{ec} - \Delta Q - E_\nu^{ec} \tag{10}$$

and at the low density:

$$\Delta E^- = \Delta Q - E_\nu^- - \mu^- \tag{11}$$

Here $\mu = \varepsilon_f + m_ec^2$ is the chemical potential of the medium. The index ec denotes that it must be evaluated at the point at which the e-capture takes place and vice versa, μ^- is the chemical potential at the place of e-emission. Consider now the full cycle. The mass unit starts at the high density side by absorbing an electron. It then moves upward where it decays as soon as the density falls. We find that the downward moving mass unit has one extra electron compared to the upward moving mass-unit. Consequently there is a net transfer of electrons downward. (The upward moving electrons are "hidden" in the form of a neutron.) Couch and Arnett add therefore the two energy equations to get:

$$\Delta E^{ec} + \Delta E^- = \mu^{ec} - \mu^- - (\varepsilon_\nu^{ec} + \varepsilon_\nu^-) \tag{12}$$

Theynow reason that the difference between the fermi energies of the two locations must be invested in maintaining the convective flow since the electron must be brought back to the original place. Note that the electrons move downward and hence

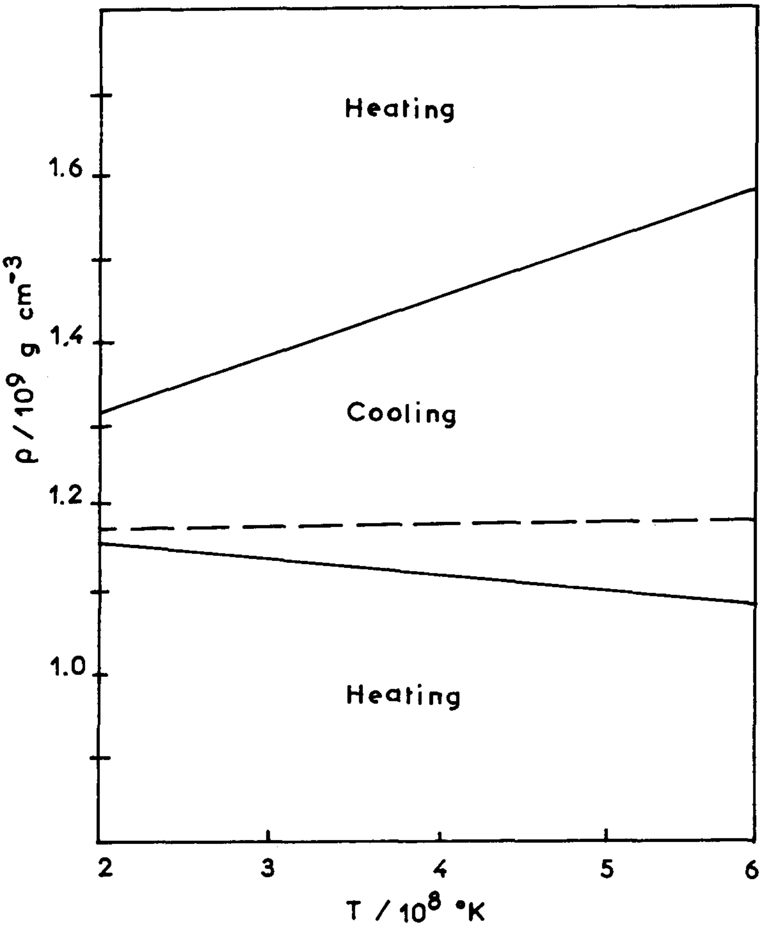

Figure 3.

The regions in the ρ T plane for which heating occurs. The calculation is carried for the Mg^{25} - Na^{25} URCA pair with uniform abundance ($X = 10^{-4.1}$). The broken line indicates the place where $\Delta Q = m_e c^2$ - the URCA shell.

https://doi.org/10.1017/S0252921100112485 Published online by Cambridge University Press

release potential energy rather than absorb. Moreover, in hydrostatic equilibrium the total chemical potential must be constant, hence $\mu = m_p\ \phi$ = const or $\Delta\mu = m_p \Delta\phi$ where ϕ is the gravitational potential and m_p the mass of the proton. The gravitational potential is determined by the C and O nuclei and not by the electrons and much less so by the URCA pair.

