A Linear Acceleration Emission Mechanism

E. T. Rowe, School of Physics, University of Sydney, NSW 2006

Abstract: A theory of linear acceleration emission, with possible application to pulsars, is developed. The equation of motion for a relativistic charged particle accelerated in the field of a longitudinal wave is solved to first order in the electric potential and used to derive the first order current. The power emitted and the absorption coefficient are calculated and the conditions under which masing can occur are discussed.

1. Introduction

In this paper we develop a theory of emission by charged particles in large amplitude longitudinal waves, with possible application to pulsars, as an alternative to emission mechanisms in the literature. In several of these existing mechanisms a strong magnetic field plays an active role. For example, gyromagnetic emission has been examined in detail (e.g. Melrose 1980, Vol. I, p. 98); however, this cannot explain emission from pulsars because in their superstrong magnetic fields of up to $10^8 \text{T}$ (e.g. Smith 1977, p. 47), electrons and positrons fall quickly into the ground state with motion only along the field. Curvature emission by electrons and positrons moving along curved field lines was suggested by Radhakrishnan (1969) and Sturrock (1971) but while this is consistent with a one-dimensional distribution of particles, the mechanism does not allow masing (Blandford 1975); masing is one way in which the coherence of emission necessary to explain the high brightness temperatures ($10^{20} - 10^{20} \text{K}$) observed, can be attained. To by-pass this dilemma, coherent curvature emission by bunches was proposed by Sturrock, Petrosian and Turk (1975), Elsässer and Kirk (1976) and Asseo, Pellat and Sol (1981), and used by Gil (1986), but this has been criticised by Melrose (1978) and many of the details of bunching remain unclear. In other mechanisms (as in the one presented here) the super-strong magnetic field plays only a passive role, guiding the particles as they radiate, and is otherwise not involved in the emission process. For instance, Rees' mechanism (Rees 1971) involves emission by particles in the field of transverse waves. An exact calculation for the particle orbit in this case has been given by Gunn and Ostriker (1971) and Melrose (1980, Vol. I, p. 176) wrote down the absorption coefficient for this process but masing, and its application to pulsars, is yet to be considered. As another alternative a linear acceleration mechanism involving time varying parallel electric fields was looked at by Melrose (1978). Despite all these ideas it is agreed (e.g. Rankin and Gil 1989) that a suitable coherent emission mechanism has not yet been found.

The linear acceleration mechanism we consider in this paper is at an early stage of development so we present only a preliminary account of it and no attempt is made here to fit values to our parameters. It is considered to be a possible pulsar emission mechanism on general theoretical grounds. We know that in a super-strong magnetic field we effectively have a one-dimensional particle distribution of electrons and positrons which, in a pulsar, will be relativistic along the magnetic field direction (e.g. Hewish 1981; Wunner, Herold and Ruder 1981) and longitudinal waves in such a plasma should propagate primarily along the field. So while we consider the mechanism to have possible application to pulsars, and this provides the chief motivation for the calculations, we do not discuss this any further here.

The organisation of the paper is as follows. In Section 2 we derive the single particle current to first order in the electric potential. The method we use involves writing the Hamiltonian for the particle in the waveframe and solving for the particle orbit, which we can find to any order. The usual method, involving the second order equation of motion, is intrinsically recursive and hence only the lowest orders can readily be considered. In Section 3 we derive the total power emitted in the special case where the refractive index is constant and in Section 4 we write down an expression for the absorption coefficient for the process and consider the possibility of maser action. Throughout we use natural units with $\hbar = c = 1$.

2. The Single Particle Current

Here we consider a particle moving along the $z$ axis, parallel to a longitudinal wave. The Hamiltonian is then

$$H = m\gamma + q\Phi_0 \cos(K(z - z_0)),$$

in the waveframe, where $m$ is the particle mass, $q$ the particle charge, $\Phi_0$ the amplitude of the electric potential, $K$ the wave vector and $z_0$ is an arbitrary initial position. Since there is no explicit time dependence, the Hamiltonian is a constant of the motion and we can take its value to be given by $m\gamma_0$, the zero order value of the particle energy.

Solving for the particle velocity, $v$, which we take to be positive, gives

$$v = \frac{\sqrt{(m\gamma_0 - q\Phi_0 \cos(K(z - z_0)))^2 - m^2}}{m\gamma_0 - q\Phi_0 \cos(K(z - z_0))},$$

which can be integrated exactly to yield time, $t$, as a function of position, $z$, involving elliptic functions. However, for simplicity we choose instead to expand Equation (1) in powers of $q\Phi_0/m\gamma_0$. The expansion converges as long as $|q\Phi_0|/m\gamma_0 < 1$ and this holds for untrapped particles which satisfy the more strict inequality

$$(\gamma_0 - 1) > \frac{|q\Phi_0|}{m},$$

(we do not treat the case of trapped particles here). Retaining only the zeroth and first order terms of (1) and integrating gives

