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The perennial problem of geometry

The vocabulary of mathematics—with its sample spaces mapped onto the real number line, functions discontinuous at points, orthogonal polynomials and price vectors—is evidence enough of the motivating power of spatial ideas throughout the subject. What would we know of integration without area, complex numbers without the Argand diagram, π without circles? To step some years down the educational ladder, we have come to appreciate in recent years the importance of spatial work in the education of young children—both because objects of different shapes and sizes, related to each other in various ways, are amongst the first experiences lending themselves to investigation in mathematical terms; and also because of the illumination that comes from bringing together space and number, informally but deliberately, in measurement and in graphical representation of many kinds.

But all this is, of course, to skirt round the problem: what should we teach, if anything, by way of "geometry" in schools, colleges of education and university undergraduate courses? Is there still a place for the vestiges of the euclidean tradition? Is the answer to re-structure the subject matter in terms of transformations or vectors? Should school geometry be treated as a branch of applied mathematics, alongside mechanics, as a means to model the physical world of our experience? Is geometry any more than a subset of algebra, and what do presentations which adopt this line signify to undergraduates who lack the background of euclidean, analytical and projective geometry that their teachers enjoyed as students? Where does topology come in?

This *Gazette* does not purport to offer answers to these questions; but in devoting an issue largely to this one area of mathematics, we seek to draw fresh attention to the debate. Our three opening articles respectively take a historical view, offer some suggestions for teaching today and give a glimpse

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of current interests in one area of geometrical research. We would welcome correspondence on the issues they raise. And whatever conclusions we come to, let us hope that the symposium of articles and notes which follows will re-kindle in our readers the pleasure which geometry has brought to many generations of mathematicians; surely ours will not be the last!

D.A.Q.

Milestone or Millstone?

A. G. HOWSON

1973 sees the jubilee of the Mathematical Association's first report on *The teaching of geometry in schools* and the centenary of the birth of one of its most influential authors, Charles Godfrey. It would seem appropriate, therefore, to look back at that report, at its origins and its effects, and to pay especial attention to the part played by Godfrey in the development of the teaching of geometry.

The position in 1922 when the General Teaching Committee of the Mathematical Association appointed a sub-committee to prepare the geometry report was not unlike that today. For, despite those modern innovators who believe that curriculum innovation began in the 1960s [1], great changes in the mathematical curriculum had taken place in schools early this century. Thus by 1912, Godfrey [2] could say of mathematics teaching in England that

"The use of graphical methods in elementary algebra is universal and entirely a 20th-century development. Other aspects of the same movement are the adoption of descriptive geometry by the mathematicians, the use of handy 4-figure tables [3], and of graphical methods in statics."

D. E. Smith [4] was also able to report that

"about 90% of the schools (in England) state that the graphical study of statistics is given ... for a new subject it has made rapid progress having both the encouragement of the mathematicians and an abundant opportunity for application. The graphical representation of functions is taught in all public schools The use of vectors is found in a large majority of schools, in connection with mechanics (velocities, accelerations, forces) ... (and) ... in some schools ... in connection with complex numbers."

Changes then were being made, but nowhere was this more apparent than in the teaching of geometry. In 1903 Cambridge ruled that in its examinations "Any proof of a proposition will be accepted that appears to the examiners to form part of a systematic treatment of the subject" [5]. What effect this had on the examiners, faced with what appeared to be a well-nigh