The fact that the change in chemical potential of the electrons must be equal to the change in gravitational potential due to the nuclei gives rise to the high degeneracy of the electrons. The difference $\mu^{ec} - \mu$ is therefore not equal to the difference in potential energy of the electrons $m_e\ \Delta\phi$ but to the difference in potential energy of the nuclei - and this in turn is not so relevant (see later).

Couch and Arnett argue that the convective blob (of given mass) contracts upon e-capture (pressure equilibrium - just the term ignored by Bruenn) and becomes denser than the surroundings while the downward moving blob emits an electron and becomes lighter than the surroundings. Thus the convective flow has to carry denser matter upward and lighter downward. This difference costs the extra energy that appears in the term $\Delta\mu = \mu^{ec} - \mu^{-}$. A similar argument to the previous one shows that this is not the case. Consequently, the final result of Bruenn remains valid.

Regev and Shaviv (1975) considered the question of convective stability of the URCA process. If heating is important, then convection may start before the Schwarzschild criterion is violated because a small perturbation in the bubble may heat it. It was found that convection may start earlier than assumed before (according to the Schwarzschild criterion). However, the rise times are quite long and it is impossible to give a final answer as to what will happen without a detailed stellar evolution calculation. The analysis was a local one.

Lazareff (1975) considered the details of the convection process with URCA heating and concluded that no stationary convective core can exist. This conclusion is correct but for completely different reasons. Lazareff assumed mixing-length theory in which only rising bubbles exist and considered the details of this motion. He assumed that during this motion the URCA process releases heat and concluded that if this process is integrated over the whole convective zone the total entropy will increase in time and hence no stationary state exists. The error in this kind of treatment is readily seen in the case of no URCA pair. One finds that the entropy density continues to increase even in the case of normal stellar convection. The problem appears because (a) the downward bubbles are not included,

(b) the work done by the buoyancy force (absorbed by the bubble but lost by the surrounding) is not accounted for and

(c) the mixing-length theory discusses the perturbed quantities and not the actual quantities and hence integral theorems are unavailable.

Shaviv and Regev (1976) proceeded in two steps. First the motion of a single bubble was analyzed and then the global properties of the convective zone were discussed.

https://doi.org/10.1017/S0252921100112485 Published online by Cambridge University Press

The change in temperature of a blob in surroundings containing an URCA pair is given by (Regev (1975)) :

$$v(r,\xi)\ \frac{dT^{*}}{dr}(r,\xi) = v(r,\xi)\ \frac{dT}{dr}\Big|_{ad} + \frac{\varepsilon(\rho^{*},\ T^{*},\ X_{\alpha}^{*})}{c_p} \quad (13)$$

when ξ is the place of formation, v the velocity, X_α the composition at time t. Quantities with asterisk denote the values inside the blob. C_p is the specific heat at constant pressure. The composition of the blob changes in time according to

$$v(r,\xi)\ \frac{dX_1^{*}(r,\xi)}{dr} = -\lambda_1^{*} X_1^{*}(r,\xi) + \lambda_2 (X - X_1^{*}(r,\xi)) \quad (14)$$

where X_1^{*} is the mass-fraction of the first species of the URCA pair and X is the total mass-fraction of the pair.

The equations of motion are then integrated in order to find the motion. A typical result is shown in figure 4. We find that (a) for most cases of rising blobs the URCA heating "helps" the buoyancy forces and the velocity is increased. (b) downward moving blobs are somewhat disturbed by the heating (c) a subadiabatic gradient may lead to unstable upward-moving blobs (in agreement with the local stability analysis of Regev and Shaviv (1975)) but prevents blobs moving downward.The most important result is: (d) the URCA losses have a negligible effect on the motion of the rising blob.Actually, as the blob starts to move, its velocity is small and the effect of the URCA on acceleration very large, but as soon as the velocity becomes large, the time scale becomes too short to have any effect on the motion. We find therefore that the URCA losses are the result of spreading the URCA isotopes uniformly over the whole convective core.

