ON LOW MASS AGB STARS

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ABSTRACT. Recent results on low mass AGB stars are presented. Observed amounts of AGB mass loss imply that thermal pulses will only be encountered for stars of initial mass less than about $4\,M_\odot$ for Pop I and $3\,M_\odot$ for Pop II. $M_c - L$, $M_c - \tau_{ij}$, and $M_c - T_b$ relations are summarized. Carbon dredge-up has been found in low mass stars of both Pop I and Pop II; the mixing length parameter α is crucial to dredge-up, and its value must be normalized according to each author's opacities and mixing length treatment (e.g., via the Sun's T_e and L). The "carbon star mystery" is nearing a solution, but a new "*s*-process mystery" has appeared: only in a narrow range of mass and metallicity have theoretical models been found that encounter the semiconvective ^{13}C *s*-process mechanism.

1. Introduction

Historically, low mass stars were among the first to be investigated in detail theoretically. However, much of the attention was soon shifted to intermediate and high mass stars, which do not encounter the helium core flash and therefore were considered to be easier to compute. Nevertheless, an understanding of low mass stars is crucial, since the *majority* of stars, including our Sun, belong to this class. Low mass stars, both in the red giant and the asymptotic giant branch (AGB) stage, are luminous and can contribute significantly to the red and infrared luminosity of galaxies and of old clusters. Low (and intermediate) mass stars on the AGB display rich spectra, showing an enrichment of carbon and *s*-process elements: they are the dominant source of these elements in the interstellar medium.

Renewed attention was given to low mass stars due to the "carbon star mystery" (see, e.g., Iben 1981), in which theory could only provide massive carbon stars ($M \gtrsim 5\,M_\odot$) of high luminosity (brighter than $M_{\rm bol} \sim -6$), while observations found carbon stars only at lower luminosities ($-3.5 > M_{\rm bol} > -6$). This "carbon star mystery", and the new "*s*-process mystery", are discussed in Section 2.2.

Low mass stars may be defined as those in the mass range $M_{\rm burn} < M < M_{\rm HeF}$, where $M_{\rm burn}$ is the lowest stellar mass for which *hydrogen* ignition can take place and $M_{\rm HeF}$ is the largest stellar mass for which *helium* ignition will take place in a *degenerate* core (leading to the helium core flash). The *lower limit* is $M_{\rm burn} \sim 0.08\,M_\odot$ (see, e.g., Dorman *et al.* 1989), and the classical value of the *upper limit* is $M_{\rm HeF} \approx 2.3\,M_\odot$ for Pop I stars and

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$M_{\mathrm{HeF}} \approx 1.8\,M_\odot$ for Pop II stars (Chiosi 1986). Note that convective overshoot, which is still very uncertain (Renzini 1987), can reduce M_{HeF} to $\sim 1.6\,M_\odot$ (Bertelli *et al.* 1986).

To reach the AGB, a star must have an initial mass of at least $\sim 0.7\,M_\odot$, due to the core mass of $M_c \sim 0.4 - 0.5\,M_\odot$ required to encounter the helium core flash and due to the observed mass loss of $\sim 0.2\,M_\odot$ on the red giant branch (see, e.g., Renzini 1981). Low mass stars on the AGB shade smoothly into intermediate mass stars, defined as those with initial masses in the range $M_{\mathrm{HeF}} < M < M_{\mathrm{up}}$, where M_{up} is the largest initial stellar mass which will develop a degenerate carbon-oxygen core. The classical value is $M_{\mathrm{up}} \approx 9\,M_\odot$ for Pop I stars and $M_{\mathrm{up}} \approx 7\,M_\odot$ for Pop II stars (Becker and Iben 1979); convective overshoot could reduce M_{up} to a value as low as $\sim 6\,M_\odot$ (Bertelli *et al.* 1986).

In the last decade, considerable work has been carried out for low mass stars, although much remains to be done, as will be discussed below.

