

Supernova Remnants

S.A.E.G.Falle,
Department of Applied Mathematical Studies,
The University,
Leeds LS2 9JT, U.K.

Abstract.

In this review I will concentrate on older remnants, by which I mean those in which radiative cooling is important somewhere and the swept up mass is sufficiently large for the details of the initial explosion not to matter. For such remnants it is the optical emission which is crucial since it allows us to deduce a great deal about the physical state of the emitting gas provided we are careful about how we interpret it. Without discussing any particular remnant in detail, I will consider how large and small scale density variations in the ambient medium affect the appearance and energetics of such remnants.

Introduction.

For spherical remnants expanding in a uniform environment it is possible to divide the evolution into three phases, free expansion, Sedov-Taylor and radiative (Woltjer 1972). Such a simple picture ignores the fact that the interstellar medium is inhomogeneous on many scales, some of which correspond to the sizes of supernova remnants and that Type II supernovae can significantly modify the interstellar medium in their neighbourhood. It is nevertheless a useful way of classifying remnants even though there are many that do not fall clearly into any one category.

For the purposes of this article, I will call a remnant old if radiative cooling is important somewhere, but will not insist that a significant part of

the explosion energy has been radiated away. Observationally this means that a remnant is old if it has a filamentary structure whose emission is characteristic of radiative shocks. Examples are the Cygnus Loop, Shan 147 and Vela. As far as theory is concerned we have to look at the effects of radiative cooling on the dynamics and appearance of the remnant.

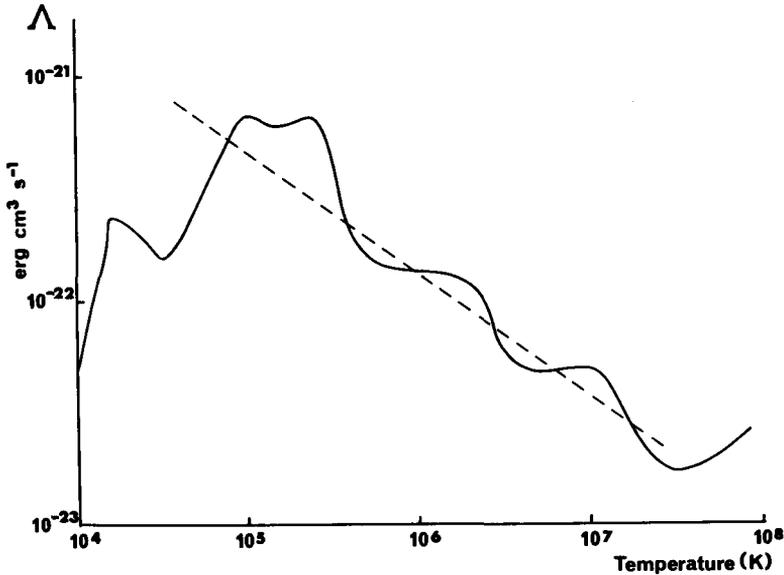


Figure 1. The radiative cooling rate per unit volume for an optically thin plasma. The straight line is the approximation (1) (Kahn 1976).

The radiative cooling rate for an optically thin gas in the relevant temperature range is shown in figure 1. Although this is not the most recent calculation, it has a maximum at about 10^5 K, which is what is important as far as the dynamics is concerned. Note that it does not include the effect of dust cooling which might well dominate above 10^5 K, depending upon the dust to gas ratio (Dwek 1987).

One of the nice things about this cooling law is that in the range 5×10^4 K $<$ T $<$ 5×10^7 K it is very well approximated by $T^{-1/2}$ power law,

$$\Lambda = A \rho^2 (p/\rho)^{-1/2} \quad (A = 3.9 \times 10^{32} \text{ c.g.s.}) \quad (1)$$

Kahn (1976) showed that this assumption makes it possible to calculate the effect of radiative cooling on the overall energetics independently of the details of the dynamics. This can not only be applied to spherical remnants, but also to those in a plane stratified medium (Falle, Garlick and Pidsley

1984). Unfortunately this cooling curve also has some nasty features which, as we shall see later, makes the transition to the radiative phase horribly complicated.

Spherical Remnant.