Consider now a convective core as a zone with $q_\nu(r)$ losses and $q(r)$ heating per unit mass and time. Clearly, close to the URCA shell, q_ν is greater than the heating but far away $q(r)$, which includes the nuclear and the URCA heating, is the dominant factor. The two contributions have different spatial behaviour.
The energy equation is :

$$\rho\ \frac{dE}{dt} + P\nabla\cdot\vec{v} = \rho q - \rho q_\nu + \rho q_f - \nabla\cdot\vec{F} \quad (15)$$

where F is the radiative flux through radius r and q_f the rate of heat generated by dissipation. The equation of motion is given by

$$\rho\ \frac{d\vec{v}}{dt} = -\nabla p - \vec{f} - \rho\nabla\phi \quad (16)$$

where f is the frictional force per unit volume and ϕ the gravitational potential. The integration over the whole convective zone down to the place where $\vec{v}$=o yields,after

https://doi.org/10.1017/S0252921100112485 Published online by Cambridge University Press

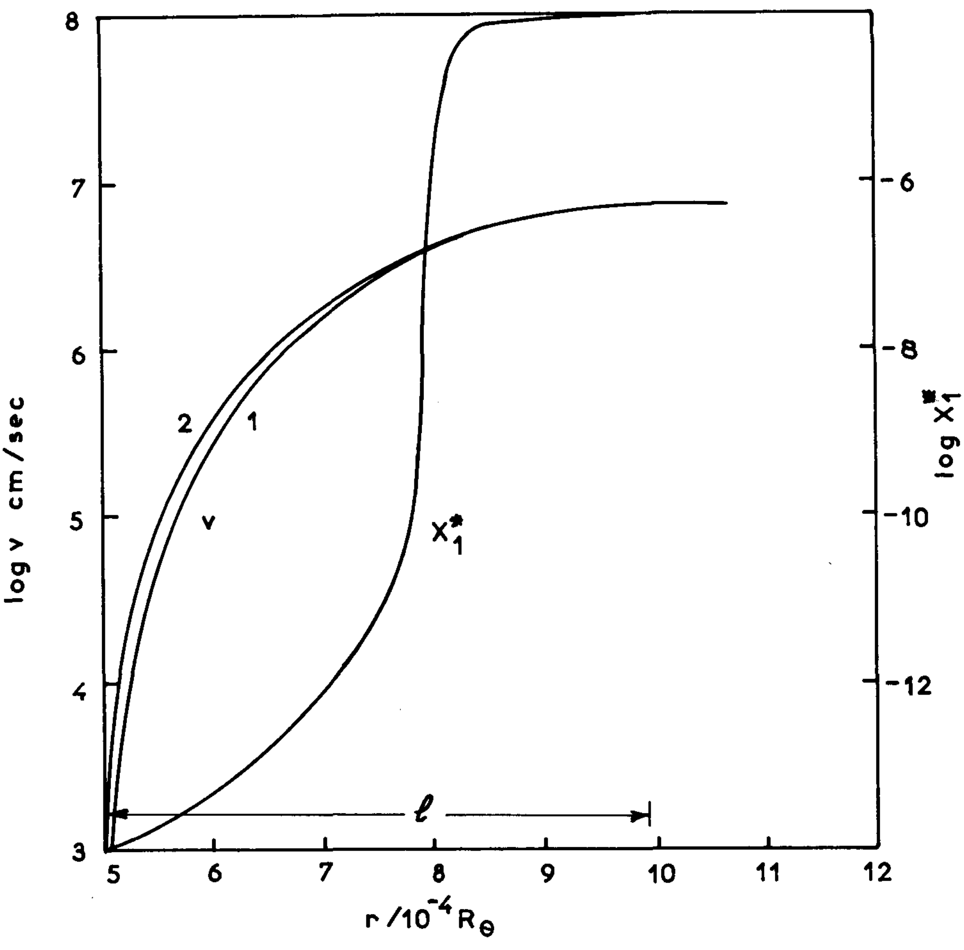

Figure 4.