2. Results and Discussion

2.1. THE ONSET OF THERMAL PULSES, MASS LOSS, AND M_c – RELATIONS

2.1.1. *Mass Loss on the AGB and the Onset of Thermal Pulses.* After helium is exhausted in a star's core, the star burns helium quietly in a shell surrounding a degenerate carbon-oxygen core; hydrogen shell burning is extinguished. The star has now left the horizontal branch, growing redder and more luminous: this phase is sometimes called the "early-AGB". A cool stellar wind is observed for these stars, with significant mass loss rates. When the helium shell has burned out far enough, the hydrogen shell re-ignites, and helium shell flashes (also called thermal pulses) begin: this double shell burning stage with a degenerate core is called the AGB phase. Fig. 1 displays the core mass M_c^{TP} at which the thermal pulses begin, for stars of Pop I ($Y = 0.28$, $Z = 0.02$: dotted line) and Pop II ($Y = 0.24$, $Z = 0.001$: short-dashed line), based on the detailed evolutionary models of Boothroyd and Sackmann (1988*c*). These results agree well with the independent detailed calculations of Lattanzio (1986). However, they do *not* agree with the interpolated relation of Iben and Renzini (1983) (long-dashed line labelled IR in Fig. 1), which was the only one available until recently. Note that, for a given initial stellar mass, Pop I stars begin their thermal pulses at considerably lower core masses than Pop II stars.

If *mass loss* on the early-AGB is sufficiently severe as to remove the entire envelope before the hydrogen shell re-ignites, *the star will never encounter helium shell flashes.* Observations indicate that this indeed happens for stars above a critical initial mass M_W. Weidemann and Koester (1983) first pointed out the existence of an observational initial-mass–final-mass ($M_i - M_f$) relation, showing surprisingly low final masses. The Weidemann and Koester (1983) $M_i - M_f$ relation, obtained from white dwarf masses in open clusters, has since been improved (for $M_i \lesssim 4\,M_\odot$) using nuclei of planetary nebulae (Weidemann 1984): this is the solid line marked W in Fig. 1. Aaronson and Mould (1985) also obtained an $M_i - M_f$ relation, using observed luminosities of the tip of the AGB in Magellanic Cloud clusters and the Iben and Renzini (1983) core-mass–luminosity relation. We used the Aaronson and Mould (1985) luminosities with the Boothroyd and Sackmann (1988*b*) core-mass–luminosity relation of eq. (1) below (more appropriate for these relatively low-mass stars); the resulting $M_i - M_f$ relation is the solid line marked AM′ in Fig. 1. The uncertainty of these $M_i - M_f$ relations is probably about $0.1\,M_\odot$. The separation of the two $M_i - M_f$ relations is probably due, at least in part, to the different metallicities of the

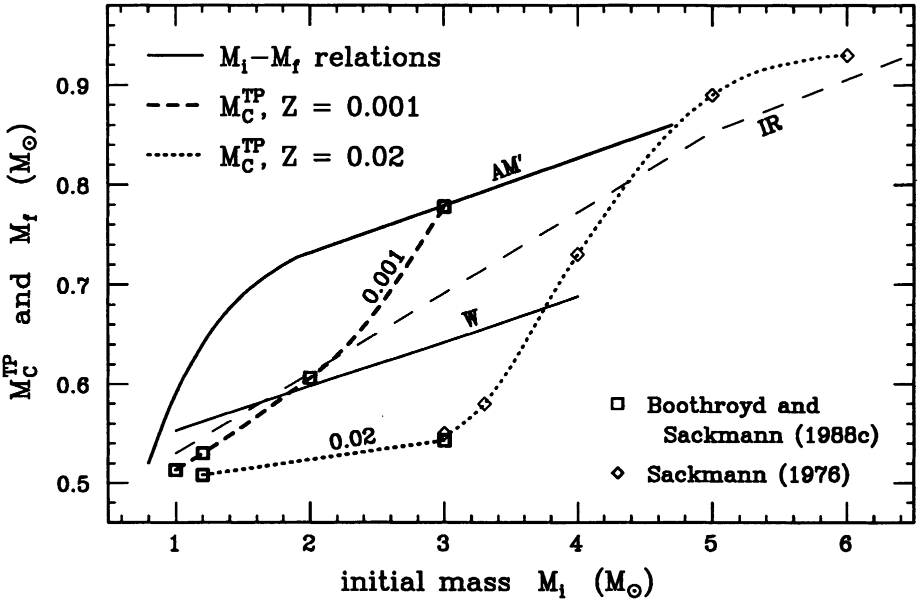

Figure 1. Core mass M_c^{TP} at the first thermal pulse, and final stellar mass M_f implied by observations, as a function of initial stellar mass M_i.