Suppose that a supernova explosion has energy E_0 , ejects mass M_e and occurs in a uniform medium with density ρ_0 . Then there will be a Sedov-Taylor phase provided

$$\left(\frac{E_0}{10^{51}} \right)^{-0.74} \left(\frac{M_e}{M_\odot} \right)^{5/6} \left(\frac{\rho_0}{10^{-24}} \right)^{0.2} < 4. \tag{2}$$

This is simply the condition that the remnant enters the Sedov phase before radiative cooling becomes important. It is based on Gull's (1973) calculations, which show that it looks like a Sedov solution once it has swept up about $50 M_e$, combined with Cox's (1972) estimate of when radiative cooling becomes important.

One would prefer a Sedov phase to exist, because then all the details of the original explosion can be ignored and only the energy E_0 matters. Condition (2) is satisfied for all plausible values of E_0 , M_e and ρ_0 , but unfortunately it ignores the fact that a Type II supernova can modify its surroundings, either because of its ionizing radiation (Shull, Dyson, Kahn & West 1985), or its stellar wind (Charles, Kahn & McKee 1985). There is a good deal of observational evidence that this occurs (Braun 1987).

I am going to ignore these complications and assume that the original state of the ambient medium is more important than the details of the explosion. Then in a uniform medium radiative cooling becomes important when the post shock temperature is

$$T = T_{sg} = 1.2 \times 10^6 \left(\frac{E_0}{10^{51}} \right)^{0.1} \left(\frac{\rho_0}{10^{-24}} \right)^{0.2} \text{ K} \tag{3}$$

Cox (1972).

If we now ignore magnetic fields and assume that all shocks are strong, then as long as $T \geq 5 \times 10^4$ K everywhere, things only depend on E_0 , ρ_0 and A . From these we can form a characteristic mass, length and time given by

$$\begin{aligned}
m_c &= \frac{(2.02E_0)^{6/7}}{\rho_0^{2/7} A^{3/7}} = 7.3 \times 10^{36} \left(\frac{E_0}{10^{51}} \right)^{6/7} \left(\frac{\rho_0}{10^{-24}} \right)^{-2/7} \text{ gm,} \\
l_c &= \frac{(2.02E_0)^{2/7}}{\rho_0^{3/7} A^{1/7}} = 1.9 \times 10^{20} \left(\frac{E_0}{10^{51}} \right)^{2/7} \left(\frac{\rho_0}{10^{-24}} \right)^{-3/7} \text{ cm,} \\
t_c &= \frac{(2.02E_0)^{3/14}}{\rho_0^{4/7} A^{5/14}} = 1.2 \times 10^{13} \left(\frac{E_0}{10^{51}} \right)^{3/14} \left(\frac{\rho_0}{10^{-24}} \right)^{-4/7} \text{ s.}
\end{aligned} \tag{4}$$

We expect radiative cooling to become important when the swept up mass is about m_c and the radius and age will then be approximately l_c and t_c respectively. Notice that these numbers are about right for the Cygnus Loop and IC443.

Radiative Instabilities.

We can write the cooling rate shown in figure 1 in the form

$$\Lambda = A\rho^2\Phi(p/\rho c_*^2), \tag{5}$$

where

$$c_*^2 = \frac{kT_*}{\mu m_h}, \tag{6}$$

and T_* is a reference temperature. T_* can be chosen to be the temperature at the maximum of Φ ($T_* = 10^5$ K). If we then set $\Phi(T_*) = 1$, we get $A = 2 \times 10^{26}$ c.g.s.

For a spherical remnant the flow is now governed by the parameters E_0 , ρ_0 , A and c_* and from these we can form a dimensionless parameter

$$\alpha = \frac{T_{sg}}{T_*} = 10 \left(\frac{E_0}{10^{51}} \right)^{0.11} \left(\frac{\rho_0}{10^{-24}} \right)^{0.22}. \tag{7}$$

Here T_{sg} is the temperature defined by equation (3). α only affects the evolution of the remnant if there is radiatively cooling gas at temperatures below T_* .

Let us now look at the stability of gas whose cooling rate is given by (5). Suppose that $\Phi(T) \propto T^s$. Then if cooling occurs at constant pressure, the

cooling time increases with increasing temperature if $s < 2$, while for constant density this is true for $s < 1$. This suggests that there is instability if $s < 2$ for constant pressure cooling and $s < 1$ for constant density.