$$t = t + \frac{z}{v_0} + \frac{q\Phi_0}{m} \frac{1}{(\gamma_0 v_0)^2} \frac{1}{K} \sin(K(z - z_0)),$$

where $\gamma$ is a constant and $v_0$ is the zero order particle velocity. The first order current is then (see e.g. Melrose 1980, Vol. I for the first expression)

$$j(\omega, k) = q \int dv \exp\{i[\omega t - k \cdot x]\}$$

$$= qe^{i\omega t} 2\pi v_0 \left\{ \delta(\omega - v_0 k_\parallel) + \frac{1}{2} \frac{q\Phi_0}{K} \frac{1}{(\gamma_0 v_0)^2} \left\{ e^{-iKz_0} \delta(\omega - v_0(k_\parallel - K)) + \frac{1}{2} \right\} \right\}.$$
At this point we could easily Lorentz transform to a frame in which the longitudinal wave has a non-zero frequency and, although the derivation we have used here applies only if the phase speed of the wave is less than that of light (so there is an attainable waveframe), the result could be used to describe any wave. We will, however, restrict the calculations to the waveframe.

The first term in (3) describes Cerenkov emission and the others correspond to Doppler and anomalous Doppler emission respectively, but the first two terms are not relevant for a plasma, since their resonance conditions cannot be satisfied. The emission parallels cyclotron emission, in the sense that low harmonics dominate and higher harmonics (and corrections to the low harmonics) occur as higher order terms. The resonance condition relevant to a plasma can be written as

$$\omega = \frac{Kv_0}{1 - n v_0 \cos \theta},$$

where we have set $k_B = n \omega_0 \cos \theta$ (so $\theta$ is the angle of $k$ to the direction of particle motion) and $n$ is the refractive index for transverse waves (we only consider emission of transverse waves here). Emission can occur for all angles, since $n<1$ in a plasma, with frequencies greater than $K v_0$ emitted in the forward direction and less than $K v_0$ in the backward direction. The resonance condition is reminiscent of the electric ondulator case (Ginzburg 1979), which involves a perpendicular electric field with $K v_0$ replacing the constant frequency $\omega_0$. In the ondulator case higher velocity particles emit at higher frequencies in the forward direction and lower frequencies in the backward direction whereas in the present case all emission frequencies increase with velocity.

3. The Power Radiated

We can use the single particle current derived in section 2 to obtain the total power emitted by a charged particle moving in the longitudinal wave. Taking $\omega_0$ to be the polarisation vector of the emitted radiation, in the temporal gauge, the power emitted per unit volume of $k$-space is (e.g. Melrose 1978)

$$P(k) = \frac{\mathbf{R}_E(k)}{e_0^2} |\epsilon_{M}(k) \cdot j[\omega M(k), k]|^2,$$  \hspace{1cm} (4)

where $M$ labels the mode of emission (here transverse waves), $\omega = \omega_0(k)$ is the dispersion relation, $\mathcal{T}$ is a time unit (which will cancel out) and $R_E(k)$ is the ratio of electric to total energy for the mode. Substituting the current (3) into (4) and retaining only the second order term of the first harmonic relevant to a plasma, we obtain

$$P(k) = \frac{\pi R_E(k)^2 (q_0^2 \phi_0^2)}{2e_0 |\omega M(k)|^2 |\epsilon_{M}(k)|^2 (\omega - \omega_0(k + K))},$$  \hspace{1cm} (5)

To calculate the total power emitted in transverse waves we sum over states of orthogonal polarisation, replacing $|\epsilon_{M}(k)|^2$ by $\sin^2 \theta$, and integrate over all phase space to obtain

$$P = \frac{q^2}{16 \pi e_0 K^2} \int_{-K v_0/(1 - n v_0 \omega_0)}^{K v_0/(1 + n v_0 \omega_0)} d\omega \omega^3 \left[ 1 - \frac{1}{n v_0} - \frac{K}{n \omega_0} \right]^2,$$  \hspace{1cm} (6)

where the $\phi$ integral was evaluated by assuming cylindrical symmetry and the $\theta$ integral by making use of the delta function in (5).

In general it may be difficult to solve for the limits of the integration; for instance, the super-strong magnetic field of a pulsar results in a complicated dependence of $n$ on the angle $\theta$ (Melrose and Stoneham 1977). Supposing instead that the refractive index is independent of frequency over our frequency band (given any refractive index, we can always find suitable values for the parameters $K$ and $\omega_0$ such that it is approximately constant for frequencies in the vicinity of the peak in the emitted power), we readily obtain

$$P = \frac{q^2 K^2 n}{12 \pi e_0} \left( \frac{\phi_0}{m} \right)^2 v_0 (1 - v_0^2)^3 (1 - n v_0^2)^3.$$

The three main features of the emitted power are as follows.