objects from which they were obtained: the metallicity of the Magellanic Cloud clusters ($Z \sim 0.002-0.01$: see, e.g., Chiosi *et al.* 1986) is several times smaller than that of planetary nebulae in the galactic disk ($Z \sim 0.02$). Higher mass loss is to be expected for higher metallicity, if radiation pressure on dust grains is the predominant cause of AGB mass loss.

The intersection of the $M_i - M_f$ relations with the curves showing the onset of flashes determines the value of M_W, the critical initial stellar mass above which flashes do not occur. Fig. 1 shows that $M_W \sim 4\,M_\odot$ for Pop I stars, and $M_W \sim 3\,M_\odot$ for Pop II stars (note that Magellanic Cloud metallicities are intermediate between the two cases shown in Fig. 1). Thus higher-mass stars ($M > M_W$) can never become carbon stars, although the uncertainty in the $M_i - M_f$ relations results in a fairly large uncertainty in M_W.

More relevant to observations are the *luminosities* L_{TP} at which thermal pulses begin (rather than the core masses M_c^{TP}). Fig. 2 displays the results of detailed calculations: the heavy solid line is for low mass Pop II objects and the heavy short-dashed line is for low mass Pop I objects (Boothroyd and Sackmann 1988*c*). These lines are again in good agreement with the independent detailed calculations of Lattanzio (1987), who considered objects of extreme helium content ($Y = 0.2$ and $Y = 0.3$) for the same metallicities (light lines of Fig. 2); he finds a fairly strong helium-dependence for the onset of flashes. Also shown is the Iben and Renzini (1983) relation (long-dashed line labelled IR), the only one available until recently; this relation *greatly overestimates the luminosity at the onset of flashes* (by as much as $\Delta M_{bol} \approx -1.7$ for a $3\,M_\odot$ star, or $\Delta M_{bol} \approx -0.8$ for a $1\,M_\odot$ star).

The recent detailed theoretical models of low mass stars are consistent with the observed range of carbon star luminosities (also shown in Fig. 2): the onset of flashes is found to be at luminosities below the lower end of the observed carbon star distribution.

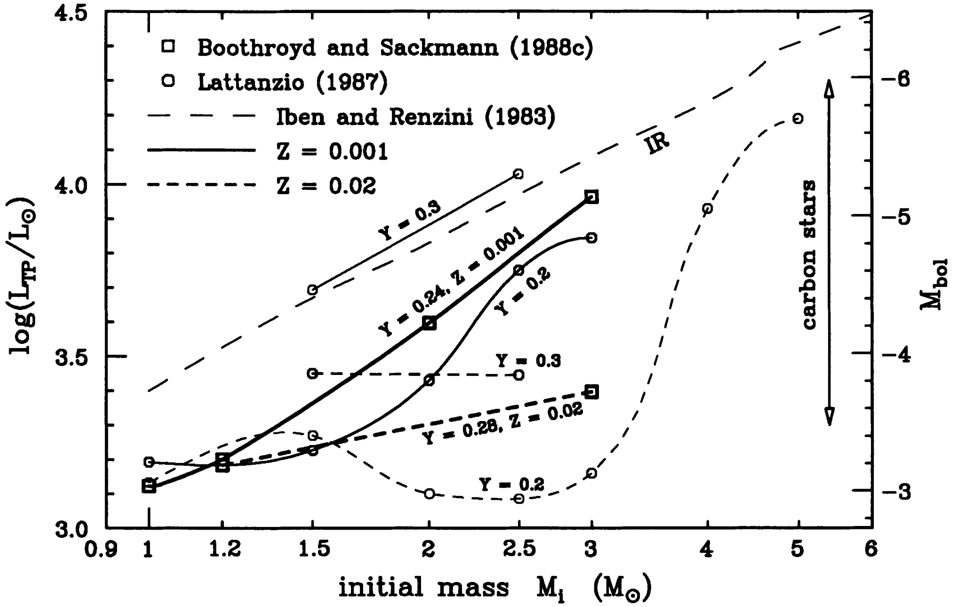

Figure 2. Stellar luminosity L_{TP} at the first thermal pulse, as a function of initial mass M_i.