Now the pressure will remain roughly constant if the cooling time $t_{\text{cool}} \gg t_{\text{dyn}}$ where t_{dyn} is some dynamical timescale. Conversely the density will remain constant if $t_{\text{cool}} \ll t_{\text{dyn}}$. Suppose that a region of initial size ℓ begins to cool and that the resulting compression is one dimensional. Then

$$\ell(t) \propto \frac{1}{\rho(t)} .$$

The relevant dynamical time is obviously the sound crossing time, so

$$t_{\text{dyn}} = \frac{\ell}{c} \propto \frac{1}{\rho T^{1/2}} .$$

On the other hand we have for the cooling time

$$t_{\text{cool}} \propto \frac{p}{\rho^2 T^s} \propto \frac{1}{\rho T^{s-1}} .$$

Hence

$$\frac{t_{\text{dyn}}}{t_{\text{cool}}} \propto T^{s-3/2} , \tag{8}$$

and so increases as the gas cools if $s < 3/2$. If t_{dyn} ever becomes much smaller than t_{cool} , then we expect a large pressure imbalance to occur which will lead to the formation of shocks. A necessary condition for this is $s < 3/2$. For the interstellar cooling law this condition is satisfied for $T > T_*$ and so we expect this kind of instability for spherical remnants if $\alpha > 1$. From (7) we can see that this should happen for almost all such remnants.

Various authors have looked at radiative instabilities. Both Avedisova (1974) and McCray, Stein & Kafatos (1975) carried out a linearised stability analysis with the post shock pressure held fixed. They found that density fluctuations grow if $s < 3$ for perturbations with wavelength much greater than the cooling length, while $s < 2$ is required if the wavelength is much shorter than the cooling length. However, these are isobaric instabilities and do not lead to the formation of additional shocks.

In my numerical calculations of thin shell formation in spherical remnants (Falle 1975, 1981), I found that cooling led to the formation of multiple shocks which caused large variations in the speed of the primary shock. Langer, Chanmugam and Shaviv (1981,1982) found a similar effect in their calculations of radiative accretion onto white dwarfs.

These results have stimulated a lot of interest in such instabilities. Chevalier & Imamura (1982) used a linearised stability analysis to show that the shock speed will not be constant if $s < 0.8$, even if it is driven by a constant speed piston. To some extent this is confirmed by numerical calculations (Imamura, Wolff & Durisen 1984). Recently Bertschinger (1986) has extended this analysis to two dimensions and shown that in that case instability occurs if $s < 1$.

It has become common practice to deduce the velocity of radiative shocks by comparing the observed optical and UV line ratios with those predicted by steady shock models with various shock speeds (e.g. Raymond et.al. 1980). Unfortunately the above considerations suggest that radiative shocks will not be steady if the shock speed is high enough for cooling to occur in the unstable region of the cooling curve.

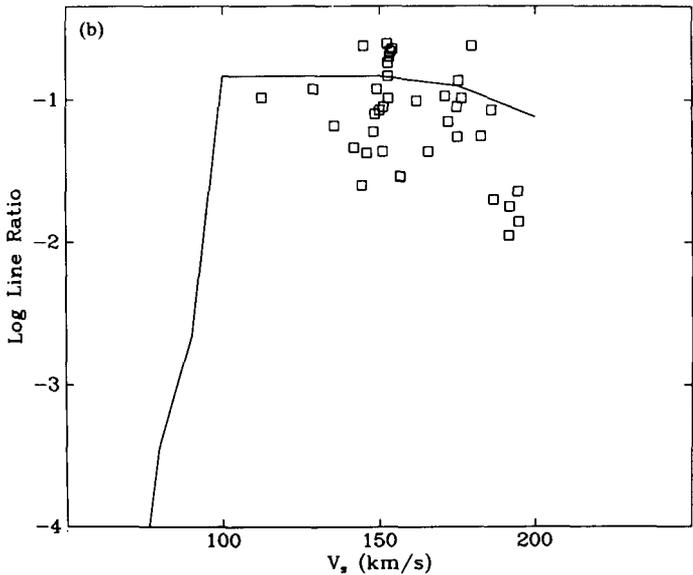


Figure 2. Instantaneous $[OIII]5008/[OII]3728$ line ratio for an unsteady shock whose mean speed is 175 km s^{-1} . The solid line is the ratio for a steady shock (Innes, Giddings & Falle 1987).