The velocity of a blob in the case of Na^{23} URCA pair. Line 1 is log v for an adiabatic blob and line 2 is log v for a blob with the URCA pair. Line a is the equilibrium abundance of Na^{23} in the surrounding medium, ℓ is the distance of one mixing length.

https://doi.org/10.1017/S0252921100112485 Published online by Cambridge University Press

some manipulation ,the following result :

$$\overline{\frac{\partial}{\partial t}\left(1/2\,\rho v^2 + E\right)} = \overline{\rho q} - \overline{\rho q_\nu} + L_{in} - L_{out} \qquad (17)$$

where L_{in} and L_{out} are the radiative flux into and out of the convective zone respectively. The bar denotes an integral over the whole convective zone. Consider first the case of no URCA process, i.e. $q_\nu = 0$ and q is the energy generation due to nuclear reactions. In a steady state, the time derivative must vanish and we find that the net outcome of convection is to spread the nuclear energy generation over a large volume so that radiative flux can carry the energy from the boundary. When the URCA pair is present and the steady state is preserved, it follows that the total heating by the URCA process (added to q) must be equal to the total neutrino losses, q_ν. If this balance is not maintained the convective core will not be stationary. A detailed balance can exist only if the convective core has a definite extent. Moreover, even if such a steady state convective core exists, it is unstable. The analysis of the URCA losses shows that q_ν dominates near the URCA shell but the heat gain dominates elsewhere. Thus if the nuclear reactions increase their energy production and the convective core expands, the URCA process will increase the heating even more unless the radiative losses increase faster, which is not the case. We conclude therefore that steady state convection cannot control the carbon burning and the problem of the fate of these models and the progenitors of pulsars remains.

A question of principle remains : how come that a process which conserves material has as its outcome net heating ? The solution is that the URCA process is out of equilibrium. The net heating is given by (Regev (1975))

$$q = \frac{No}{A}\, m_e c^2 \left([\Delta Q - m_e c^2 - e_f]\, \lambda_2 X_2 - X_2 L_2 + [\varepsilon_f + m_e c^2 - \Delta Q]\lambda_1 X_1 - X_1 L_1 \right) \qquad (18)$$

where L_1 and L_2 are the neutrino loss rates by the e.c. and β decay rates per nucleus respectively. N_o and A are the Avogadro number and atomic weight respectively. When the URCA pair is spread uniformly, $q > 0$ and we have heating, but at equilibrium $X_1\lambda_1 = \lambda_2 L_2$ and the expression for q becomes

$$q = - \frac{No}{A}\, m_e c^2 \left(X_1 L_1 + X_2 L_2\right) \qquad (19)$$

and we have cooling only.

We are led finally to the question of the distribution of the URCA pair. Two time-scales affect the distribution : the convection mixing time $\tau_{conv} = \ell / v_{conv}$ and $\tau_{URCA} = (\lambda_1 + \lambda_2)^{-1}$, which is the decay time. Define a new parameter by

$$\alpha_{mix} = \tau_{conv}\, \tau_{URCA}^{-1} \simeq 3.5 \times 10^7 \, (\lambda_1 + \lambda_2) v_{conv}^{-1} \qquad (20)$$

where the convective velocity is given in cm/sec. The limit of complete mixing is obtained for $\alpha_{mix} << 1$ while equilibrium is reached for $\alpha_{mix} >> 1$. In reality we find $\alpha_{mix} \approx 1$. Consequently, at the beginning of the convection the process is close to equilibrium but as time goes on the URCA pair is driven away from equilibrium and the heating appears. The entropy added into the convective zone is due, as pointed out by Lazareff (75), to the non-equilibrium state of the URCA pair.

Acknowledgement : It is a pleasure to thank Mr O. Regev for discussions that made this analysis and presentation possible.

BIBLIOGRAPHY

1. Bruenn S. W. (1973) Ap J. Lett. 183, L125
2. Couch R. G. and Arnett W. D. (1974) Ap J. 194, 537
3. Gamow G. and Schoenberg M. (1940) Phys. Rev 58, 1117
4. Gamow G. and Schoenberg M. (1941) Phys. Rev 59, 539
5. Tsuruta S. and Cameron A. G. W. (1970) Astr & Space Sci. 7, 314
6. Gunn J. E. and Ostriker J. P. (1971) Ap J. 160, 979
7. Lazareff B. (1975) Ast. & Astrophys. 45, 141
8. Paczynski B. (1972) Astrophys. Lett 11, 53
   (1973) ibid 15, 147
   (1973) Acta Astronomica 23, 1
9. Regev O. (September 1975) On the Interaction Between Convection and the URCA Process M.Sc. thesis Tel Aviv University
10. Regev O. and Shaviv G. (1975) Ast. & Space Sci 37, 143
11. Shaviv G. and Regev O. (1976) Ast. & Astrophys. (in press)

https://doi.org/10.1017/S0252921100112485 Published online by Cambridge University Press