2.1.2. *The Core-Mass-Luminosity Relation.* Paczyński (1970) and Uus (1970) discovered that there is a relation between the core mass M_c of an AGB star and its luminosity L in the interflash period (after flashes have built to "full amplitude", which takes 5–10 flashes). Until recently, this $M_c - L$ relation had been computed in detail only for Pop I stars (Paczyński 1970; Iben 1977; Havazelet and Barkat 1979; Wood and Zarro 1981); these authors concentrated on the higher core masses. The detailed AGB computations of Boothroyd and Sackmann (1988*a*, *b*) concentrated on the lower core masses, for stars of Pop II as well as Pop I; Fig. 3 displays the maximum interflash luminosity values as a function of core mass for these models, for a number of successive flashes. These were fitted by the heavy solid lines of Fig. 3, obtaining an $M_c - L$ relation which can be expressed as

$$L = 238{,}000\,\mu^3 (Z_{\mathrm{CNO}})^{1/25} (M_c^2 - 0.0305 M_c - 0.1802)\,, \quad 0.5 < M_c < 0.67 \qquad (1)$$

(L in $L_\odot$, M_c in $M_\odot$). Note that the molecular weight μ is roughly 0.6, and the mass fraction Z_{CNO} of CNO elements is about $0.75 \times Z$; the value of $\mu^3 (Z_{\mathrm{CNO}})^{1/25}$ is about 0.16 for the $Z = 0.001$ case and about 0.20 for the $Z = 0.02$ case.

Also shown in Fig. 3 are the relations of Wood and Zarro (1981) (short-dashed line labelled WZ) and Paczyński (1970) (dot-dashed line labelled P); their their relations should *not* be used below about $0.65 M_\odot$, where the $M_c - L$ relation becomes somewhat non-linear. Note that our $Z = 0.02$ line approaches the the Wood and Zarro (1981) line, as it should; a reasonably accurate fit to the combined $Z = 0.02$ data of Boothroyd and Sackmann (1988*b*) with that of Wood and Zarro (1981) gives

$$L = \begin{cases} 51{,}010\,(M_c - 0.453)\,, & 0.5 \lesssim M_c \le 0.755 \\ 59{,}250\,(M_c - 0.495)\,, & 0.755 < M_c \lesssim 0.9\,. \end{cases} \qquad (2)$$

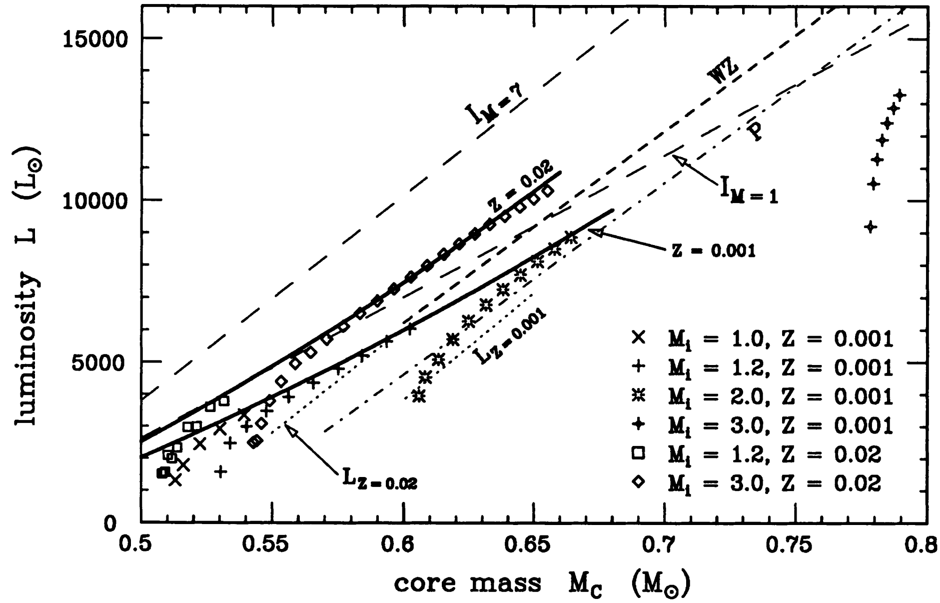

Figure 3. The $M_c - L$ relation for the AGB, for Pop I and Pop II stars. Symbols are interflash luminosities for the models of Boothroyd and Sackmann (1988*b*, *c*).