Recently Innes, Giddings & Falle 1987 have shown that, if the detailed atomic physics is included, then radiative shocks will be unsteady if their speed is greater than 130 km s^{-1} . The line ratios then do not correlate with the primary shock speed, nor even with the mean fluid speed, but vary dramatically on the cooling timescale. This effect can be seen in figure 2

which shows the instantaneous [OIII]5008/[OII]3728 line ratio plotted against the instantaneous primary shock speed for a shock driven by a constant speed piston such that the mean shock speed is 175 km s^{-1} . The variations in shock speed were induced by a single sinusoidal density perturbation upstream. The perturbation had an amplitude of 50% of the upstream density and a wavelength 1.4 times the thickness of the cooling region.

Small Scale Inhomogeneities.

The appearance of remnants like the Cygnus Loop suggests that the blast wave is interacting with irregularities with quite small scales. Indeed McKee & Cowie (1975) have argued that in the Cygnus Loop we only see optical filaments when shocks propagate into small clouds.

The interaction of a plane shock with density inhomogeneities has been looked at by many authors (e.g. Sgro 1975; Chevalier & Theys 1975; Woodward 1976; Nittmann, Falle & Gaskell 1982; Hamilton 1985; Heathcote & Brand 1983). Although we have a rough idea of what happens, at least in the adiabatic case, there are a number of important details which are not clear.

Rather than looking at deformable clouds, I will consider what happens when a shock hits a rigid object. Of course supernova remnants do not encounter rigid objects, at least not of significant size, but from pressure balance we have

$$V_c = V_e \left(\frac{\rho_e}{\rho_c} \right)^{1/2}, \tag{9}$$

where V is a shock velocity and the subscripts e and c refer to the exterior and cloud respectively. So if the cloud is much denser than its surroundings, it deforms slowly compared to the timescale of the exterior flow. The cloud therefore behaves like a rigid body, at least as far as the transient stage of the exterior flow is concerned.

Because of this it makes sense to treat clouds which are much denser than the ambient medium as rigid bodies. This is more efficient from a computational point of view since it avoids the large disparity in timescales between the interior and exterior flows. Another advantage is that there is a wealth of experimental data upon which we can draw.

An obvious case to look at is that of a plane shock hitting a rigid sphere. Figures 3 and 4 show the results obtained with a second order Eulerian

Godunov scheme on a high resolution spherical polar grid. The advantages of such a grid are that not only can the sphere be represented exactly, but also that the highest resolution is near the surface of the sphere where most of the action takes place. There is, however, some degradation of the solution due to the non-uniformity of the grid.

Figure 3 shows the sequence of events. Initially there is a regular reflection at the surface which evolves into a Mach reflection at $\theta \approx 135^\circ$ (here $\theta = 0$ corresponds to the direction of motion of the incident shock). The reflected shock associated with this eventually becomes a stationary bow shock in this case since the flow behind the incident shock is supersonic in the frame of the obstacle.

Further round the sphere the incident shock is diffracted into a funnel shape before reflecting off the symmetry axis. Because of the axial symmetry this is a Mach reflection from the very beginning. The Mach disc which moves down the axis initially has zero size, but it grows enough to become significant before the triple point associated with it disappears and it merges with the incident shock.

The solution at the latest time is shown in greater detail in figure 4. This should be compared with the experimental results obtained by Bryson & Gross (1961) for shock interaction with a rigid sphere in air. Most of the features of the experiment are well reproduced by the simulation, the exception being the vortex formed when the slip line produced by the first Mach reflection rolls up. It would, however, be too much to expect to get this sort of thing right at this resolution.

We must now consider how much of this is relevant to supernova remnants. Clearly the flow is essentially the same as the one that arises when a non-radiative shock encounters a very dense spherical cloud and so our results can be used to interpret observations of such shocks (Raymond, Davis, Gull & Parker 1980). The first thing to note is that large portions of both the reflected and diffracted shocks are oblique. This means that the usual practice of interpreting the observations in terms of normal shocks can be very misleading. Fortunately, for non-radiative shocks it is possible to calculate the flow as we have done here and then use the appropriate physics to deduce the spectrum and velocity dispersion. In this way we ought to be able to get a good idea of the nature of some of the filaments in, for example, the Cygnus Loop.