Firstly, the power is emitted in a band of frequencies, peaked at $\omega > K v_0$ (in the forward direction) and widening with particle velocity. Secondly, the total power radiated increases quadratically with the electric field amplitude, $\phi_0 K$, and thirdly, there is a value of $v_0$, dependent only on refractive index, for which the efficiency of the mechanism in converting particle energy into radiated energy is maximised. Maximum efficiency is attained when the widening of the frequency band is balanced by the overall drop in power for higher velocities.

4. The Absorption Coefficient

The absorption coefficient for waves in mode $M$ with wave-vector $k$ is (c.g. Melrose 1980, Vol. 1, p. 161)

$$\gamma_M(k) = - \int d^3 p_d \frac{P(k)}{\omega M(k)} \Delta p_d \frac{\partial f(p_d)}{\partial p_d},$$  \hspace{1cm} (7)

where $f(p_d)$ is the particle distribution function. The integration variable must be a constant of the single particle motion and $\Delta p_d$ is the change in this constant on emission of a photon. To first order we have $p_d = m v_0 \gamma_0$ (as can be seen from Equation (2)). In the Rees’ mechanism case the constant of the motion was given by Arons (1972) and in the present case it is also possible to determine $p_d$ exactly but we do not need to do this here.

In analogy with Rees’ mechanism (see e.g. Melrose 1980, Vol. 1, p. 176) we have $\Delta p_d = (k_x, k_y, K + k_z)$ and assuming a one dimensional particle distribution, $f(p_d) = \int \delta(p_d) \delta(p_d)$, substituting Equation (5) into Equation (7) and performing the integrals over delta functions we obtain

$$\gamma_M(k) = \frac{- \pi R_E(k)^2 (q_0^2 \phi_0^2)}{2e_0 K} (k_z + K) |\epsilon_{M}(k)|^2 \frac{k_z + K}{\omega_+^2} \frac{\omega}{\omega_+^2},$$  \hspace{1cm} (8)

with

$$\omega_+ = \frac{1}{\sqrt{1 - (\omega/(k_z + K))^2}}.$$

This expression is valid provided that any perpendicular velocity after emission is small compared with the parallel velocity (since we have assumed $v_+ = 0$ in our calculations). Here we can average over two orthogonal polarisations and, with the resonance condition and $k_B = n \omega_0 \cos \theta$, write an average absorption coefficient

$$\gamma_M(k) = \frac{- \pi R_E(k)^2 (q_0^2 \phi_0^2)}{4e_0 K} \frac{\omega}{\gamma_+^2} \frac{1}{\gamma_+^2} \frac{\omega}{\gamma_+^2},$$  \hspace{1cm} (9)

with

$$\gamma_+ = \frac{1}{\sqrt{1 - (\omega/(k_z + K))^2}}.$$
If the absorption coefficient is negative, maser action is allowed and waves grow exponentially, whereas if it is positive we have exponential damping of the waves. Here maser action is easily achieved for one dimensional distributions; all we require is an inverted population in the sense $\frac{df}{dp} > 0$, such as is the case in the 'bump in the tail' instability. All that remains is to determine suitable values for the parameters in our mechanism and to see whether the implied growth rates are sufficient to explain pulsar emission, but we will not attempt that here. The growth rate has almost the same functional form as the power per unit frequency range, (6), apart from a factor $\frac{\omega^2}{\gamma_b}$. Consequently, if we ignore the distribution function (i.e. set it to unity), then the frequency for which $\gamma_M(k)$ is maximized is slightly less than that for which the power radiated per unit frequency is maximized.

5. Conclusion

The purpose of this paper has been to outline early work on an emission mechanism involving a charged particle moving parallel to a longitudinal wave, the motivation being its possible application to pulsars. The mechanism has been presented as a possible pulsar emission mechanism on general grounds and we have not attempted to fit values to our parameters as yet. We have given expressions for the single particle current, power radiated in transverse waves and the absorption coefficient, all calculated in the wave-frame. The total power radiated at the first harmonic was calculated for the simple case of constant refractive index and we found that it depends on the square of the electric field amplitude, that most power is radiated at a frequency greater than $K_{\nu_0}$ and that the efficiency of the mechanism is maximised for some value of particle velocity, $v_0$. In section (4) we found that maser action simply requires an inverted population or rising distribution function over some range of velocities.

An obvious next step is to fit suitable values to the parameters $K$, $\phi_0$ and $v_0$ in order to explain observed frequencies, bandwidths and intensities of pulsar radiation. We can also introduce a longitudinal-wave frequency, $\Omega$, into the calculations simply by Lorentz transforming to another frame moving along the particle trajectory. This would give us another variable to consider and, since the results of the transform would be valid for any values of $K$ and $\Omega$, we could look at the added possibility of longitudinal waves with phase speed greater than light.

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Melrose, D. B., 1980, Plasma Astrophysics (Gordon and Breach, 2 Vols.)