This is valid for $Z = 0.02$ (Pop I); it should be reduced by a factor of about 0.8 for $Z = 0.001$ (Pop II), according to the composition dependence of eq. (1).

Note that our $Z = 0.02$ line and that of Wood and Zarro (1981) lie at *higher* luminosity than the Paczyński (1970) relation, which was computed using the stationary shell burning approximation, giving the *average* interflash luminosity. From full flash computations, one sees that the average luminosity is lower than the maximum interflash luminosity due to the post-flash luminosity dip (see, e.g., Boothroyd and Sackmann 1988*a*). The $M_c - L$ relations of Lattanzio (1986) are also shown in Fig. 3 for Pop I and Pop II compositions (dotted lines marked L); the steepness and lower luminosity of his relations suggest that his flash computations had not quite reached "full amplitude" (this would also account for the overly flat slope of his core-mass–interflash-period relation: see Section 2.1.3).

For completeness, we include the Iben and Truran (1978) relation, for higher-mass stars:

$$L = 63{,}400\,(M_c - 0.44)(M/7)^{0.19}\,, \quad 0.96 \lesssim M_c < 1.4 \qquad (3)$$

This relation has been widely used, though it is not appropriate for lower mass stars, for which there is *no dependence on the total stellar mass* M, as confirmed by a number of investigators (Paczyński 1970; Havazelet and Barkat 1979; Wood and Zarro 1981; Boothroyd and Sackmann 1988*b*). Fortunately, for low mass stars of low core mass, the Iben and Truran (1978) formula was constructed to fall close to the $M_c - L$ relation of eq. (1): for example, see the line labelled $I_{M=1}$ in Fig. 3, which is eq. (3) with a mass $M = 1\,M_\odot$ inserted. On the other hand, use of a higher stellar mass in this formula does not work except for relatively high core masses: for example, see the line labelled $I_{M=7}$ in Fig. 3, which is eq. (3) with a mass $M = 7\,M_\odot$ inserted.

2.1.3. *The Core-Mass–Interflash-Period Relation.* Paczyński (1975) discovered the core-mass–interflash-period ($M_c - \tau_{if}$) relation for Pop I stars ($Z = 0.03$), which he gave as

$$\log \tau_{if} = 4.5\,(1.678 - M_c)\,, \quad 0.5 \lesssim M_c < 1.4\,. \tag{4}$$

A slightly less steep relation was given by Wood and Zarro (1981). Boothroyd and Sackmann (1988*c*) obtained an $M_c - \tau_{if}$ relation for Pop II as well as Pop I stars:

$$\log \tau_{if} = \begin{cases} 4.5\,(1.689 - M_c)\,, & 0.5 \lesssim M_c \lesssim 0.7 \quad (Z = 0.02) \\ 4.95\,(1.644 - M_c)\,, & 0.5 \lesssim M_c \lesssim 0.8 \quad (Z = 0.001) \end{cases} \tag{5}$$

This relation is in agreement with the Paczyński (1975) Pop I relation of eq. (4), but the interflash period differs by as much as a factor two between Pop I and Pop II stars. The Lattanzio (1986) $M_c - \tau_{if}$ relation shows a similar composition dependence, but has too flat a slope, suggesting that his flashes had not quite reached "full amplitude" (see Boothroyd and Sackmann 1988*c*).