If the shocks are radiative, then life becomes much more difficult. Raga & Böhm (1987) have computed the flow past a hemisphere using a McCormack

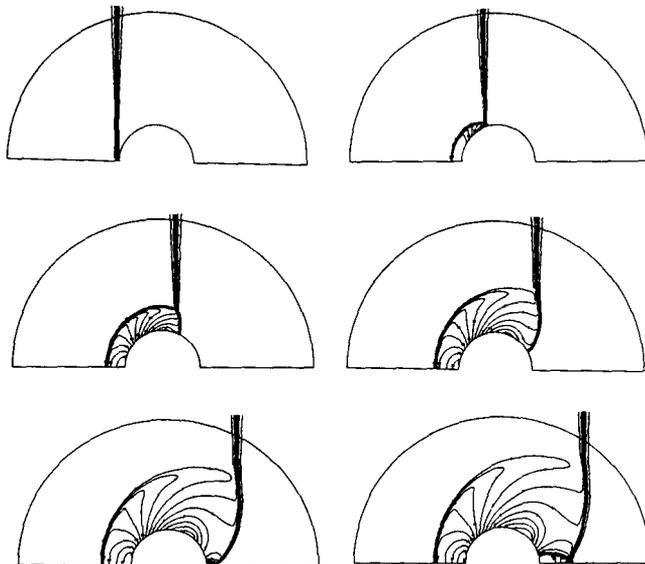
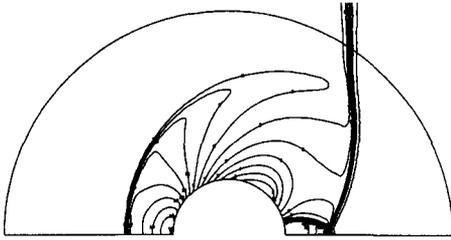


Figure 3. Pressure contours for a plane shock with an initial Mach number of 2.81 interacting with a rigid sphere ($\gamma = 5/3$). Calculated on spherical polar grid with 100 cells in the radial direction and 180 in polar angle.

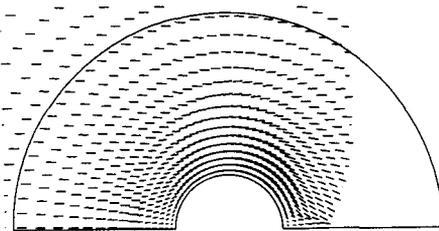
scheme. Since they had a priori knowledge of where the radiative shock would be, they were able to resolve the cooling region by an ingenious choice of grid so that their results are almost certainly reliable. Clearly this kind of thing is much harder to do for unsteady problems and for the moment it is probably better to try and guess how cooling modifies the flow. We have little chance of doing this if the cooling length is of the same order as the size of the obstacle or if the shock speeds are in the range in which the radiative instability is important, but we can do something if the cooling length is very short and the shock is stable. The shocks are then isothermal and to some extent one can get an idea of the trends by looking at lower values of the ratio of specific heats γ . Reducing γ increases the shock compression so that the bow shock is closer to the surface of the sphere. Its shape is therefore similar to that of the obstacle and, since this is true generally, we should be able to deduce something about the shape of a dense cloud if we can identify the bow shock. Another consequence of the greater compression is that the transition to Mach reflection is less likely. For a sphere a Mach shock will always occur both on the surface and on the axis downstream, but as the



Pressure Contours



Density Contours



Velocity Vectors

Figure 4. Details of the flow at the latest time calculated.

shock compression tends to infinity the point at which Mach reflection starts approaches $\theta = 90^\circ$ and the size of the Mach disc on axis tends to zero.

Conclusions.

I have discussed some of the effects that radiative cooling and density inhomogeneities can have on the evolution of supernova remnants. I have indicated that radiative instabilities must exist in radiative remnants and that these make it very difficult to interpret the spectra of radiative shocks. They may also be responsible for at least some of the complex structure seen in old remnants.

Interactions with small scale inhomogeneities are more difficult to deal with, but we can use laboratory experiments, numerical simulations and perhaps Whitham's area rule (Whitham 1974) to deduce how clouds of various sizes and densities affect the appearance of remnants. We clearly need a reliable quantitative theory of such interactions in order to make the proper use of the detailed observations that are now possible.

References.

