

Each chapter ends with exercises which test the understanding of the text and indicate extensions of the theory. A very substantial number of topics are dealt with, and the treatment of some is, perhaps necessarily, somewhat sketchy. Some readers would no doubt prefer a more intrinsic treatment of parts of the book, especially Chapter IV; moreover, the book contains a number of inaccuracies. However, the author has succeeded in giving the reader a good commentary on the subject as a whole, and a useful list of references to original sources.

T. J. WILLMORE

LANG, S., *Introduction to Differentiable Manifolds* (John Wiley & Sons, 1962), vii+126 pp., 53s.

The purpose of this book is to fill the gap which exists in the literature dealing with that branch of mathematics which borders on differential topology, differential geometry and differential equations. The essential contribution of this book is to show that nothing is lost in clarity of exposition if a manifold is defined by means of charts of Banach or Hilbert Spaces rather than finite dimensional spaces. In fact, the claim is that there is a positive gain from the indiscriminate use of local coordinates x_1, \dots, x_n and their differentials dx_1, \dots, dx_n . Moreover such a treatment is necessary when dealing with infinite-dimensional spaces, and there is every indication that the systematic introduction of infinite-dimensional topological spaces will have successful results in the theory of differentiable manifolds.

Chapter I gives a brief resumé of differential calculus, following the viewpoint of Dieudonné's *Foundations of Modern Analysis*, Chapter VIII. Chapter II describes manifolds by means of charts of Banach spaces. Chapter III describes vector-bundles, and exact sequences of bundles. Chapter IV, on "Vector fields and differential equations", collects a number of results which make use of the notion of differential equations and solutions of differential equations. In particular, there is an interesting account of "sprays". Chapter V is about differential forms, exterior differentiations and the Poincaré Lemma. Chapter VI gives a proof of a generalisation of the Frobenius Existence Theorem. Chapter VII, about riemannian metrics, shows how a riemannian metric determines a spray and hence geodesics. In this chapter use is made of the standard spectral theorem for (bounded) symmetric operators, and a proof of this theorem is given in Appendix I. Although his treatment avoids the use of local coordinates, the author recognises that in the finite-dimensional case, they constitute an effective computational tool. In Appendix II he interprets differential forms, sprays and the riemannian spray in terms of these local coordinates.

I think that few readers will find the book easy reading, even though it is largely self-contained. There is little doubt, however, that this is an important contribution to the literature, and that it will have an important influence on workers in the fields of differential topology, differential geometry and differential equations.

T. J. WILLMORE

WILLIAMSON, J. H., *Lebesgue Integration* (Holt, Rinehart and Winston, London, 1962), viii+117 pp., 26s.

Although this book was conceived by its author as an introduction to more advanced texts on measure and integration, it is not aimed, as for example is J. C. Burkill's Cambridge tract on the subject, towards readers who may have no wish to plumb the depths of the theory of real functions; it is a book on Lebesgue integration for those who have an interest in functional analysis, and in this field it is undoubtedly a good book. The treatment is in the general setting of n -dimensional Euclidean space.

Measure is first defined for bounded intervals in R^n , then for sets which are countable unions of intervals within some fixed bounded interval; inner and outer measure are defined in terms of the measure of such sets. The integral of a function $f(x)$ over a set E is defined by means of upper and lower approximating sums $S_{\mathcal{E}}, s_{\mathcal{E}}$ corresponding to a dissection \mathcal{E} of E . By allowing \mathcal{E} to contain a countable infinity of sets and by introducing the idea of an *admissible* dissection (one for which $\sum h_r m(E_r)$ is finite, where E_r is a set in \mathcal{E} and $h_r = \sup_{x \in E_r} |f(x)|$) the author avoids a separate treatment of functions which are sometimes negative or of unbounded functions or sets. The final chapter is on more general measures and includes, for example, a proof of the Radon-Nikodym theorem for Borel measures. Exercises are given at the end of each chapter.

The text gives frequent references to alternative methods of developing the subject. In general the material is presented clearly, but proofs of theorems are rather terse and the reader is often expected to bear in mind the incidence of overriding hypotheses which are not mentioned explicitly in the statements of individual theorems. Sometimes extreme terseness has meant that a passage which is unambiguous to a person familiar with the subject will occasion unnecessary and distracting mental effort in the less expert reader. It seems unfortunate that a little more space was not allowed to a text which is intended as an introduction; no doubt this would have necessitated a higher price, but the value of the book might have been enhanced out of all proportion.

The type is small but clear and the layout is pleasing, but the incorporation in the text of formulae which are carried over from one line to the next does contribute to the general impression of terseness in the proofs.

P. HEYWOOD

MACHOL, R. E. (ED.), *Information and Decision Processes* (McGraw-Hill Book Company, New York, 1960), xi+185 pp., 46s.

Papers by the ten speakers at a symposium on the title subject held at Purdue University in 1959 and two from a similar symposium held a year earlier make up this volume.

To make a very crude classification, there are papers by H. Chernoff, M. Flood, L. Weiss, W. Hoeffding and M. Sobel concerned with statistical decision problems (mostly sequential); by J. Wolfowitz and C. Shannon on coding and channel problems; by J. L. Doob and M. Rosenblatt on stochastic processes; by P. Suppes concerned with subjective probabilities; by G. W. Brown about computers in decision making; and by D. Rosenblatt on models of certain general behavioural systems. The papers vary from rather detailed consideration of a specific problem to very broad exposition.

Apart from particular results set forth, no small part of the interest this book holds lies in its display of some of the diverse areas in which the application of mathematics is being seriously studied.

R. N. BRADT

ALDER, H. L. AND ROESSLER, E. B., *Introduction to Probability and Statistics* (2nd edition) (W. H. Freeman & Co., San Francisco & London, 1962), xii+289 pp., 32s.

The first edition of this book was reviewed in this journal, June 1961. The second edition differs from the first by the addition of a brief chapter introducing the F-distribution and one on elementary analysis of variance (in which only one-way classification problems are considered). The additions are on the same clear, elementary level as the earlier parts and serve to make a much more useful text.

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