2.1.4. *The Core-Mass–Base-Temperature Relation for Flash-Driven Convection.* Iben and Truran (1978) presented a relation between the core mass M_c and the maximum temperature T_b reached at the base of the flash-driven convective tongue during a shell flash, namely

$$T_b = 310 + 285\,(M_c - 0.96)\,, \quad M_c \gtrsim 0.96\,, \tag{6}$$

where T_b is in units of $10^6\ K$. Using the results of Boothroyd and Sackmann (1988*c*), Malaney and Boothroyd (1987) presented an $M_c - T_b$ relation for low core masses, with a flatter interpolated segment joining it to the Iben and Truran (1978) relation:

$$T_b = \begin{cases} 250 + 305\,(M_c - 0.53)\,, & 0.5 \lesssim M_c < 0.65 \\ 290 + 67\,(M_c - 0.65)\,, & 0.65 < M_c < 0.96\,. \end{cases} \tag{7}$$

(Actually, T_b was *lower* than this relation for Pop I models and *higher* for Pop II models, by about $10 \times 10^6\ K$.) *This relation should be used with caution*, since there is some indication that there may in fact be no unique $M_c - T_b$ relation: T_b follows a similar slope for different stars as a function of core mass, but perhaps at a different level depending on the starting core mass M_c^{TP}. This is consistent with the finding by Boothroyd and Sackmann (1988*c*, *d*) that the flash strength appears to grow linearly with flash number, and does not seem to level off to an asymptotic value.

2.2. DREDGE-UP, CARBON STARS, AND THE S-PROCESS

2.2.1. *New Opacities and the Mixing Length.* It is only recently that molecular opacities have been included in opacity tables, increasing the opacities greatly at low temperatures. These new molecular opacities greatly decrease the stellar effective temperature T_e (increasing the radius), when other stellar parameters are held fixed (Sackmann and Boothroyd 1990). In order to match the observed T_e of the Sun, one must increase considerably the value used for the mixing length parameter α (the ratio of the mixing length l to the pressure scale height H_p). For example, Guenther *et al.* (1989) required $\alpha = 1.26$ to match the Sun, using Los Alamos Opacity Library (LAOL) opacities in the interior but Cox and Stewart

(1970) opacities without molecules near the surface. On the other hand, Sackmann *et al.* (1990) required $\alpha = 2.1$ to match the Sun, using LAOL opacities from Keady (1985) which included some molecular opacity near the surface. This same value of $\alpha = 2.1$ was also found to give the correct T_e for the base of the red giant branch. Note that different authors use both different mixing length algorithms and different molecular opacity tables: for example, VandenBerg (1983) needed to increase his value of α from 1.0 to 1.6 when he included Alexander (1975, 1981) molecular opacities, in order to fit isochrones to globular clusters having a wide range of metallicites. In general, the old opacities (without molecules) required mixing length parameters in the range $1 \lesssim \alpha \lesssim 1.5$; but larger values are required when molecular opacities are included, in the range $1.5 \lesssim \alpha \lesssim 2.3$. It is important that each author *normalize* his mixing length to his particular opacities and mixing length algorithm, preferably by matching the observed T_e (and L) of the Sun.

As shown in Sackmann *et al.* (1990), the new molecular opacities with the new mixing length result in the *same* depth of solar convection as do the old opacities with the old mixing length. However, this is *not* the case on the AGB: the new opacities and mixing length result in a convective envelope that reaches *deeper* in temperature T_{CE}. For example, Sackmann and Boothroyd (1990) show that, for a Pop I star of mass 1.2 $M_\odot$, core mass $M_c = 0.6844\ M_\odot$, and luminosity $\log L = 4.0715$, the Guenther *et al.* (1989) value of $\alpha = 1.26$ with the Cox-Stewart (1970) opacities results in $\log T_{\mathrm{CE}} \approx 5.3$, while the value of $\alpha = 2.1$ with the LAOL opacities (including molecules) results in $\log T_{\mathrm{CE}} \approx 5.7$, a factor of 2.5 hotter. It is also clear that use of an inappropriate value of α will give a large error in the depth of convection on the AGB, resulting in erroneous conclusions as to dredge-up.

