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Energetic Solar Electrons – Whistler Bootstrap, Magnetic Knots and Small-scale Reconnection

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Abstract. The (near) relativistic electrons, emanating from the solar corona in long-lasting, gradual events, are generally observed at 1 AU as delayed vs the less energetic, type-III beams. The observations are consistent with the delayed electrons being energized along the stretched post-CME coronal field lines, when the tail of an anisotropic seed population, which is injected in conjunction to the observed radioheliograph bursts, interacts with the self-excited whistler waves (bootstrap mechanism). These bursts indicate efficient processes where suprathermal seed electrons are injected as a result of magnetic reconnection at the marginally stable coronal configuration left behind the emerging CME. The dependence of the bootstrap mechanism on the electron injection raises the general question of the MHD description and its deviation over the small electron skin-depth scale. The similarity between MHD and knot theories allows one to characterize any turbulent magnetic configuration through topological invariants, while deviation over electron skin-depth scale, characterized by the generalized vorticity, which is enhanced due to density inhomogeneity, creates the conditions for the potential injection sites.

Keywords. relativistic electrons, solar flares, knots, reconnection.

1. Introduction

Formation of non-thermal electron populations in solar environment has been investigated experimentally through (a) remote sensing of electromagnetic waves emitted by the accelerated electrons via their local interaction with plasma or with magnetic field and (b) in situ measurements by satellites traversing various heliospheric regions during active solar periods. In this paper we address briefly two physical aspects of electron energization with potential implications beyond solar physics: (1) the injection of a suprathermal seed population through reconnection process and (2) the successive energization of the electron tail to relativistic energies. We discuss the bootstrap acceleration process, specify the MHD description in terms of knot theory invariants and show its violation over small scales, where the electrons are frozen into the generalized vorticity.

2. Bootstrap model

The near-relativistic electrons are observed at 1 AU with a solar injection timing significantly delayed with respect to the lower energy electrons, pointing out to different inception and energization mechanisms. Precise timing of the observed electron fluxes (Krucker *et al.*, 1999) showed that the injected electrons could be characterized into low-energy, injected instantaneously with the electromagnetic radiation, and more energetic with a delay of up to 30 minutes. Klassen *et al.* (2005) investigated the intense Halloween 2003 event and found long-lasting high energy relativistic electron oppulation with an onset of 25 minutes after the type III initialization. Maia et al (2007) concluded from the April 15 2001 event that the energization process operates at the very low corona and the energetic electrons, observed through illuminated loops, are formed behind the CME, in the disturbed, turbulent, marginally unstable corona. These observations lead to the bootstrap energization model described below.

At active solar times, after CME uplift, intense spectroscopic signals are observed by radio-heliographs (RH), indicating coronal injection. The energization occurs behind the CME leading edge, often at low solar altitudes, on closed field lines, without direct correlation to the CME. It has been therefore conjectured that reconnection process injects non-isotropic electrons, which destabilize whistler waves, resulting in an efficient tail energization (Roth, 2008). The energization proceeds via resonant interaction between the waves and the electrons: $k_\| v_\| = \omega - n\Omega/\gamma$. The bootstrap model requires injection of a seed population in tandem with the RH emissions, which is facilitated by unstable, turbulent magnetic configurations; their description is presented in the next sections.

3. Magnetized plasma descriptions

General description of plasma consists of a fluid or Boltzmann model. Since we are interested in magnetic structures which are supported mainly by the bulk distributions, and because various thermal effects can be incorporated a posteriori, we shall adhere here to a fluid approximation. Plasma can then be divided into electron fluid which follows changes due to electric field, Lorentz and Hall forces, electron pressure gradient and plasma resistivity (with the standard notation)

$$-(m_e/e)[du/dt] + J \times B/nec - \nabla p_e/ne + \eta j = E + u \times B/c \tag{3.1}$$

and center of mass fluid which is advanced under the effects of the (total) kinetic and magnetic pressures and magnetic tension,

$$\rho[dV/dt] = J \times B/c - \nabla p = -\nabla(p + B^2/8\pi) + (B.\nabla)B/4\pi \tag{3.2}$$

while the electromagnetic fields are related via Maxwell equation:

$$\nabla \times E = c^{-1}\partial B/\partial t. \tag{3.3}$$

The set of the above equations allows one to consider approximations at different scales and with different physical implications:

a. On a scale larger than ion skin depth the lowest order approximation for the electron fluid ignores all the terms on the lhs of Eq 3.1 and substitutes this result into the Maxwell equation. The electrons now are tied almost entirely to the ions, $u \sim V$, and the magnetic field is frozen into the plasma: $\partial_t B = \nabla \times (V \times B)$. This constitutes the evolutionary equation for the (turbulent) magnetic field in the MHD approximation where both ion fluids move together, dragging the magnetic field (Section 4).

b. On the scale smaller than the ion skin depth one chooses various approximations to the "ion" fluid in eq 3.2 and attempts to include as many effects as possible in the electron dynamics, Eq. 3.1. Over scale much below ion skin depth, in the lowest order, the ion motion is completely neglected and using vector identities Eq 3.1 becomes

$$\partial_t G = \nabla \times (u \times G); \quad G = B - (mc/e)\nabla \times u. \tag{3.4}$$

Here the electrons are dragging the G field, termed generalized vorticity, indicating that on the small scale they decouple from the magnetic field. The electron drift takes the role of a vector potential.