- Avedisova, V.S. 1974. *Sov. Astron.*, 18, 283.
- Bertschinger, E. 1986. *Astrophys. J.*, 304, 154.
- Braun, R. 1987. IAU Colloquium 101, Penticton, Canada, p.363.
- Bryson, A.E. & Gross, R.W.F. 1961. *J. Fluid Mech.*, 10, 1
- Charles, P.A, Kahn, S.M. & McKee, C.F. 1985. *Astrophys. J.*, 295, 456.
- Chevalier, R.A. & Imamura, J.N. 1982. *Astrophys. J.*, 261, 543.
- Chevalier, R.A. & Theys, J.C. 1975. *Astrophys. J.*, 195, 53.
- Cox, D.P. 1972. *Astrophys. J.*, 178, 159.
- Dwek, E. 1987. IAU Colloquium 101, Penticton, Canada, p.363.
- Falle, S.A.E.G. 1975. *Mon. Not. R. astr. Soc.*, 172, 55.
- " 1981. *Mon. Not. R. astr. Soc.*, 195, 1011.
- Falle, S.A.E.G., Garlick, A.R. & Pidsley, P.H. 1984. *Mon. Not. R. astr. Soc.*, 208, 925.
- Gull, S.F. 1973. *Mon. Not. R. astr. Soc.*, 161, 47.
- Hamilton, A.J.S. 1985. *Astrophys. J.*, 291, 523.
- Heathcote, S.R. & Brand, P.W.J.L. 1983. *Mon. Not. R. astr. Soc.*, 203, 67.
- Imamura, J.N., Wolff, M.T. & Durisen, R.H. 1984. *Astrophys. J.*, 276, 667.
- Innes, D.E., Giddings, J.R. & Falle, S.A.E.G. 1987. *Mon. Not. R. astr. Soc.*, 226, 67.
- Kahn, F.D. 1976. *Astr. & Astrophys.*, 50, 145.
- Langer, S.H., Chanmugam, G. & Shaviv, G. 1981. *Astrophys. J.*, 245, L23.
- " 1982. *Astrophys. J.*, 258, 289.
- McCray, R., Stein, R.F. & Kafatos, M. 1975. *Astrophys. J.*, 196, 565.
- McKee, C.F. & Cowie, L.L. 1975. *Astrophys. J.*, 195, 715
- Nittmann, J., Falle, S.A.E.G. & Gaskell, P.H. 1982. *Mon. Not. R. astr. Soc.*, 201, 833.
- Raga, A.C. & Böhm, K.H. 1987. *Astrophys. J.*, 323, 193.

- Raymond, J.C., Black, J.H., Dupree, A.K. & Hartmann, L. 1980. *Astrophys. J.*, 238, 881.
- Raymond, J.C., Davis, M., Gull, T.R. & Parker, R.A.R. 1980. *Astrophys. J.*, 238, L21.
- Sgro, A.G. 1975. *Astrophys. J.*, 197, 621.
- Shull, P., Dyson, J.E., Kahn, F.D. & West, K.A. 1985. *Mon. Not. R. astr. Soc.*, 212, 799.
- Whitham, G.B. 1974. *Linear and Nonlinear Waves*, Wiley Interscience, chapter 8.
- Woltjer, L. 1972. *Ann. Rev. Astron. Astrophys.*, 10, 129.
- Woodward, P.R. 1976. *Astrophys. J.*, 207, 484

Discussion:

ZINNECKER: Given these impressive numerical calculations, can you tell us under which conditions gaseous clumps can be imploded after being hit by a supernova remnant shock, i.e. for which parameters of the clumps (e.g. size, density) can star formation be triggered?

FALLE: I can't tell you off the top of my head, but basically the denser the cloud the smaller it can be and still be induced to collapse. This is not only because of the dependence of the Jeans' mass on density, but because a dense cloud can more easily survive until it reaches the low velocity region in the interior of the remnant.

PALOUS: Even if the SNR is non spherical for any reason, entering the Sedov phase of evolution, it will be more spherical after, since the parts moving with a higher velocity will be more decelerated from the ambient medium than the other parts. This was pointed out in a paper by Bisnovatyg-Kogan and Biliunikov (1982).

FALLE: You are quite right. In the Sedov-Taylor phase the remnant has lost all memory of the details of the explosion, including any asymetry. One can therefore conclude that if an old remnant is not spherically symmetric then this must be due to the surrounding medium.