2.2.2. *Carbon Production and Dredge-Up.* Most of the carbon in the universe is created not in the Big Bang, nor in supernovae, but in AGB stars. The two key reactions are the triple-α reaction $^4\mathrm{He}(2\alpha,\gamma)^{12}\mathrm{C}$ which produces carbon and the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction which destroys it. The rate of the first reaction is fairly well established, but the rate of the second reaction is uncertain by a factor of about two (Caughlan and Fowler 1988). However, during helium shell flashes on the AGB, the triple-α reaction dominates, producing primarily carbon with little oxygen; uncertainties in the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction have little effect. The flash drives a convective tongue outwards from the center of the helium-burning shell, reaching almost to the position of the hydrogen-burning shell. This results in a *carbon pocket* containing (for low mass stars) about 25% carbon, about 2% percent oxygen, and the rest mainly helium (see, e.g., Iben 1976, 1982; Lattanzio 1987; Boothroyd and Sackmann 1988*c*). It is this pocket which is the source of carbon for carbon stars on the AGB. All AGB stars have deep convective envelopes. Due to the energy deposited by the helium shell flash, the regions outward from the helium shell expand; the hydrogen shell cools and hydrogen burning is extinguished. The expansion causes the convective envelope to reach deeper in mass; in some cases it reaches the carbon pocket, *dredging up* carbon to the surface. Early theoretical models found such dredge-up for higher-mass stars but not for lower-mass stars.

It is only relatively recently that dredge-up has been found in theoretical models of low mass stars. Sackmann (1978, 1980), using $\alpha = 1.0$, was the first to find dredge-up in a low mass star (of Pop I composition, $Z = 0.03$): it had a high core mass ($M_c = 0.8\ M_\odot$) and luminosity ($M_{\mathrm{bol}} \approx -6$), and a tiny envelope mass ($M_{\mathrm{env}} = 0.015\ M_\odot$: starting to move off the AGB). It was also the first time that a single dredge-up episode produced a carbon star. A very strong helium shell flash resulted in expansion pushing the carbon pocket out to very low temperatures: it was pointed out here for the first time that carbon opacities at such low temperatures could contribute significantly to the total opacity and therefore possibly drive convection even further.

Wood (1981), using $\alpha = 1.0$, carried out extensive flash computations for low mass models. He found carbon dredge-up in a $2\,M_\odot$ star of Pop II ($Z = 0.001$) with a core mass of $M_c = 0.674\,M_\odot$; however, his corresponding Pop I models did not produce dredge-up. In other words, Wood (1981) was the first to demonstrate that a low metallicity favored dredge-up. By comparing static envelope calculations to his full evolutionary computations, Wood (1981) also demonstrated that increased envelope mass and increased mixing length parameter α favored dredge-up.

Iben and Renzini (1982*a*, *b*) found carbon dredge-up in a $0.7\,M_\odot$ star of $Z = 0.001$ with a core mass $M_c = 0.624\,M_\odot$ and a luminosity $M_{\rm bol} \sim -4.8$. Hollowell (1987) and Hollowell and Iben (1988) also found dredge-up when they recomputed this model with improved opacities and $\alpha = 1.5$, using a more sophisticated time-dependent treatment of convection which included overshoot beyond the formal boundary of convection. Iben (1983), using $\alpha = 1.5$ and the *same* model and opacities as Iben and Renzini (1982*a*, *b*) but modifying the envelope mass and composition, found carbon dredge-up in a Pop I model ($Z = 0.02$) of low mass ($1.0\,M_\odot$).

Lattanzio (1987), using $\alpha = 1.5$, found dredge-up and carbon star production in Pop II stars of mass $1.5\,M_\odot$ ($Z = 0.003$ and $Z = 0.006$), with luminosities in the range where carbon stars are observed: his core masses were as low as $M_c = 0.62\,M_\odot$ and luminosities as low as $M_{\rm bol} = -4.4$ (in the post-flash luminosity dip). Lattanzio (1990) also found dredge-up in a $1.0\,M_\odot$ star of $Z = 0.001$, and also obtained dredge-up in a low-mass Pop I model ($1.5\,M_\odot$, $Z = 0.02$).