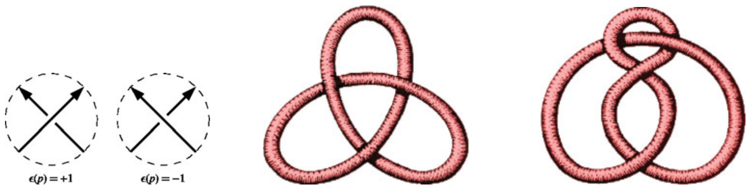

Figure 1. (a)-(b) Writhe operations; (c) Trefoil knot; (d) Figure Eigth knot.

4. MHD Knot description

MHD is an approximate description of the magnetic field evolution as it is immersed in a plasma fluid; although the field may form complicated structures, the magnetic flux through surface intersecting B lines is conserved. The plasma flow drags the magnetic field lines such that they may only stretch and bend. MHD turbulence forms then a collection of non-intersecting, entangled fields. Similarly, a mathematical knot is depicted as a closed loop of a non-self-intersecting curve, whose evolution is determined via continuous deformation in R^3, following laws of knot topology - smooth changes in the surrounding viscous fluid, allowing only stretching or bending. Hence, MHD field evolution may be viewed as a topological deformation and its dynamics forms equivalent configurations with a set of invariants; similarly, knots are distinguished by a variety of topological invariants. Such invariants are crucial in obtaining topological information about the knot or a link (collection of non-intersecting, entangled knots), and equivalently about the (turbulent) magnetic field configuration. The topological information about knots may be obtained from their diagrams - 2D projections which preserve the over/under crossing of the 3D curve. The general deformations which satisfy this equivalency were described in the three link moves R_j, j = 1,3 (Reidemeister, 1926, Hass and Lagarias, 2001). Then, to each knot or link one can attribute a set of mathematical operations and check if the result is preserved under the R_j moves. Figures 1a-b show two basic configurations with assigned value of a writhe at crossing p, $\epsilon(p)$, which takes the values of ± 1. More general knot characterization is obtained where each intersection with undercrossing arcs a, b and overcrossing arc c is assigned algebraic relation c(t) = ta + (1-t)b for variable t; summarizing over the whole knot forms a consistency matrix whose determinant results in a Polynomial P(t) (Alexander, 1928). This characteristic feature becomes an important invariant of each knot. For instance, the Trefoil and Figure Eight Knot have the Polynomial invariants $t^{-1} - 1 + t$ and $t^{-1} - 3 + t$, respectively, showing their inequivalent character (Fig 1c-d). For 60 years Alexander Polynomial was the only known invariant until Jones (1985), using skein relations discovered more powerful invariant in the form of Laurent polynomial, which is able to distinguishes between a knot and its mirror. Hence, the topological information contained in the knot invariant may be useful in description of the MHD (turbulent) field. The description of magnetic configuration through its topological invariants is valuable in characterizing the complexity of the magnetic field and in assessing possibility of conversion between the various magnetic structures in the realm of MHD. Magnetic helicity was shown to satisfy the R_j moves.

Space observations and laboratory measurements indicate that some physical processes violate the smooth knot evolution. To allow pinching and reconfiguration the knot theory invoked a procedure of the Connected Sum of Knots, which joins two knots near a chosen point on each one of them. This mathematical operation is commutative and includes the unit element (unknot) forming a semigroup (group without inverse), while in physics this process violates the frozen-in condition and requires a modified approach.

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5. Electron-MHD

At time scales much shorter than the ion gyrofrequency and on spatial scale smaller than ion skin depth, plasma dynamics is determined mainly by electrons. This one-species regime, commonly designed as EMHD, is dominated by a helicon/whistler mode, which replaces the role of Alfven wave in MHD. Ampere's law relates the electron velocity u to the magnetic field where ($\alpha = (c/4\pi e\hat{n})$ and $\hat{n}$ denotes an average density,

$$u = -[c/4\pi e n_o(x)]\nabla \times B = -(\alpha/\nu)\nabla \times B \tag{5.1}$$

which casts the evolution of the magnetic field into Eq 3.4 with an extended expression for the generalized vorticity

$$G = [1 - ({d_e}^2/\nu)\nabla^2]B + ({d_e}^2/\nu)(\nabla \times B) \times \nabla ln\nu \tag{5.2}$$

where $\nu = n_o(x)/\hat{n},\ d = c/\omega_e$ (electron skin depth). Eq (5.2) combines the effects of the current concentration on the small electron skin depth scale with density dips, which are observed in data (Mozer *et al.*, 2003; Mozer, 2005). When the generalized vorticity differs significantly from the magnetic field, the violation of the frozen-in condition becomes conducive to the formation of electron vortices; it is conjectured, based on preliminary simulations, that regions with large field and density gradients are susceptible to form sites of enhanced current modifications which lead to electron injection.

6. Summary

The bootstrap model of energization in solar gradual events requires injection of an electron seed population into the marginally stable corona left behind the emerging CME. Formation of this seed population in magnetic configuration raises a general question of turbulent magnetic field description, which is here given as a collection of knots moving smoothly in a viscous fluid, preserving their topological invariants, which characterize well a complex MHD magnetic field. It is suggested that this classification may be fruitful in comparing various magnetic configurations and, with physical input, assessing the timescales for their modifications. It is shown that on the electron skin depth scale the electron fluid, which decouples from the stationary ions and the magnetic field, is frozen in the generalized vorticity; it is conjectured that regions of largest deviations of G from B form the sites of local current enhancements and electron seed injection.

Acknowledgments

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