All the above models neglected mass loss, although observations show large AGB mass loss; note that mass loss tends to *oppose* dredge-up. Boothroyd and Sackmann (1988*c*, *d*) were the first to find dredge-up in low-mass stellar models which took mass loss into account. They found that dredge-up was much more difficult than the above models without mass loss would indicate. Nevertheless, dredge-up was found in a model of $1.72\,M_\odot$ (*initial* mass $2.0\,M_\odot$) with $Z = 0.001$ and $\alpha = 1.5$, with $M_{\rm bol} = -4.68$ in the post-flash luminosity dip. Dredge-up was also found for a star of $0.81\,M_\odot$ (*initial* mass $1.2\,M_\odot$) with $Z = 0.001$ and $\alpha = 3.0$, with $M_{\rm bol} = -3.59$ in the post-flash luminosity dip. With a mixing length of $\alpha = 1.0$, the classical value, no dredge-up was found for *any* model. It should be noted that a value of $\alpha = 2.1$ is appropriate for these models.

2.2.3. *Semiconvection and the s-Process Mystery.* Iben and Renzini (1982*a*, *b*) were the first to find semiconvection at the tip of the carbon pocket, due to the increased opacity (from the carbon). This semiconvection is of minor importance to dredge-up, but of major importance to *s*-process nucleosynthesis. Shell flashes in low mass stars do not reach temperatures high enough to strongly activate the $^{22}\mathrm{Ne}(\alpha,n)^{25}\mathrm{Mg}$ neutron source (requiring $T \gtrsim 3 \times 10^8\ K$; note however that this nuclear rate is highly uncertain: see Arnould 1990). The alternate neutron source, the $^{13}\mathrm{C}(\alpha,n)^{16}\mathrm{O}$ reaction, operates at lower temperatures (requiring only $T \gtrsim 1.5 \times 10^8\ K$), but requires a source of $^{13}\mathrm{C}$ (see Hollowell and Iben 1989). When CNO burning goes to completion, most carbon is converted to nitrogen, and very little $^{13}\mathrm{C}$ is left. However, semiconvection can mix small amounts of hydrogen into the top of the carbon pocket; when hydrogen re-ignites later, this small amount of hydrogen burns on $^{12}\mathrm{C}$, resulting in $^{13}\mathrm{C}$. There is thus a $^{13}\mathrm{C}$-enriched pocket just below the hydrogen burning shell, which is engulfed and mixed down by the intershell convective tongue in the next helium shell flash, where it burns via $^{13}C(\alpha,n)^{16}O$ (generating the neutrons for the *s*-process). When Hollowell (1987) and Hollowell and Iben (1988) repeated this work with improved carbon opacities, they continued to find this semiconvection (and resulting *s*-process). Their time-dependent convective formalism, with overshoot beyond the formal boundary of convection, resulted in dredge-up of *s*-processed material (although another convective formalism, *completely* without overshoot in that it did not even test whether mixing a layer next to a convective region would make it convective, did *not* result in dredge-up). However, Iben (1983) found that this semiconvection and ^{13}C-production occurred only in his $0.7\,M_\odot$, $Z = 0.001$ model and not in his $1.0\,M_\odot$ or $Z = 0.02$ models: in other words, this mechanism may be confined to a relatively narrow range in mass and metallicity. Of course, due to AGB mass loss, most low and intermediate mass stars will pass through this mass range, near the end of their AGB evolution. However, unless the initial mass was fairly small, by the time the envelope mass is reduced to such a small mass, the star will be an OH/IR star or infrared carbon star in the final stages of mass loss, coccooned by its circumstellar dust shell. The *s*-process enhancements are observed in optically visible AGB stars below the tip of the AGB, *before* they enter an extreme mass loss phase. This provides a new "*s-process mystery*": *s*-process enhancements are observed in stars with core masses too low to have the ^{22}Ne neutron source but with envelope masses too large to encounter the semiconvective ^{13}C production via the Iben and Renzini (1982*b*) mechanism.

Note that Lattanzio (1987, 1990) did *not* find semiconvection in any of his 1.0 and $1.5\,M_\odot$ models. On the other hand, Boothroyd and Sackmann (1988*d*) found semiconvection in *all* of their Pop II models of $M_i \leq 2\,M_\odot$, but the amount of hydrogen mixed down was negligible: at most, two orders of magnitude less than that found by Hollowell and Iben (1988), and usually none at all. Thus the extent of this carbon-pocket semiconvection is still very uncertain. The observed enrichment of *s*-process elements in low mass stars remains a puzzle, to be solved (like the carbon star mystery) by improved stellar models (perhaps by new reaction rates for neutron-producing reactions).

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